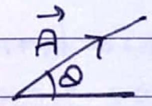
Motion in Two Dimensions

Introduction to vectors

vector: is defined as a physical quantity which has both magnitude and direction (Forces, velocity, acceleration, ...)

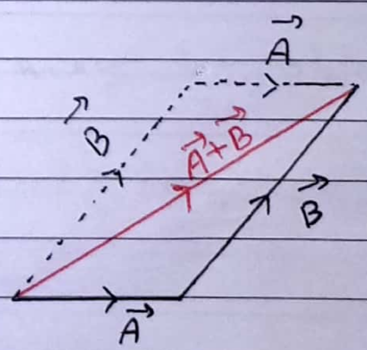
Scalar: is defined as a physical quantity which has magnitude only (mass, time, energy, ...)

* vectors are denoted by $\vec{A}$ or **A** (highlighted)
* magnitude of vector $|\vec{A}|$ or A
* A vector is pictured in a diagram by an arrow with a length is proportional to the magnitude and an angle for the direction

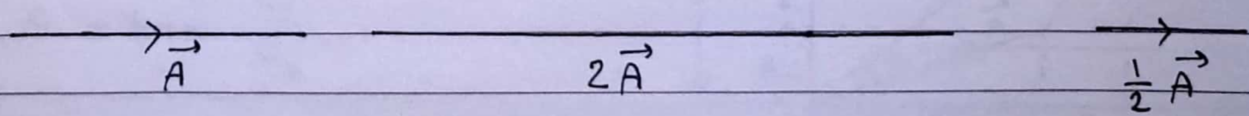


Addition of vectors (graphically)

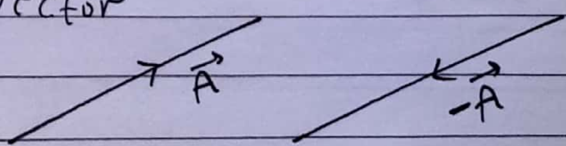
note that $\vec{A}+\vec{B}=\vec{B}+\vec{A}$



Multiplication of a vector by a scalar



Negative of a vector



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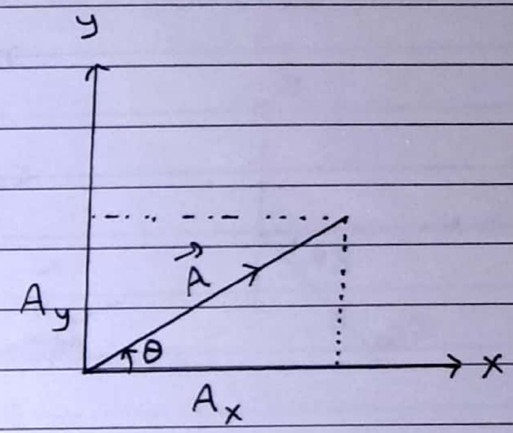
- Components of a vector



$A_x = A\cos\theta$ (x-component)

$A_y = A\sin\theta$ (y-component)

where θ is the angle measured from the positive x-axis counterclockwise

- Uni vectors $(\hat{x}, \hat{y})$

The vector $\vec{A}$ can be written interms of unit vectors as

$$\vec{A} = A_x\hat{x} + A_y\hat{y}$$

where $\hat{x}$ is a unit vector in the x-direction and $\hat{y}$ is a unit vector in the y-direction

- The sum of two vectors may be obtained as

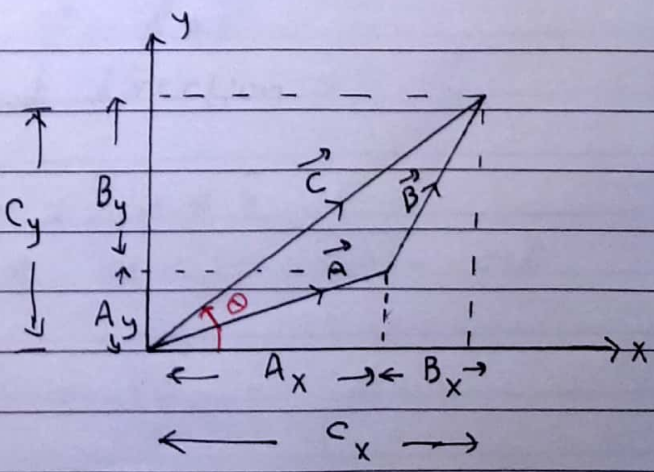
$$\vec{C} = \vec{A} + \vec{B} = (A_x\hat{x} + A_y\hat{y}) + (B_x\hat{x} + B_y\hat{y}) = C_x\hat{x} + C_y\hat{y}$$

with $C_x = A_x + B_x$

$C_y = A_y + B_y$

$$C = |\vec{C}| = \sqrt{C_x^2 + C_y^2}$$

$$\theta = \tan^{-1}\frac{C_y}{C_x}$$



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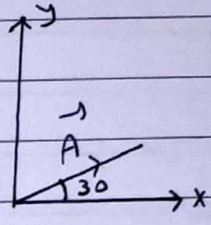
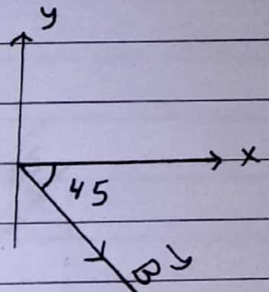
Example 2.2

a) Find the components of the vectors $\vec{A}$ and $\vec{B}$ if $|\vec{A}|=2$ and $|\vec{B}|=3$





$A_x = A\cos\theta_1 = 2\cos 30 = 1.73$

$A_y = A\sin\theta_1 = 2\sin 30 = 1$

$B_x = B\cos\theta_2 = 3\cos(-45) = 2.12$

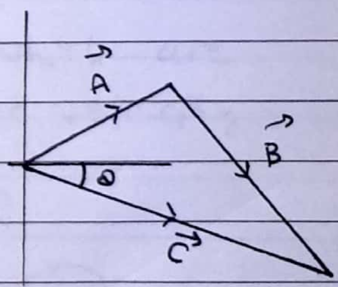
$B_y = B\sin\theta_2 = 3\sin(-45) = -2.12$

b) Find the sum (resultant) of $\vec{A}$ and $\vec{B}$

$$\vec{C} = \vec{A}+\vec{B} = (A_x+B_x)\,\hat{x} + (A_y+B_y)\,\hat{y}$$
$$= (1.73+2.12)\,\hat{x} + (1-2.12)\,\hat{y}$$
$$= 3.85\,\hat{x} - 1.12\,\hat{y}$$

$$c = |\vec{C}| = \sqrt{(3.85)^2+(-1.12)^2} = 4$$

$$\theta = \tan^{-1}\frac{C_y}{C_x} = \tan^{-1}\frac{-1.12}{3.85} = -16.2^\circ$$



Example 2.3

Given $\vec{A} = 2\hat{x}+\hat{y}$, $\vec{B} = 4\hat{x}+7\hat{y}$

a) Find the components of $\vec{C} = \vec{A}+\vec{B}$

b) Find the magnitude and direction of $\vec{C}$

a) $\vec{C} = (2+4)\,\hat{x} + (1+7)\,\hat{y} = 6\hat{x}+8\hat{y}$

thus $C_x = 6$, $C_y = 8$ $\Rightarrow |\vec{C}| = \sqrt{36+64} = 10$

b) $\theta = \tan^{-1}\frac{C_y}{C_x} = \tan^{-1}\frac{8}{6} = 53^\circ$

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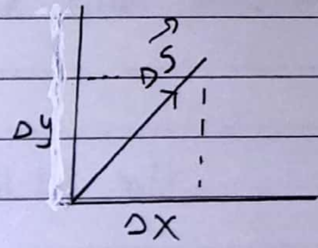
The Velocity in Two Dimensions

the displacement $\Delta\vec{S}$ is given by

$$\Delta\vec{S} = \Delta x\,\hat{x} + \Delta y\,\hat{y}$$



the average velocity is

$$\vec{\bar{V}} = \frac{\Delta\vec{S}}{\Delta t} = \frac{\Delta x}{\Delta t}\,\hat{x} + \frac{\Delta y}{\Delta t}\,\hat{y}$$

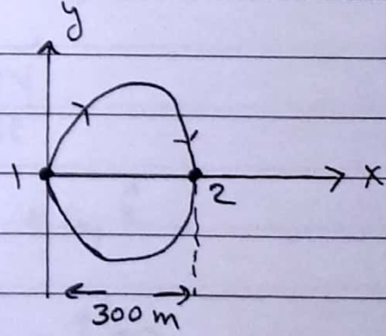
$$\vec{\bar{v}} = \bar{V}_x\,\hat{x} + \bar{V}_y\,\hat{y}$$

Example 2.4

A car travels halfway around an oval racetrack at constant speed of 30 m/s

a) what are it V_{ins} at points 1 and 2

b) it takes 40 s to go from 1 to 2 which are 300 m apart, what is the average velocity during this time interval?



a) $\vec{V}_{ins\,(1)} = 30\,\hat{y}$ m/s

$\vec{V}_{ins\,(2)} = -30\,\hat{y}$ m/s

b) $\vec{\bar{v}} = \frac{\Delta\vec{S}}{\Delta t} = \frac{\Delta x\,\hat{x} + \Delta y\,\hat{y}}{\Delta t} = \frac{300\,\hat{x} + 0\,\hat{y}}{40} = \frac{300}{40}\,\hat{x} = 7.5\,\hat{x}$ m/s

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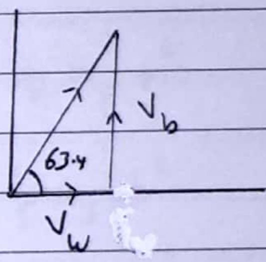
Example 2.5

A boat moves at 10 m/s relative to the water toward the shore. The velocity of the water current is 5 m/s to the right. Find the velocity of the boat relative to the shore

$$|\vec{v}| = \sqrt{v_b^2 + v_w^2} = \sqrt{10^2 + 5^2} = 11.18 \text{ m/s}$$

$$\theta = \tan^{-1}\frac{v_b}{v_w} = \tan^{-1}\frac{10}{5} = 63.4^\circ$$



Acceleration in Two Dimensions

* average acceleration: $\vec{\bar{a}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$

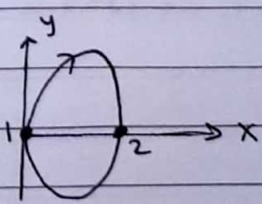
where $\vec{v} = v_x \hat{x} + v_y \hat{y}$

$$\vec{\bar{a}} = \frac{\Delta v_x}{\Delta t}\hat{x} + \frac{\Delta v_y}{\Delta t}\hat{y} = \bar{a}_x \hat{x} + \bar{a}_y \hat{y}$$

* instantaneous acceleration: $\vec{a} = \frac{d\vec{v}}{dt}$

$$\vec{a} = \frac{dv_x}{dt}\hat{x} + \frac{dv_y}{dt}\hat{y} = a_x \hat{x} + a_y \hat{y}$$

Example 2.6



In example 2.4, the velocity of the car changes from $\vec{v}_1 = 30 \text{ m/s } \hat{y}$ to $-30 \text{ m/s } \hat{y}$, what was the average acceleration?

$$\vec{\bar{a}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{-30\hat{y} - 30\hat{y}}{40} = -1.5 \hat{y} \text{ m/s}^2$$

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Finding The Motion of An Object

Motion in two dimensions can be considered as two separate motions, the first in the x-direction and the second in the y-direction.
Using the same kinematical equations for one dimensional motion

$$V_x = V_{ox} + a_x t \qquad \Delta x = V_{ox} t + \frac{1}{2} a_x t^2$$

$$V_y = V_{oy} + a_y t \qquad \Delta y = V_{oy} t + \frac{1}{2} a_y t^2$$

Projectiles

* two conditions must be satisfied
  1) air resistance is neglected
  2) $\vec{g}$ is constant and the only force acting on the object is the gravitational force.

* Projectile motion can be considered as two separate motions:
  1) uniform motion in the x-direction ($a_x = 0$)
  2) Free falling motion in the y-direction ($a_y = -g$)

$$\left.\begin{aligned} V_x &= V_{ox} \\ \Delta x &= V_{ox} t \end{aligned}\right\} \text{uniform motion}$$

$$\left.\begin{aligned} V_y &= V_{oy} - g t \\ \Delta y &= V_{oy} t - \frac{1}{2} g t^2 \end{aligned}\right\} \text{free falling}$$

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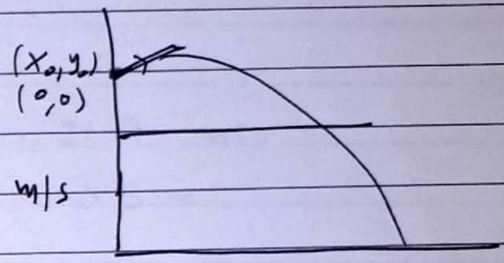
Example 2.7

a) A diver leaps from a tower with $V_{ox} = +7$ m/s and $V_{oy} = +3$ m/s.
Find the components of her position and velocity 1 s. later.



$V_x = V_{ox} = 7$ m/s

$V_y = V_{oy} - gt = 3 - 9.8(1) = -6.8$ m/s

$x = V_{ox} t = 7(1) = 7$ m

$y = V_{oy} t - \frac{1}{2} g t^2 = 3(1) - \frac{1}{2}(9.8)(1)^2 = -1.9$ m

b) the position and velocity

$v = \sqrt{V_x^2 + V_y^2} = \sqrt{(7)^2 + (-6.8)^2} = 9.7$

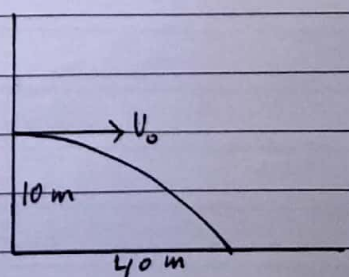
$\theta = \tan^{-1} \frac{V_y}{V_x} = -44.1^\circ$

$|\vec{r}| = \sqrt{x^2 + y^2} = \sqrt{(7)^2 + (-1.9)^2} = 7.25$ m

$\theta = \tan^{-1} \frac{y}{x} = -15^\circ$

Example 2.8

A ball is thrown horizontally from a window 10 m above the ground and hits the ground 40 m away. How fast was the ball thrown?



$y = v_{oy} t - \frac{1}{2} g t^2 = 0 - \frac{1}{2}(9.8)(t^2)$

$-10 = -\frac{1}{2} 9.8 t^2 \Rightarrow t = 1.429$ s.

$x = V_{ox} t \Rightarrow V_{ox} = \frac{x}{t} = \frac{40}{1.429} = 28$ m/s

note that $V_{ox} = V_o = 28$ m/s

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Example 2.9

A ball is kicked with $V_0 = 25$ m/s at an angle of 30° to the horizontal.

a) When does it reach its greatest hight ?

b) where is it at that time ?

a) $V_{0x} = V_0 \cos\theta = 25 \cos 30 = 21.7$ m/s

$V_{0y} = V_0 \sin\theta = 25 \sin 30 = 12.5$ m/s

At the heighest distance $V_y = 0$

$V_y = V_{0y} - gt$

$0 = 12.5 - 9.8t \Rightarrow t = 1.28$ s.

b) $x = V_{0x} t = 21.7 (1.28) = 27.8$ m

$y = V_{0y} t - \frac{1}{2} g t^2 = 12.5 (1.28) - \frac{1}{2} (9.8)(1.28)^2 = 7.97$ m

Position: $\vec{r} = x\hat{x} + y\hat{y}$

$= (27.8\,\hat{x} + 7.97\,\hat{y})$ m

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