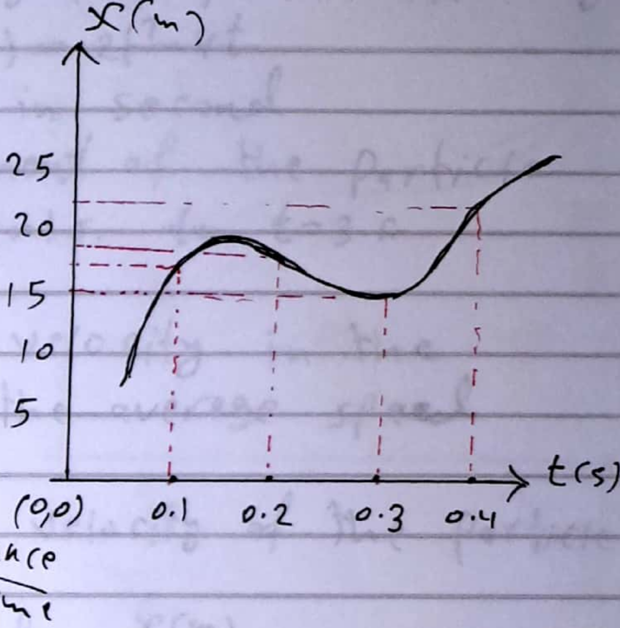
Motion in a Straight Line



Position (x) : it is the distance from the origin

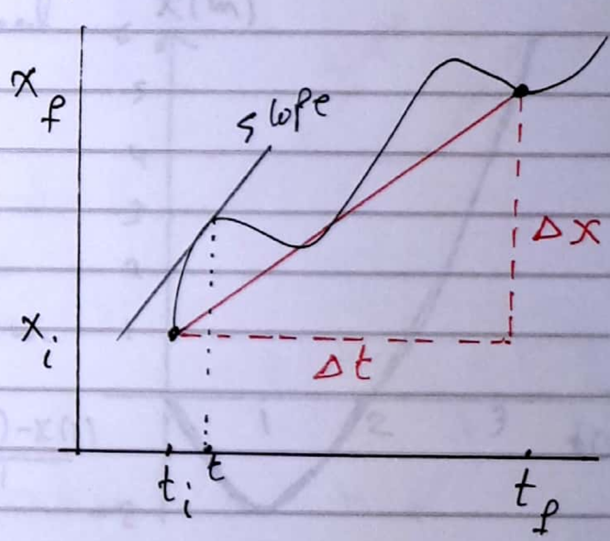
Displacement (Δx) : $\Delta x = x_f - x_i$

Average velocity ($\bar{v}$) : $\bar{v} = \frac{\Delta x}{\Delta t}$

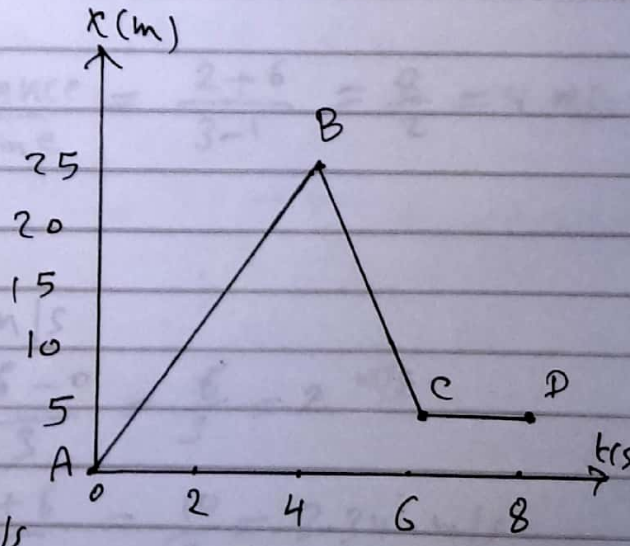
Average speed (speed) : $\text{speed} = \frac{\text{total distance}}{\text{total time}}$

Instantaneous velocity (v):

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \equiv \frac{dx}{dt}$$



Example



i) $x(4) = 25$, $x(6) = 5$

ii) on the interval $[0, 8]$:

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x(8) - x(0)}{8 - 0} = \frac{5-0}{8} = \frac{5}{8} \text{ m/s}$$

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}} = \frac{25+20+0}{8} = \frac{45}{8} = 5.6 \text{ m/s}$$

iii) $v(2) = \left.\frac{dx}{dt}\right|_{t=2} = \text{slope} = \frac{25-0}{4-0} = 6.25 \text{ m/s}$, $v(5) = \frac{5-25}{6-4} = -10 \text{ m/s}$

Example : A particle moves along the x-axis according to the expression $x(t) = 2t^2 - 4t$

where x in meter and t in second

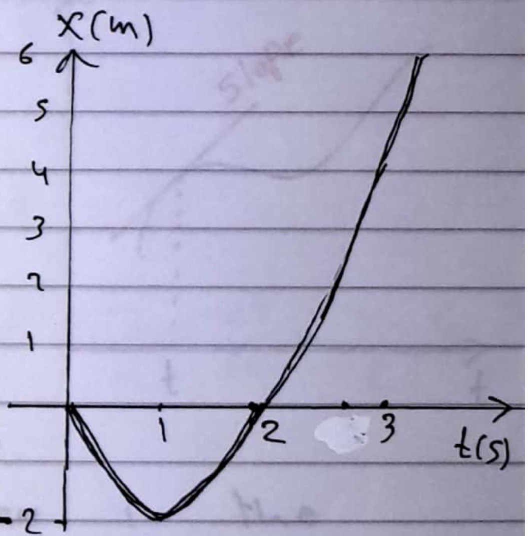
a) determine the displacement of the particle in the time interval $t = 1\,s.$ to $t = 3\,s.$

b) calculate the average velocity in the same interval and the average speed

c) Find the instantaneous velocity of the particle at $t = 2.5\,s.$

d) Find the average velocity and speed in the interval $[0,3]$

Solution



a) $x(1) = 2 - 4 = -2\ m$

$x(3) = 2(9) - 4(3) = 6\ m$

$\Delta x = x(3) - x(1) = 6 - (-2) = 8\ m$

b) average velocity : $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x(3) - x(1)}{3 - 1}$

$\bar{v} = \frac{8}{2} = 4\ m/s$

average speed $= \frac{\text{total distance}}{\text{total time}} = \frac{2 + 6}{3 - 1} = \frac{8}{2} = 4\ m/s$

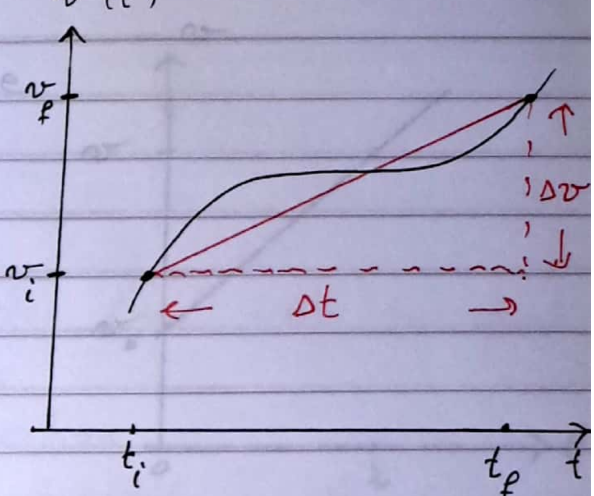
c) $v = \frac{dx}{dt} = 4t - 4$

$v(2.5) = 4(2.5) - 4 = 6\ m/s$

d) $[0,3]$: $\bar{v} = \frac{x(3) - x(0)}{3 - 0} = \frac{6 - 0}{3} = \frac{6}{3} = 2\ m/s$

average speed $= \frac{2 + 2 + 6}{3} = \frac{10}{3} = 3.34\ m/s$

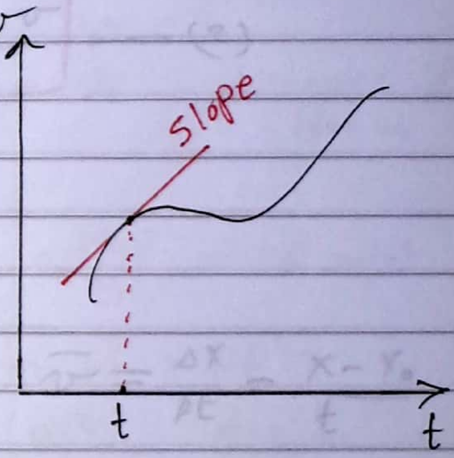
Acceleration



- average acceleration $(\bar{a})$: $\bar{a} = \frac{\Delta v}{\Delta t}$
- instantaneous acceleration: $a = \frac{dv}{dt}$

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

$$a = \frac{dv}{dt} \equiv \text{slope}$$



Example: The velocity of a particle moving along the x-axis varies in time according to the expression

$$v(t) = (40 - 5t^2) \text{ m/s}$$

a) Find the average acceleration in the interval $t=0$ to $t=2$ s.

b) what is the acceleration at $t=2$ s.

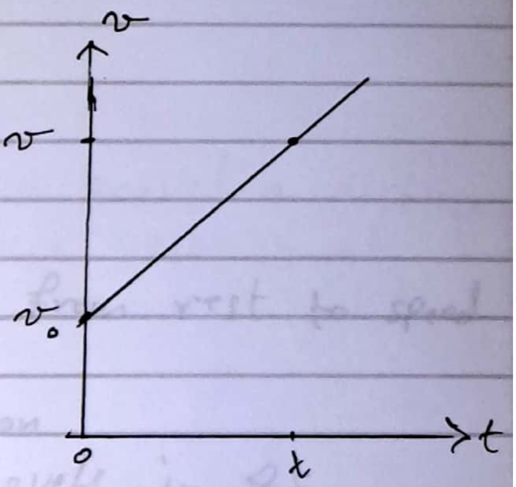
solution

a) $$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v(2) - v(0)}{2-0} = \frac{20-40}{2} = -10 \text{ m/s}^2$$

b) $$a(t) = \frac{dv}{dt} = -10t$$

$$a(2) = -20 \text{ m/s}^2$$

One Dimensional Motion with Constant Acceleration



For constant acceleration in the interval $[0, t]$

$$\bar{a} = a = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t}$$

$$a = \frac{v - v_0}{t}$$

$$\boxed{v = v_0 + at} \quad \text{—— (1)}$$

in the same interval: $\boxed{\bar{v} = \frac{v_0 + v}{2}}$ —— (2)

Substitute eq. (1) in eq. (2)

$$\bar{v} = \frac{v_0 + v_0 + at}{2} = v_0 + \frac{1}{2}at$$

but from the definition of $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t}$ we find

$$\bar{v} = \frac{x - x_0}{t} = v_0 + \frac{1}{2}at$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2$$

$$\boxed{x = x_0 + v_0 t + \frac{1}{2}at^2} \quad \text{—— (3)}$$

Again $\bar{v} = \frac{v + v_0}{2} = \frac{x - x_0}{t}$

$$x - x_0 = \frac{t}{2}(v + v_0)$$

and substitute for t from eq. (1)

$$x - x_0 = \frac{1}{2}\frac{v - v_0}{a}(v + v_0) = \frac{1}{2a}(v - v_0)(v + v_0) = \frac{v^2 - v_0^2}{2a}$$

$$\Rightarrow \boxed{v^2 = v_0^2 + 2a(x - x_0)} \quad \text{—— (4)}$$

Results: for constant acceleration

$$v = v_0 + at$$
$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$
$$v^2 = v_0^2 + 2a(x - x_0)$$

Example: An object accelerates from rest to speed of 128 m/s in 8 s.

a) determine the acceleration
b) find the distance it travels in 8 s.
c) what is the velocity after 10 s.
d) after how long time it will travels a distance of 1600 m

solution

a) $v = v_0 + at$

$128 = 0 + a(8) \Rightarrow a = \frac{128}{8} = 16 \ m/s^2$

b) distance $\equiv \Delta x = x - x_0 = v_0 t + \frac{1}{2} a t^2$

$\Delta x = 0 + \frac{1}{2}(16)(8)^2 = 512 \ m$

c) $v = v_0 + at$

$= 0 + 16(10) = 160 \ m/s$

d) $\Delta x = v_0 t + \frac{1}{2} a t^2$

$1600 = 0 + \frac{1}{2}(16) t^2$

$t^2 = \frac{1600}{8} = 200$

$t = \sqrt{200} = 14.14 \ s.$

Example: A particle moves from rest with a constant acceleration $5\ m/s^2$. Find

a) its velocity after 3 s.

b) its displacement after 3 s.

c) after how long time it will travel a distance of 100 m and what is the velocity at this time?

Solution

a) $v = v_0 + at$

$v = 0 + 5(3) = 15\ m/s$

b) $\Delta x = v_0 t + \frac{1}{2} a t^2$

$= 0 + \frac{1}{2}(5)(9) = 22.5\ m$

c) $\Delta x = v_0 t + \frac{1}{2} a t^2$

$100 = 0 + \frac{1}{2}(5) t^2$

$t = \sqrt{\frac{200}{5}} = 6.3\ s.$

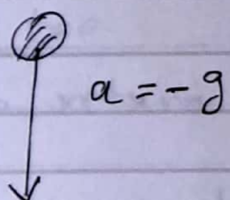
$v = v_0 + at$

$= 0 + 5(6.3) = 31.5\ m/s$

Freely Falling Bodies

In this case the object moves under the influence of gravity ($F = -mg$) with a constant acceleration of ($-g$).

Therefore the equations of motion can be obtained as



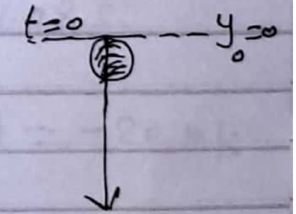
$$v = v_0 - gt$$
$$y = y_0 + v_0 t - \frac{1}{2} g t^2$$
$$v^2 = v_0^2 - 2g(y - y_0)$$

$\boxed{g = 9.8\ m/s^2}$

note: choose $y_0 = 0$ as the initial position at $t = 0$

Example: A Freely falling body starts its motion from rest. calculate its position and velocity at ~~t~~

a) $t = 1$ s. b) $t = 2$ s. c) $t = 3$ s.



solution

a) at $t = 1$ s.

$$y = v_0 t - \frac{1}{2} g t^2$$
$$= 0 - \frac{1}{2}(9.8)(1) = -4.9\ m$$
$$v = v_0 - gt = 0 - 9.8(1) = -9.8\ m/s$$

b) at $t = 2$ s.

$$y = v_0 t - \frac{1}{2} g t^2 = 0 - \frac{1}{2}(9.8)(4) = -19.6\ m$$
$$v = v_0 - gt = 0 - 9.8(2) = -19.6\ m/s$$

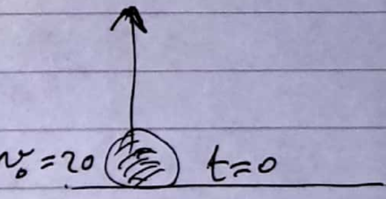
c) at $t = 3$ s.

$$y = v_0 t - \frac{1}{2} g t^2 = 0 - \frac{1}{2}(9.8)(9) = -44.1\ m$$
$$v = v_0 - gt = 0 - 9.8(3) = -29.4\ m/s$$

Example: A stone is thrown upward with initial velocity of 20 m/s. Find

a) the maximum height

b) the time needed to reach the maximum height



c) the time needed for the stone to return to the level of thrower

d) the velocity of the stone at this instant

e) the velocity and position at $t = 2.5$ s.

solution

a) $v^2 = v_0^2 - 2gy \Rightarrow 0 = (20)^2 - 2(9.8)\,y \Rightarrow y = 20.4\,m$

b) $v = v_0 - gt \Rightarrow 0 = 20 - 9.8t \Rightarrow t = 2.04$ s.

c) $y = v_0 t - \frac{1}{2} g t^2 \Rightarrow 0 = 20t - \frac{1}{2}(9.8)t^2$

$0 = (20 - 4.9t)t \Rightarrow \cancel{t = 0}, \quad t = 4.08$ s.

d) $v = v_0 - gt \Rightarrow v = 20 - 9.8(4.08) = -20$ m/s

e) $v = v_0 - gt \Rightarrow v = 20 - 9.8(2.5) = -4.5$ m/s

$y = v_0 t - \frac{1}{2} g t^2 \Rightarrow y = 20(2.5) - \frac{1}{2}(9.8)(2.5)^2 = 19.37$ m