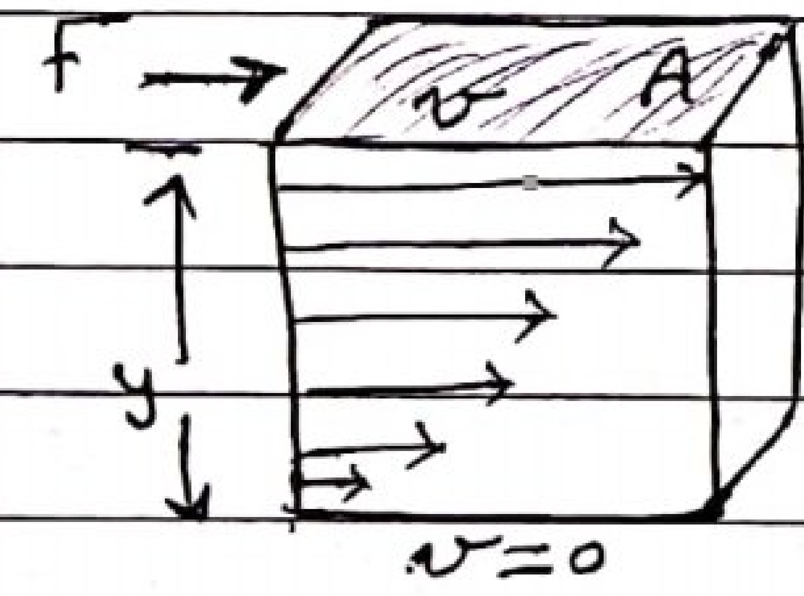
Chapter 14

Viscous Fluid Flow

□ **Viscosity**: it is a measure of the fluid resistance

the Force applied on the upper plate is given by



$$F = \eta A \frac{v}{y}$$

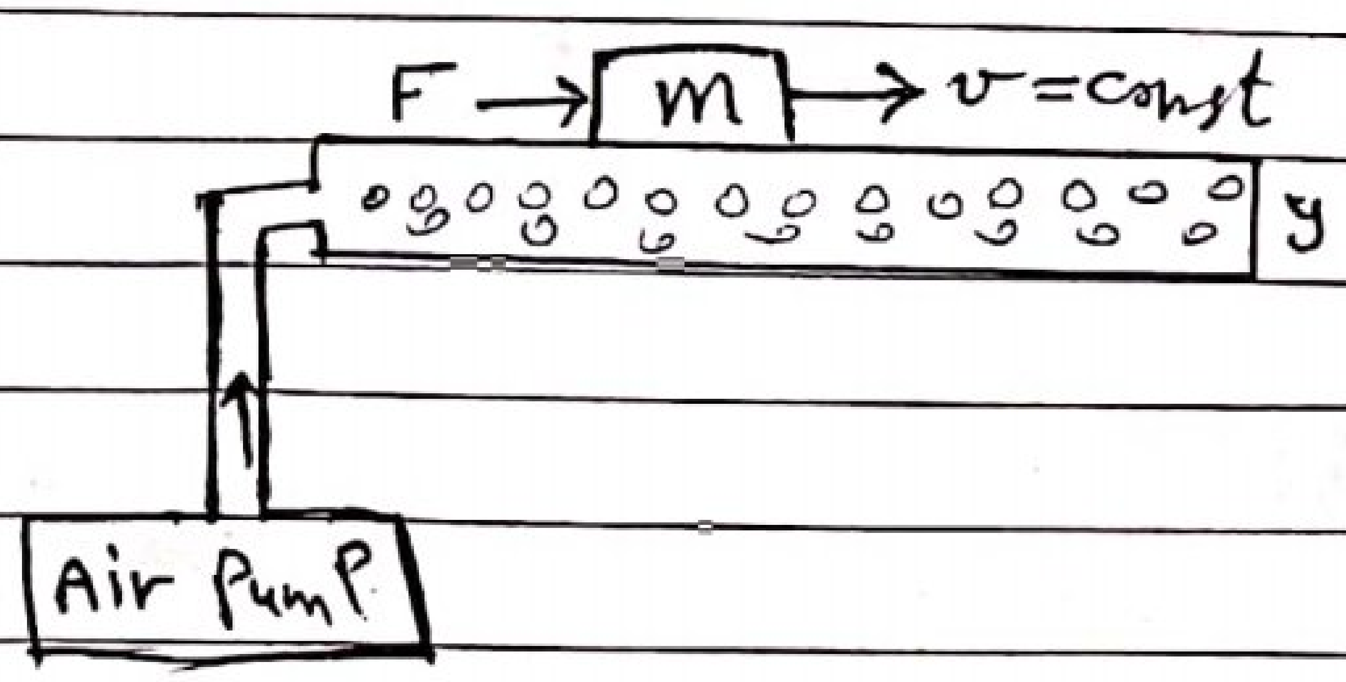
where

- η: is the coefficient of viscosity
- v: velocity of the upper plate
- A: Area of the surface
- y: thick of the fluid

units of η is $(P_a \cdot s)$

Example: (An air track)
a thin amount of air 1 mm thick and $0.04\ m^2$ in area. Find the Force required to move the cart at constant speed of $0.2\ m/s$. ($\eta_{air} = 1.8\times10^{-5}\ P_a \cdot s$)

solution



$$F = \eta A \frac{v}{y}$$

$$= (1.8\times10^{-5})(0.04)\frac{0.2}{1\times10^{-3}}$$

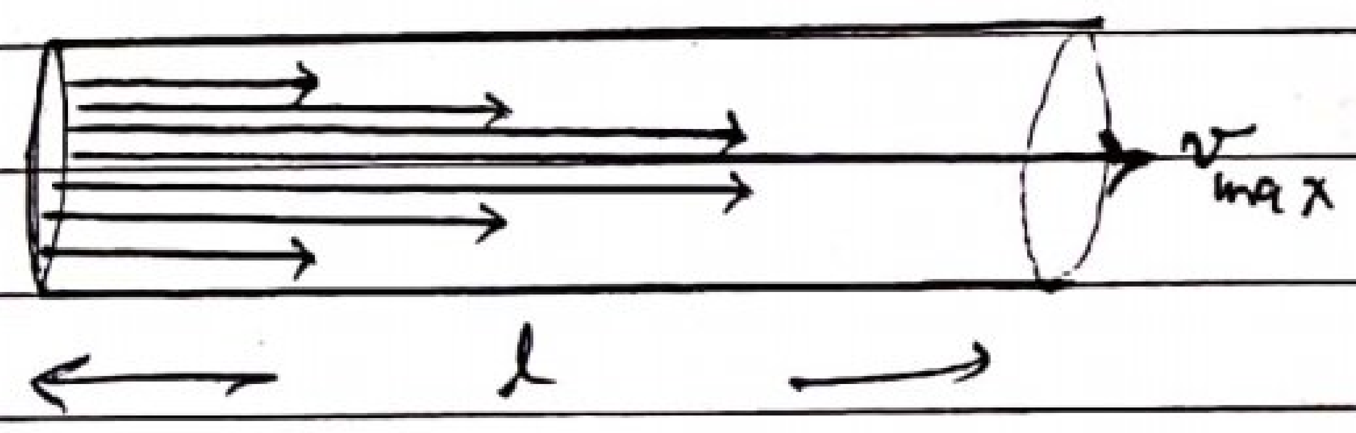
$$F = 1.44\times10^{-4}\ N$$ (nearly frictionless)

Laminar Flow in A tube

The flow of fluid between the two layers is a laminar flow (like streamline flow)
velocity decreases with lower sections of the fluid

if we consider the laminar ~~tube~~ flow in cylindrical tube with v_{max} at the center, then the average velocity of the fluid

$$\bar{v} = \frac{1}{2} v_{max}$$



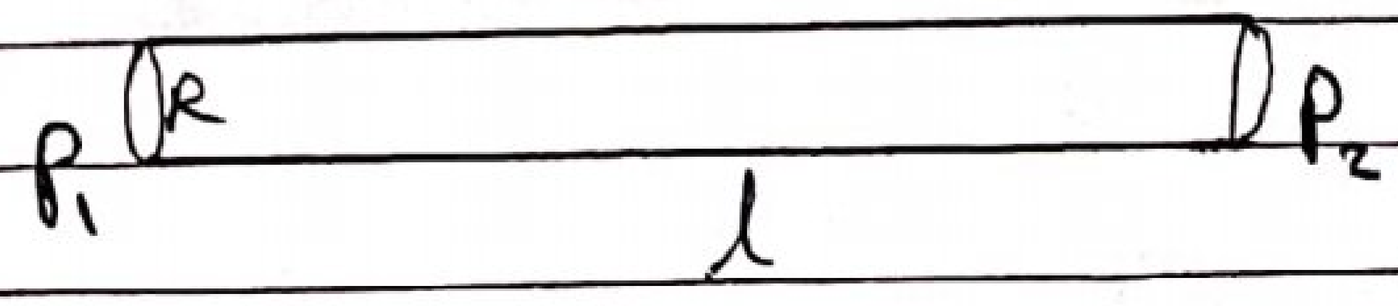
The pressure drop ΔP as the fluid moves which is due to the work done against the viscous Forces is given as

$$\Delta P = \frac{8 \eta l \bar{v}}{R^2}$$

or

$$\boxed{\bar{v} = \frac{\Delta P R^2}{8 \eta l}}$$

ΔP is the pressure drop $= P_2 - P_1$
R is the radius of the tube
l is the length of the tube



and the Flow rate is given by

$$Q = A\bar{v} = \frac{\Delta P R^2 \pi R^2}{8\eta l}$$

$$\boxed{Q = \frac{\Delta P \pi R^4}{8\eta l}} \quad \text{(Poiseuille's law)}$$

the high the viscosity the low the flow rate
the longer the tube " " " " "
the large the radius " higher " " "
" " " pressure drop " " " " "

Example: A large artery ( ) in a dog has $r_{inner} = 4\times10^{-3}$ m. Blood flows at a rate $1\ cm^3/s$. Find ($\eta_{blood} = 2.084\times10^{-3}$ Pa.s)

a) $\bar{v}$ b) v_{max} c) ΔP if $l = 0.1$ m

solution

a) $\bar{v} = \frac{Q}{A} = \frac{Q}{\pi R^2} = \frac{1\times10^{-6}}{\pi(4\times10^{-3})^2} = 1.99\times10^{-2}$ m/s

b) $v_{max} = 2\bar{v} = 3.98\times10^{-2}$ m/s

c) $\Delta P = \frac{8\eta l \bar{v}}{R^2} = \frac{8(2.084\times10^{-3})(0.1)(1.99\times10^{-2})}{(4\times10^{-3})^2}$

$$\Delta P = 2.07\ P_a$$

Power Dissipation

- power dissipated due to viscous force must equal to power that must be supplied to maintain the flow

$$P = F\bar{v} = \Delta P A \bar{v} = \Delta P Q$$

$$P = \Delta P Q = \Delta P \pi R^2 \bar{v}$$

this formula is suitable for blood vessels (أوعية)

Example:

What is the power required to maintain the blood flow in the dog's artery in the previous example

$\bar{v} = 1.99 \times 10^{-2}$ m/s
$\Delta P = 2.07$ Pa
$R = 4 \times 10^{-3}$ m

$$P = \Delta P \pi R^2 \bar{v}$$

$$= (2.07)(\pi)(4 \times 10^{-3})^2 (1.99 \times 10^{-2})$$

$$= 2.07 \times 10^{-6} \text{ W}$$

Flow Resistance

The flow resistance R_f is defined as the ratio of the pressure drop to the flow rate

$$R_f = \frac{\Delta P}{Q} = \frac{\Delta P}{\frac{\Delta P \pi R^4}{8\eta l}} = \frac{8\eta l}{\pi R^4} \quad \text{(laminar flow)}$$

the unit for R_f is $Pa \cdot s/m^3$

Example:

The aorta ([illegible]) of an average adult human has $r = 1.3 \times 10^{-2}$ m. What are R_f and ΔP over a 0.2 m distance? Assuming the flow rate of 10^{-4} m^3/s

a) $$R_f = \frac{8\eta l}{\pi R^4} = \frac{8(2.084\times 10^{-3})(0.2)}{\pi (1.3 \times 10^{-2})^4} = 3.72 \times 10^{4} \; \frac{Pa \cdot s}{m^3}$$

b) $$\Delta P = R_f \, Q$$

$$= (3.72 \times 10^{4})(10^{-4}) = 3.72 \; Pa$$

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Chapter 15

Cohesive Forces In Liquids

Surface Tension:

is a phenomena in which the surface of a liquid in contact with gas (air) acts like a thin elastic sheet.

It is defined as the ratio of the surface Force F to length l along which the force acts

$$\gamma = \frac{F}{l} \quad \text{For each surface}$$

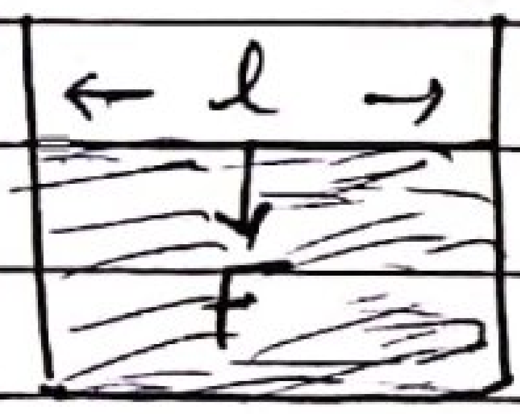
and

$$\gamma = \frac{F}{2l} \quad \text{for the surface and its opposite}$$

γ: is the surface tension and it has a unit $\frac{N}{m}$

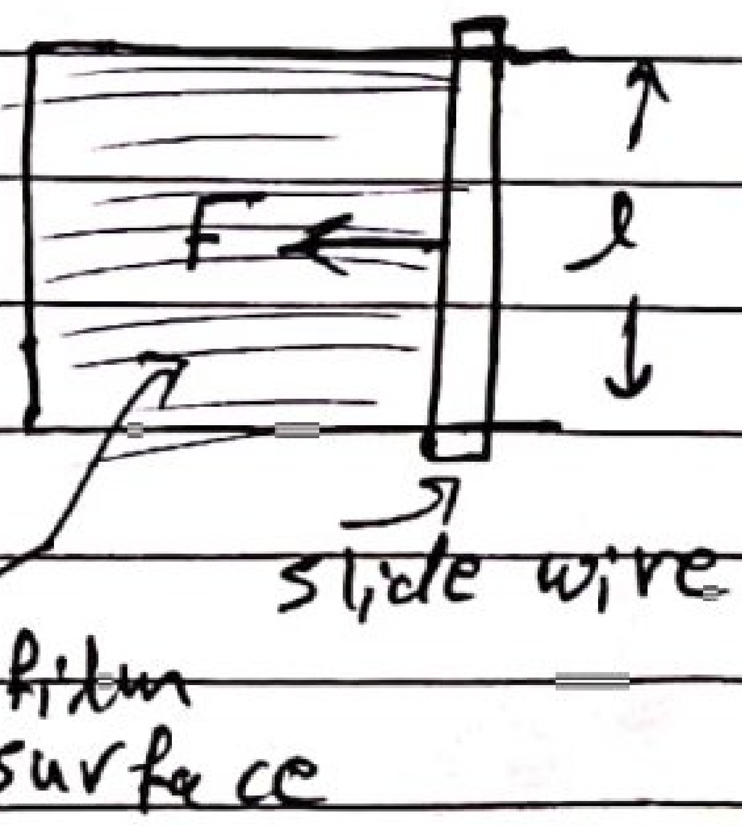
Example:

Calculate the force on the slide wire if it is 3.5 cm long and the fluid is ethyl alcohol ($\gamma = 0.022$)



$$F = 2\gamma l = 2(0.022)(3.5\times10^{-2})$$

$$F = 1.54\times10^{-3}\ N$$



Example:

The U-shaped loop is dipped into water at 20°C, the slide wire is 0.1 m long and has a mass $m_1 = 1$ g

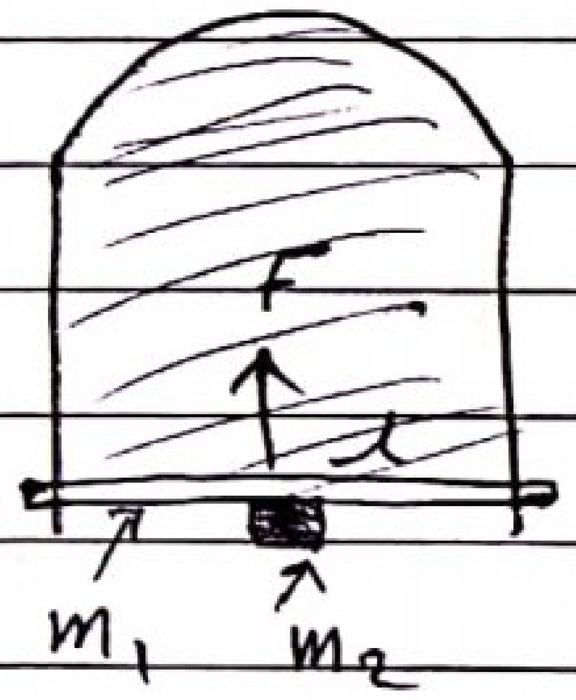
a) How large is the surface tension Force ($\gamma = 7.28 \times 10^{-2}$ N/m)

b) if the wire is in equilibrium, how large the mass m_2 is suspended from the wire?

solution

a) $F = 2\gamma l = 2(7.28 \times 10^{-2})(0.1)$

$F = 1.46 \times 10^{-2}$ N



b) $F = m_1 g + m_2 g$

$$m_2 = \frac{1}{g}[F - m_1 g]$$

$$= \frac{1}{9.8}\left[1.46 \times 10^{-2} - (10^{-3})(9.8)\right] = 0.49 \times 10^{-3} \text{ kg}$$

$$= 0.49 \text{ g}$$

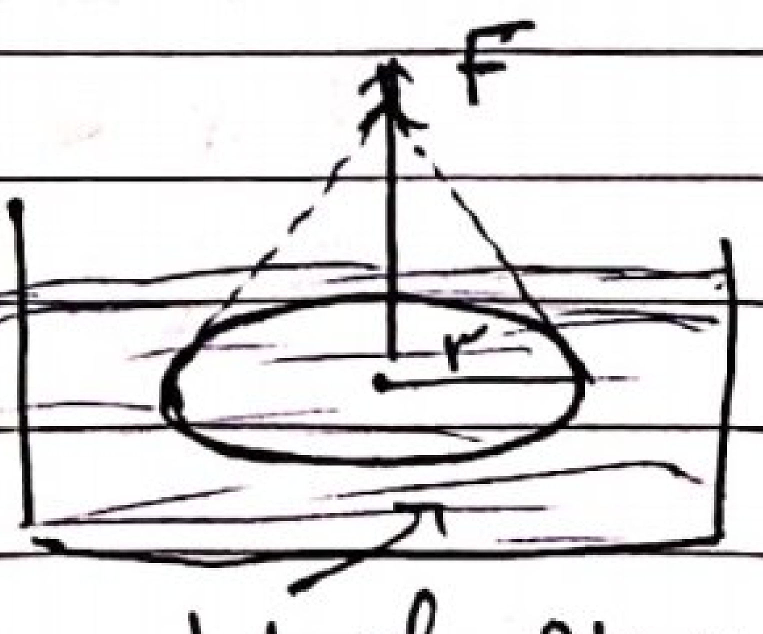
Example:

In order to lift a wire ring of radius 1.75 cm from the surface of a container of blood plasma, a vertical Force of 1.6×10^{-2} N greater than the weight of the ring is required. Calculate the surface tension of blood plasma

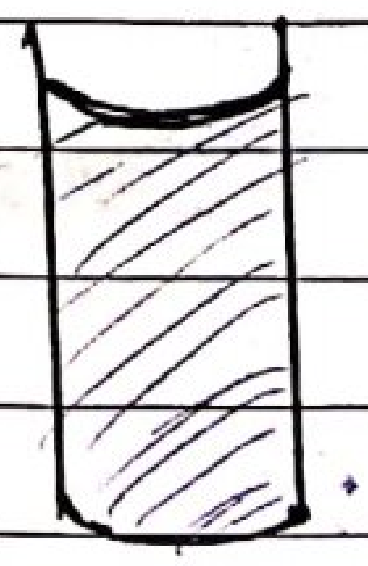
$$F = \gamma l = \gamma(2\pi r)$$

$$\gamma = \frac{F}{2\pi r} = \frac{1.6 \times 10^{-2}}{2\pi(1.75 \times 10^{-2})}$$

$$\gamma = 0.146 \text{ N/m}$$



☐ Cohesive and Adhesive Forces



Adhesive
تلاصق

Water

it surface has
a concave (مقعر) shape
because water wets the
surface

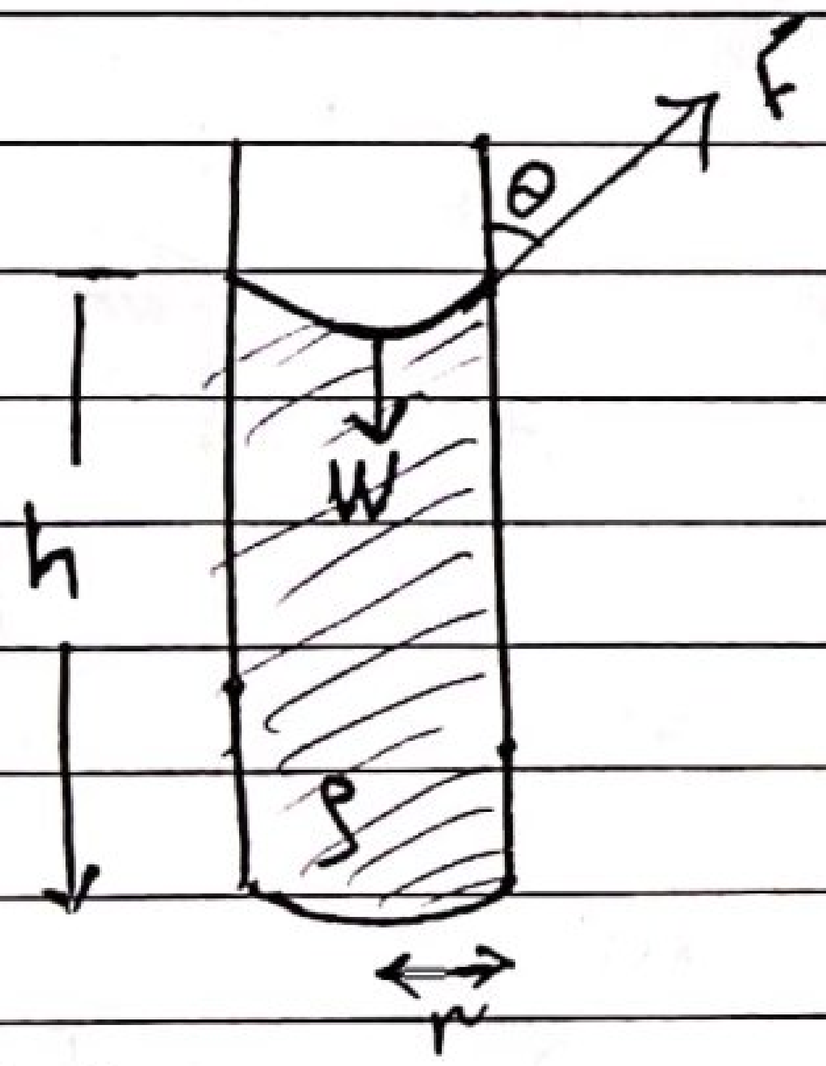
cohesive
تماسك

mercury

it surface has a
convex (محدب) shape
because mercury does
not wet glass

Contact angles and capillarity

The surface of a liquid in contact with solid surface forms an angle θ with respect to the solid surface.



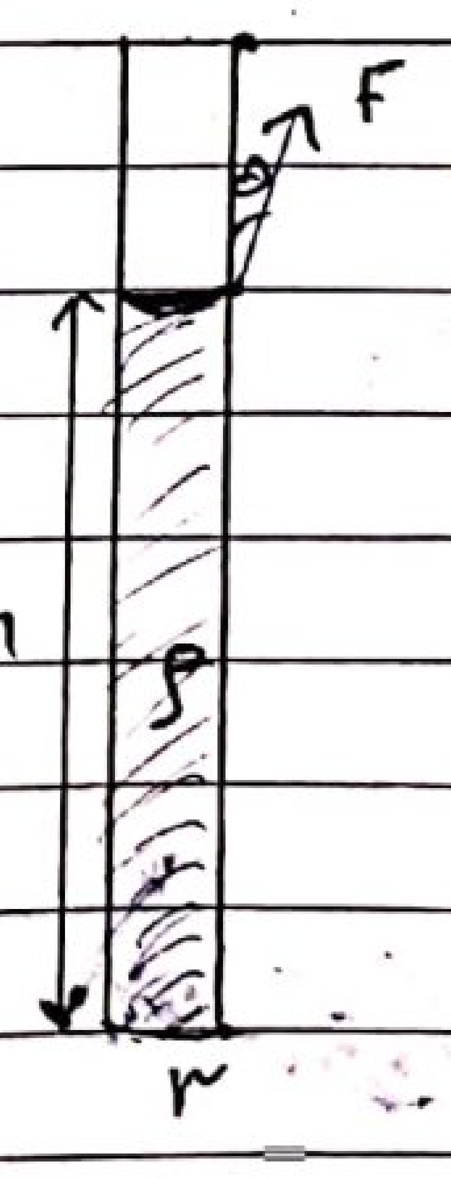
This contact force determined by the relative magnitude of the cohesive force between molecules of liquids and adhesive force between liquid and solid, and it depends on the particular solid and liquid

if $\theta < 90$, water will rise in a narrow tube
if $\theta > 90$, water will depressed in a tube
if $\theta = 90$, water will neither rise nor fall

Now

$$F\cos\theta = W$$
$$\gamma l \cos\theta = mg$$
$$\gamma(2\pi r)(\cos\theta) = \rho V g$$
$$2\pi\gamma r \cos\theta = \rho(r^2\pi h) g$$

$$\boxed{h = \frac{2\gamma\cos\theta}{\rho r g}}$$



$$h \sim \begin{cases} + & \theta < 90 \quad \text{the fluid rises} \\ - & \theta > 90 \quad \text{" " depressed} \\ 0 & \theta = 90 \quad \text{Neither rises nor depressed} \end{cases}$$

Capillarity Action

الأنابيب الشعرية

Example:

Find the height to which water would rise in a capillary tube with a radius equal to 5×10^{-5} m. Assume the contact angle is small enough to be considered zero

$$h = \frac{2\gamma \cos\theta}{\rho g r} \simeq \frac{2(0.073)(1)}{(10^3)(9.8)(5 \times 10^{-5})} = 0.3 \text{ m}$$

Example:

The sap in trees (النسغ في الأشجار) which is water rises in a system of capillaries of radius $r = 2.5 \times 10^{-5}$ m, $\theta = 0$, $\rho_w = 10^3$ kg/m^3. What is the maximum height of rising water?

$$h = \frac{2\gamma \cos\theta}{\rho g r} = \frac{2(7.28 \times 10^{-2})(1)}{(10^3)(9.8)(2.5 \times 10^{-5})}$$

$$h = 0.59 \text{ m}$$