

Hypothesis Testing

Example: Average intake in children for dietary fat is 70 g of fat per day. Suppose we want to study children who eat a vegetarian diet. Possible hypotheses are

1. Average fat intake is 70 g per day
2. Average fat intake is less than 70 g per day

Def: One-sample problem—a single distribution.

Def: Hypothesis—statement about parameters in a population or populations. We want to know how likely this is to be true, given the evidence (data).

Def: Null hypothesis— H_0 —the hypothesis to be tested. This is usually a statement of no difference. The population value of the parameter is not different from some specified value.

Def: Alternative hypothesis— H_1 or H_A —This is the statement we will accept if we reject the null hypothesis.

H_0 : Mean fat intake in vegetarian children is 70 g per day. $H_0: \mu = \mu_0$ or $\mu \geq \mu_0$

H_1 : Mean fat intake in vegetarian children is < 70 g per day. $H_1: \mu < \mu_0$

Possible decisions:

- 1. Accept H_0 (really, fail to reject H_0)**
- 2. Reject H_0**

Possible Scenarios:

Jury Trial		
	Truth	
Verdict	Innocent	Guilty
Innocent	Correct decision	Error
Guilty	Error	Correct decision

Test of hypothesis		
	Truth	
Results of Test	H₀	H₁
Accept H₀	Correct decision	Type II error
Reject H₀	Type 1 error	Correct decision

Def: Type I error is the probability of rejecting H₀ when H₀ is true.

Def: Type II error is the probability of accepting H₀ when H₁ is true.

Def: Level of significance: α = Probability of a Type I error. This is the area under the curve below (or above) the critical value. This is the probability of rejecting H_0 when H_0 is true.

Def: β : Probability of a Type II error.

Def: $1-\beta$ —Power of a test. This is the Pr (rejecting $H_0|H_1$ is true).

Goal: Make α, β as small as possible. Usually, as $\alpha \uparrow, \beta \downarrow$ and as $\alpha \downarrow, \beta \uparrow$.

Fix α (0.05 or 0.01). Find a test to minimize β .

Best test for the fat experiment is one based on \bar{X} .

Def: Acceptance Region—These are the values of \bar{X} for which H_0 is accepted.

Def: Rejection Region—These are the values of \bar{X} for which H_0 is rejected.

Note: For this example, we are conducting a one-sided or one-tailed test. We will only reject H_0 for values of \bar{X} that are low.

Def: One-tailed—this is a test in which values of parameter under H_1 either $>$ or $<$ values under H_0 but not both.

The picture behind the two sided test

