DONE BY :
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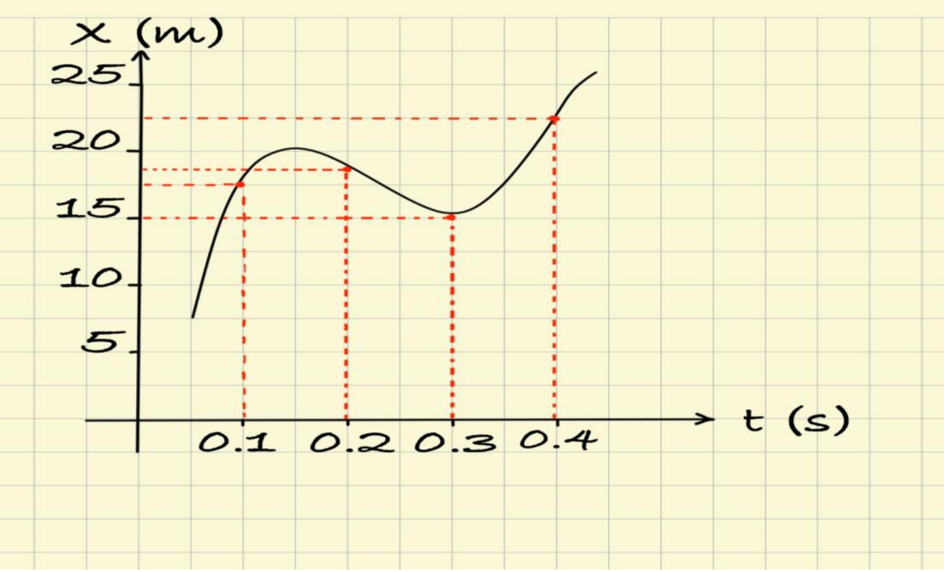
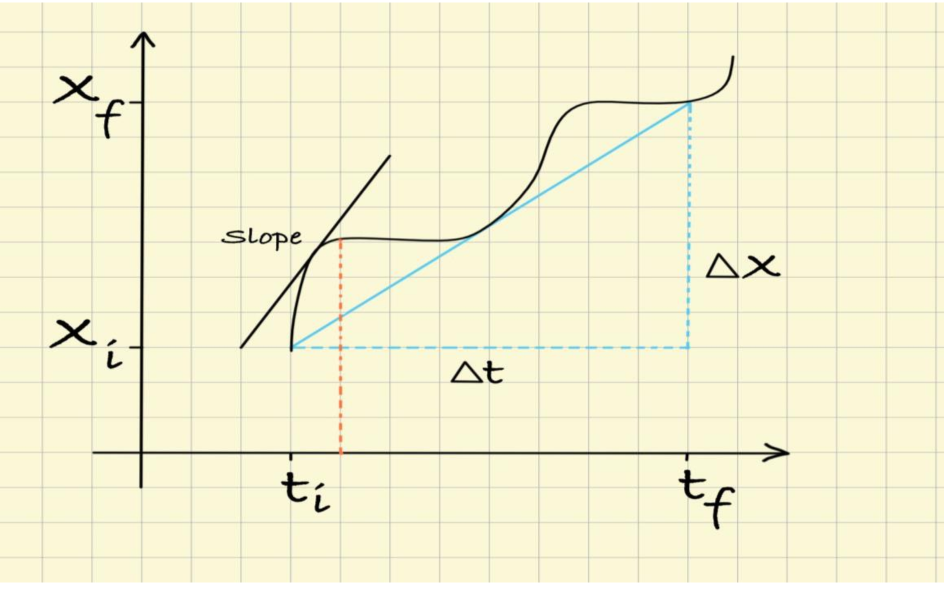
Motion in a straight line

- Position (x) : it's the distance from the origin
- Displacement (Δx) : $\Delta x = x_f - x_i$
- Average velocity ($\bar{v}$) : ($\bar{v}$) = Δx/Δt
- Average speed (speed) :
- speed= total distance/total time

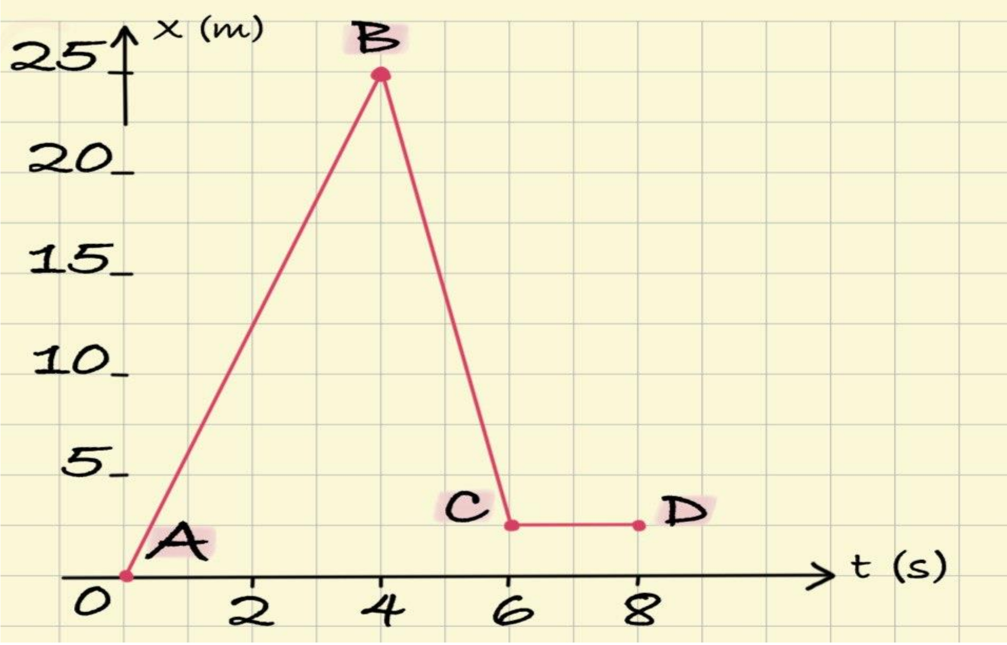


- Instantaneous velocity (v) :
- $V = \lim_{\Delta t \to 0} \Delta x/\Delta t = dx/dt$



Example :

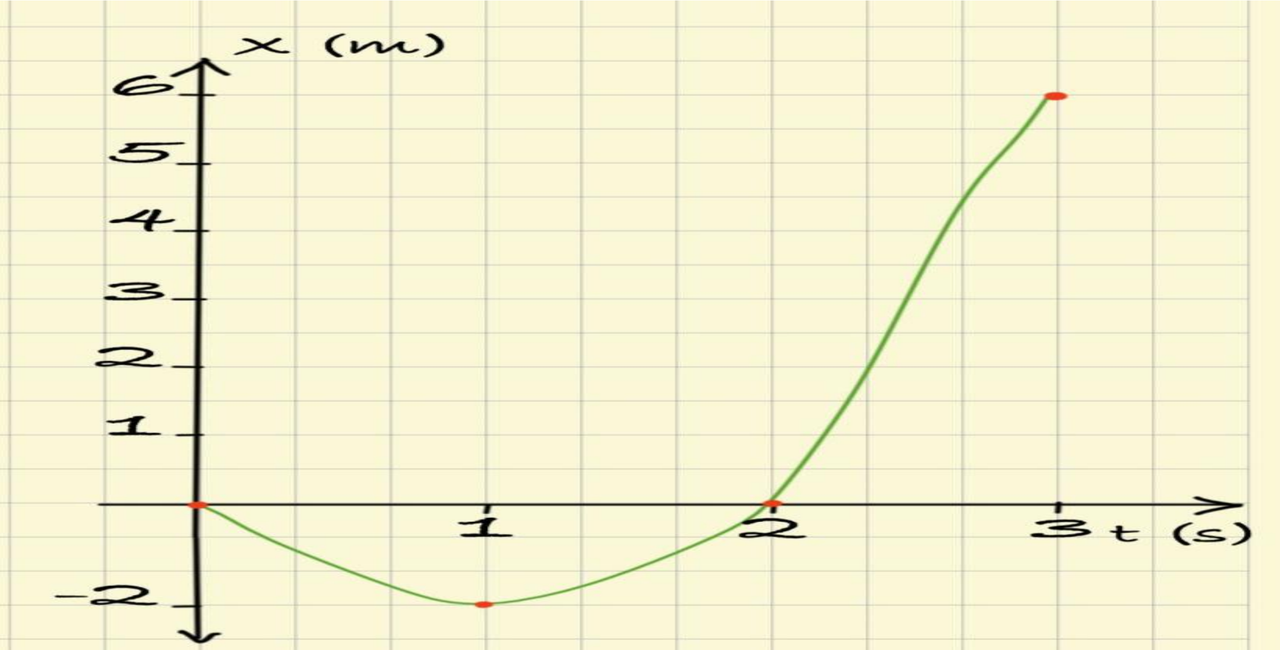
- X(4) = 25 , x(6) = 5 , On the interval [0,8] :



- **Solution :**

- $\bar{V} = \Delta x/\Delta t = x(8) - x(0)/8 - 0 = (5-0)/8 = 5/8$ m/s
- Average speed = total distance/total time = (25 + 20 + 0)/8 = 45/8 = 5.6 m/s
- $V(2) = dx/dt|_{t=2}$ = slope = (25 – 0)/4 – 0 = 6.25 m/s , v(5) = (5-25) / 6-4 = -10 m/s

- **Example :** a particle moves a long the x – axis according to the expression
- ***$x(t) = 2t^2 - 4t$*** , where *X* in meter and *T* in second :
- 1) determine the displacement of the particle in the time interval t=1s to t=3s
- 2) calculate the average velocity in the same interval and the average speed
- 3) find the instantaneous velocity of the particle at t=2.5s
- 4) find the average velocity and speed solution in the interval [0,3]



- **To be continued ……**

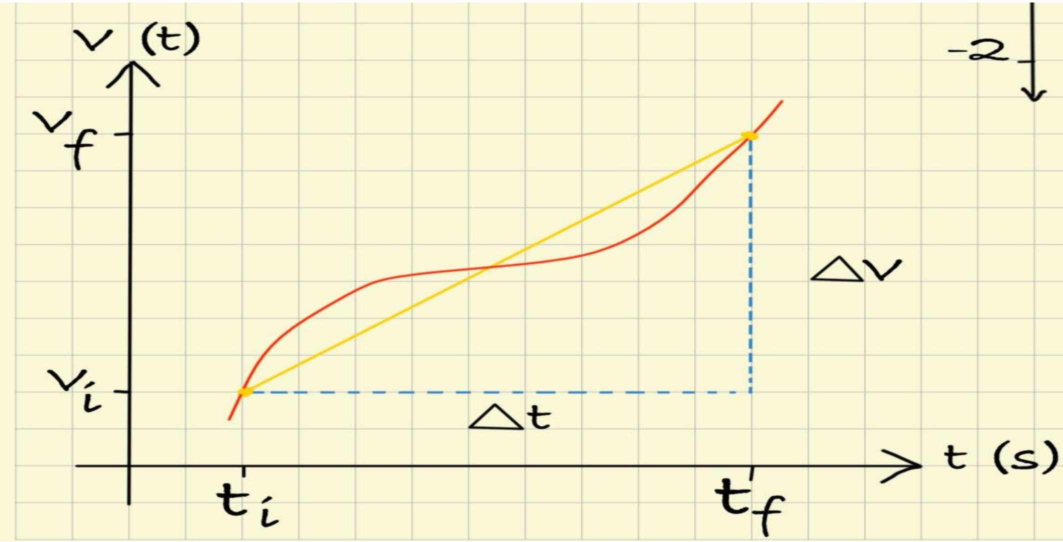
Solution :

1)

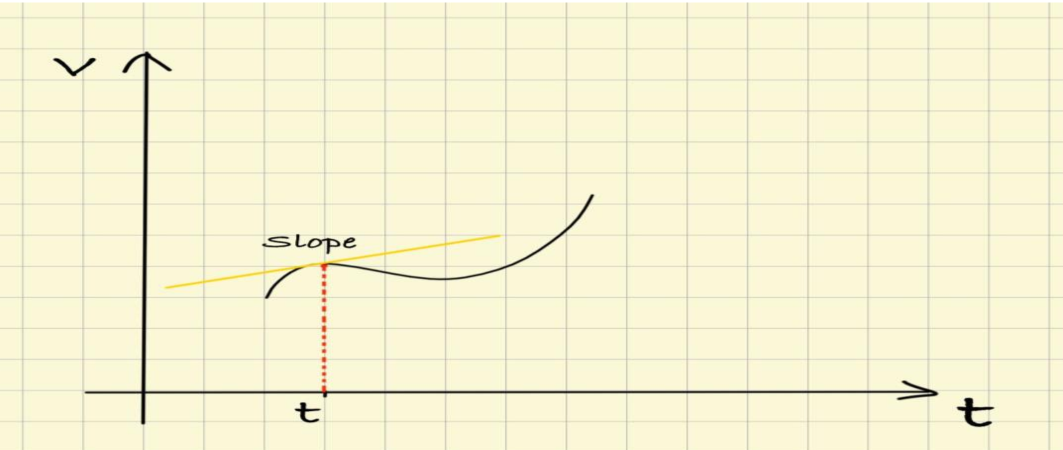
- X(1) = 2-4 = -2 m
- X(3) = (2*9) – (4*3) = 6 m
- Δx = x(3) – x(1) = 6 – (-2) = 8 m
- 2)
- Average velocity : $\bar{v}$ = $\Delta x/\Delta t$ = x(3) – x(1)/3-1 = 8/2 = 4 m/s
- Average speed = total distance/total time = (2+6)/3-1 = 8/2 = 4 m/s
- 3)
- V = dx/dt = 4t-4
- V(2.5) = (4*2.5) – 4 = 6 m/s
- 4)
- [0,3] …… $\bar{v}$ = x(3) – x(0)/3-0 = (6-0)/3 = 6/3 = 2 m/s
- Average speed = (2+2+6)/3 = 10/3 = 3.34 m/s

Acceleration

- Average acceleration ($\bar{a}$) : $\bar{a} = \Delta v/\Delta t$
- Instantaneous acceleration : $a = dv/dt$

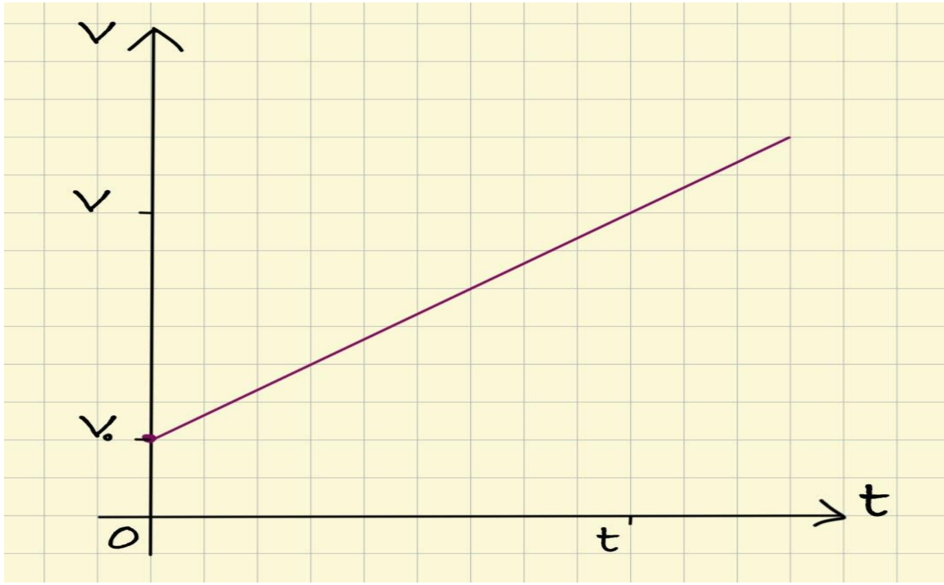


- $\bar{a} = \Delta v/\Delta t = (v_f - v_i)/t_f - t_i$
- $a = dv/dt =$ slope



- **Example :** the velocity of a particle moving a long the x-axis varies in the time according to the expression: $v(t) = (40-5t^2)$ m/s
- 1) find the average acceleration in the interval t=0s to t=2s
- 2) what is the acceleration at t=2s
- **Solution:**
- 1)
- $\bar{a} = \Delta v/\Delta t = v(2) - v(0)/2 - 0 = (20-40)/2 = -10\ m/s^2$
- 2)
- $a(t) = dv/dt = -10\ t$
- $a(2) = -20\ m/s^2$

One dimensional motion with constant acceleration



- For constant acceleration in the interval [0,t]
- $\bar{a} = a = \Delta v/\Delta t = (v - v_0)/t$
- *$V = v_0 + at$*
- In the same interval : *$\bar{v} = (v + v_0)/2$*
- Substitute eq.(1) in eq.(2)
- $V = (v_0 + v_0 + at)/2 = v_0 + ½\ at$
- But from the definition of $\bar{v} = \Delta x/\Delta t = (x - x_0)/t$ we find :
- $\bar{V} = (x - x_0)/t = v_0 + ½\ at$
- $X - x_0 = v_0 t + ½\ at^2$
- *$x = x_0 + v_0 t + ½\ at^2$*

- **Again :**
- $\overline{V} = (v + v_0)/2 = (x - x_0)/t$
- $X - x_0 = t/2 * (v + v_0)$ ..... And substitute for t from eq.1 :
- $X - x_0 = ½ * (v - v_0)/a * (v + v_0) = (v^2 - v_0^2)/2a$
- *$V^2 = v_0^2 + 2a * (x - x_0)$*

- **Results for constant acceleration** :
- $V = v_0 + at$
- $X = x_0 + v_0 t + ½ at^2$
- $V^2 = v_0^2 + 2a * (x - x_0)$

- **Example :** an object accelerates from rest to speed of 128 m/s in 8s .
- 1) determine the acceleration
- 2) find the distance it travels in 8s
- 3) what is the velocity after 10s ?
- 4) after how long time it will travels a distance of 1600 m ?

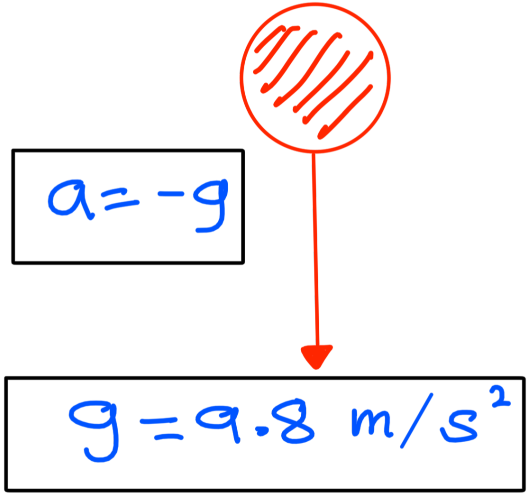
- **Solution :**
- 1) $v = v_0 + at$ .... $128 = 0 + 8a$ ...... $a = 128/8 = 16\ m/s^2$
- 2) distance = $\Delta x = x - x_0 = v_0 t + ½ at^2$ ...... $\Delta x = 0 + (½ * 16 * 8^2) = 512\ m$
- 3) $v = v_0 + at = 0 + (16 * 10) = 160\ m/s$
- 4) $\Delta x = v_0 t + ½ at^2$ ..... $1600 = 0 + (½ * 16 * t^2)$ ..... $T^2 = 1600/8 = 200$
- $T = \sqrt{200} = 14.14\ s$

- **Example :** a particle moves from rest with a constant acceleration $5 m/s^2$, find :
- 1) its velocity after 3s
- 2) its displacement after 3s
- 3) after how long time it will travel a distance of 100m and what is the velocity at this time ??

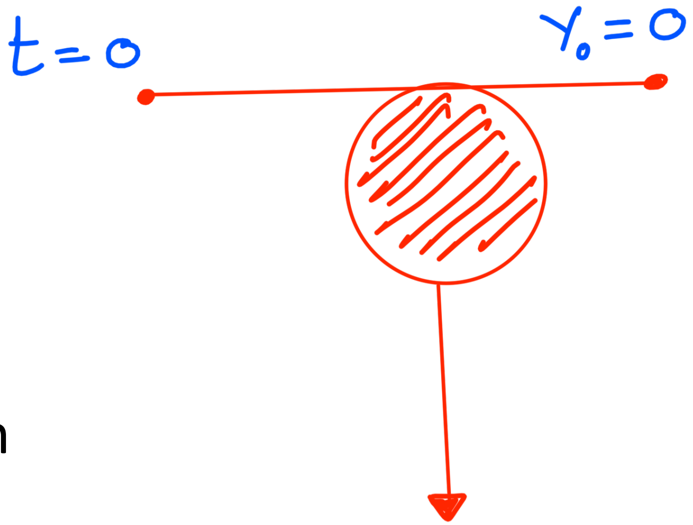
- **Solution :**
- 1) $v = v_0 + at$ ….. $V = 0 + (5*3) = 15$ m/s
- 2) $\Delta x = v_0 t + ½ at^2$ ……. $= 0 + (½ * 5 * 9) = 22.5$ m
- 3) $\Delta x = v_0 t + ½ at^2$ …… $100 = 0 + (½ * 5 * t^2)$…… $t = \sqrt{200/5} = 6.3$ s
- $v = v_0 + at$ …… $0 = 5*6.3 = 31.5$ m/s

Freely falling bodies

- In this case the object moves under the influence of gravity (F = -mg) with a constant acceleration of (-g)
- Therefore the equations of motion can be obtained as :
- 1) $v = v_0 - gt$
- 2) $y = y_0 + v_0 t - \frac{1}{2} g t^2$
- 3) $v^2 = v_0^2 - 2g * (y - y_0)$
- ***Note :*** **choose y_0=0 as the initial position at t=0**

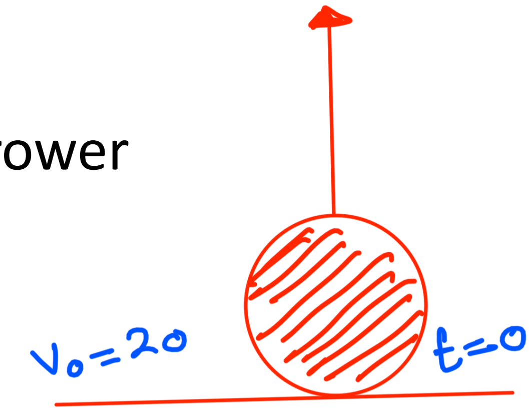


- **Example :** a freely falling body starts its motion from rest , calculate its position and velocity at :
- 1) t = 1s , 2s , 3s respectively ?



- **Solution :**
- At t = 1s
- $Y = v_0 t - ½ gt^2$ ……. = 0 – (½ * 9.8 * 1) = -4.9 m
- $V = v_0 - gt$ = 0 – (9.8*1) = -9.8 m/s
- At t = 2s
- $Y = v_0 t - ½ gt^2$ = 0 – (½ * 9.8 * 4) = -19.6 m
- $V = v_0 - gt$ = 0 – (9.8 * 2) = -19.6 m/s
- At t = 3s
- $Y = v_0 t - ½ gt^2$ = 0 – (½ * 9.8 * 9) = 44.1 m
- $V = v_0 - gt$ = 0 – (9.8 * 3) = 29.4 m/s

- **Example :** a stone is thrown upward with initial velocity of 20 m/s , find :
- 1) the maximum height
- 2) the time needed to reach the maximum height
- 3) the time needed for the stone to return to the level of thrower
- 4) the velocity of the stone at this instant
- 5) the velocity and position at t = 2.5s



- **Solution:**
- 1) $v^2 = v_0^2 - 2gy$ …… $0 = 20^2 - (2 * 9.8y)$ ….. $Y = 20.4$ m
- 2) $v = v_0 - gt$ ….. $0 = 20 - 9.8t$ ….. $T = 2.04$s
- 3) $y = v_0 t - ½ * gt^2$ ……. $0 = 20t - ½ * 9.8t^2$ ….. $0 = (20 - 4.9t) * t$ …… $t = 0$, $t = 4.08$s
- 4) $v = v_0 - gt$ …… $v = 20 - (9.8 * 4.08) = -20$ m/s
- 5) $v = v_0 - gt$ …… $v = 20 - (9.8 * 2.5) = -4.5$ m/s
- $Y = v_0 t - ½ gt^2$ ……. $Y = 20 * 2.5 - (½ * 9.8 * 2.5^2) = 19.37$ m