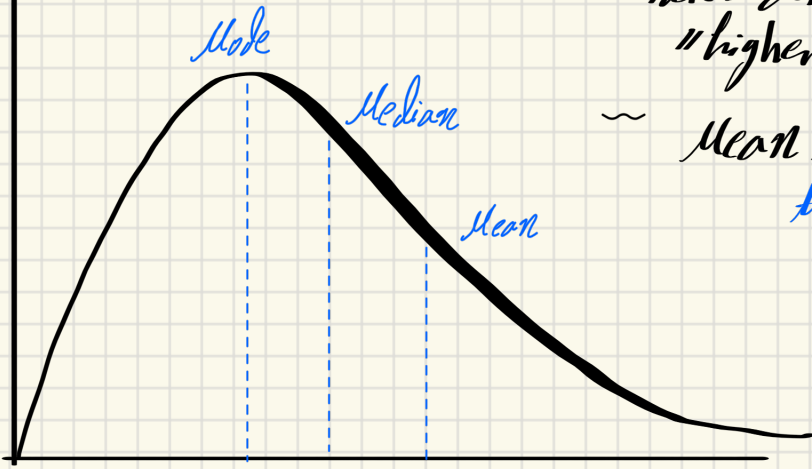


Positively skewed

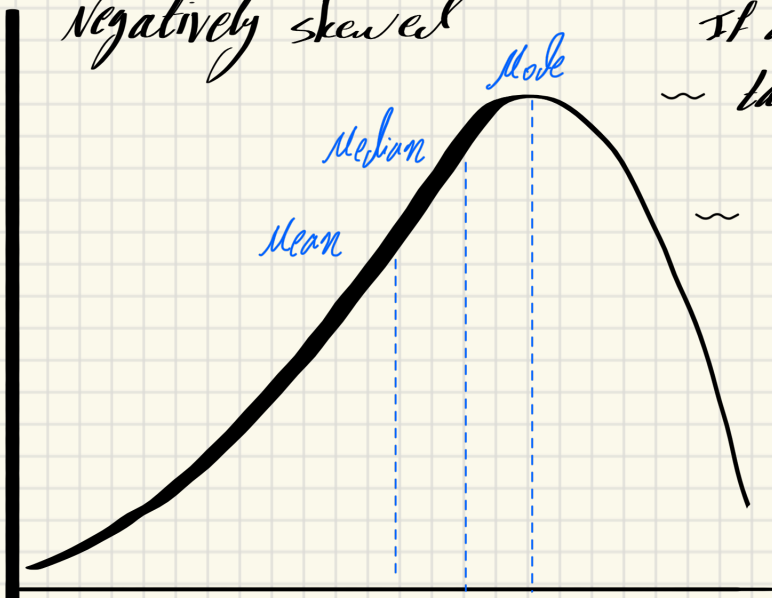


- \* If a curve skewed to the right:
  - ~ Tail's direction towards the right "higher values"
  - ~  $\text{Mean} > \text{Median} > \text{Mode}$  as a value

Notes:

- ~ Mode is always the highest point.  
If distribution is even;  $\text{Mean} / \text{Median} = \text{Mode}$ .  
↳ normal
- ~ tail direction  $\Rightarrow$  skewness direction
- ~ Mean is furthest away from Mode, towards the tail
- ~ Median  $\rightarrow$  in the middle

Negatively skewed



- If a curve is skewed to the left:
  - ~ tail's direction  $\rightarrow$  towards left "lower values"
  - ~  $\text{Mean} < \text{Median} < \text{Mode}$  as a value

Generally, if the distribution of data is skewed to the left, the mean is less than the median, which is often less than the mode. If the distribution of data is skewed to the right, the mode is often less than the median, which is less than the mean

# Median

$n$ ; odd number "one median"  
 $n$ ; even number "two medians"

$\hookrightarrow x_{(\frac{n+1}{2})}$       $\hookrightarrow \frac{x_{(\frac{n}{2})} + x_{(\frac{n}{2}+1)}}{2}$

**ex:** for sample A  
 1, 2, 5, 20, 25  
 $n \rightarrow$  odd number  
 $\rightarrow x_{(\frac{5+1}{2})} \Rightarrow x_{(3)}$   
 $x_{(3)} = 5$

**ex:** sample B  
 4, 5, 7, 8, 10, 12  
 $n \rightarrow$  even number  
 $x_{(\frac{6}{2})} + x_{(\frac{6}{2}+1)}$   
 $\rightarrow \frac{x_{(3)} + x_{(4)}}{2} \rightarrow$   
 $\frac{7+8}{2} = 7.5$

# Notes:

~ To find the median the observation should be arranged in "ascending" order

~ Numbers are sorted from smallest to largest

$x_{(\frac{n+1}{2})}, x_{(\frac{n}{2})}, x_{(\frac{n}{2}+1)}$

Indicates value at position

$\rightarrow \frac{n+1}{2}$   
 $\rightarrow \frac{n}{2} + 1$   
 $\rightarrow \frac{n}{2}$

## Median

The median is also a frequently used measure of central tendency. The **median** is the midpoint of a distribution: the same number of scores is above the median as below it.

$\hookrightarrow$  Also considered as the 50<sup>th</sup> percentile

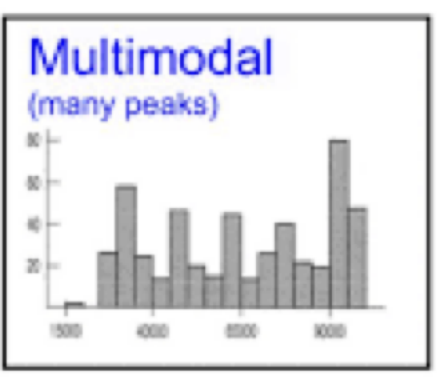
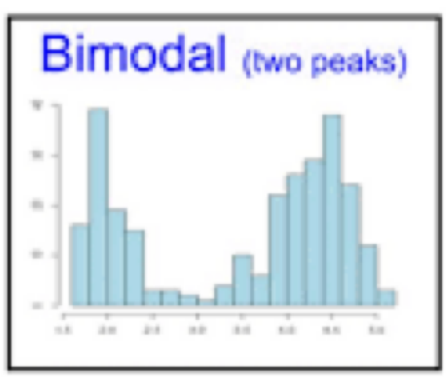
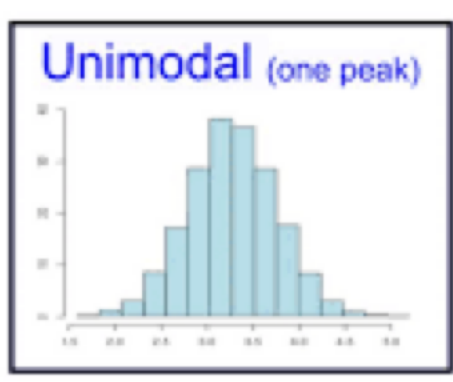
## Median

In order to calculate median:

1. Arrange the numbers in the set from smallest to largest.
2. Determine N or n (number of scores)
3. If N or n is odd then the median is the middle number.
4. If N or n is even then the median is the average of the middle two numbers

# Mode $\hat{=}$ The value with the highest frequency

- ~ uni-modal: A dataset with one mode
- ~ Bi-modal: A dataset with two modes
- ~ Multi-modal: A dataset with more than two modes
- ~ NO Mode: A dataset where no value repeats more than others



Mean

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

ex: 1, 2, 5, 20, 25

$$\sum x = 1 + 2 + 5 + 20 + 25 = 53$$

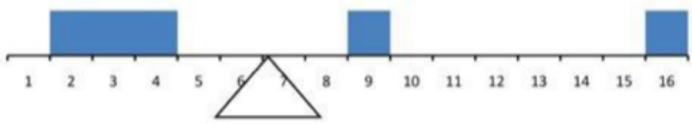
$$n = 5$$

$$\bar{x} = \frac{53}{5} = 10.6$$

concept of mean in relation to SD

Mean

One definition of central tendency is the point at which the distribution is in balance. Figure 3 shows the distribution of the five numbers 2, 3, 4, 9, 16 placed upon a balance scale. If each number weighs one pound, and is placed at its position along the number line, then it would be possible to balance them by placing a fulcrum at 6.8. The fulcrum or balancing point is calculated as the arithmetic mean or mean.

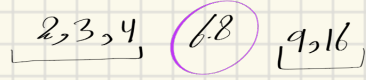


Fulcrum = نقطة الاتزان

$$\sum (x - \text{Mean}) = 0$$

However, this equation doesn't imply that the sum of the actual values above the mean equals the sum of the actual values below the mean. Instead, it implies that the sum of the positive deviations (values above the mean) is exactly canceled out by the sum of the negative deviations (values below the mean). This means that the total amount by which values exceed the mean is balanced by the total amount by which values fall short of the mean.

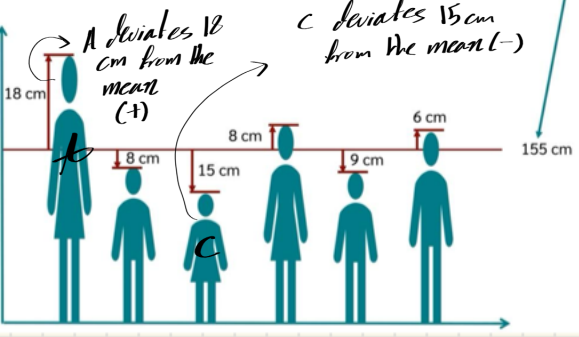
$$\text{Mean} = \frac{2+3+4+9+16}{5} = 6.8$$



the distance to all scores below the mean equals the distance to all scores above the mean. The mathematical definition of the mean is the point in a distribution at which the total distance to all the scores above that point equals the total distance to all scores below that point.

DATA tab

...how much data scatter around the mean value



\* We don't consider the deviation of every single value from the mean, but we consider how a person deviates from the mean on average

|                     |  |
|---------------------|--|
| Mean of a sample    | $\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$               |
| Range               | $R = x_L - x_s$                                      |
| Sample variance     | $s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n - 1}$ |
| Population variance | $\sigma^2 = \frac{\sum_{i=1}^N (x_i - \mu)^2}{N}$    |

|                          |  |
|--------------------------|--|
| Standard deviation       | $s = \sqrt{s^2} = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n - 1}}$ |
| Coefficient of variation | $C.V. = \frac{s}{\bar{x}} (100)\%$                                     |
| Mean of a population     | $\mu = \frac{\sum_{i=1}^N x_i}{N}$                                     |

# Measure of dispersion

~ Range = Max - Min

~~~~ Population

~~~~ Sample

Variance:  $\frac{\sum (x - \mu)^2}{N} = \sigma^2$

$s^2 = \frac{\sum (x - \bar{x})^2}{n-1}$  or  $s^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}$   
 or  $\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}$

- Standard deviation:  $\sigma = \sqrt{\sigma^2}$

$s = \sqrt{s^2}$

- coefficient of variation:

$C.V = \frac{s}{\bar{x}} \times 100\%$

comprehensive example:

Sample 6: 5, 8, 10, 14, 20

$n = 5, \sum x = 57$

~ Mean =  $\frac{\sum x}{n} \rightarrow \frac{57}{5} = 11.4$

$\therefore \bar{x} = 11.4$

~ Mode = x

~ Median  $\rightarrow n$  is odd  $\rightarrow x(\frac{n+1}{2})$

$x(\frac{5+1}{2}) \rightarrow x(3) = 10$

~ Range = Max - Min

$20 - 5 = 15$

~ Variance

Way 1

| x  | x - $\bar{x}$   | (x - $\bar{x}$ ) <sup>2</sup> |
|----|-----------------|-------------------------------|
| 5  | 5 - 11.4 = -6.4 | (-6.4) <sup>2</sup> = 40.96   |
| 8  | -3.4            | 11.56                         |
| 10 | -1.4            | 1.96                          |
| 14 | 2.6             | 6.76                          |
| 20 | 8.6             | 73.96                         |
|    |                 | Tot = 135.2                   |

$s^2 = \frac{\sum (x - \bar{x})^2}{n-1}$

$= \frac{135.2}{4} = 33.8 \rightarrow S.D = \sqrt{s^2} = \sqrt{33.8} = 5.8$

Way 2

| x         | x <sup>2</sup> |
|-----------|----------------|
| 5         | 25             |
| 8         | 64             |
| 10        | 100            |
| 14        | 196            |
| 20        | 400            |
| Tot = 785 |                |

$s^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}$

$= \frac{785 - \frac{57^2}{5}}{4} = \frac{135.2}{4} = 33.8$

|                           |                                |
|---------------------------|--------------------------------|
| Count                     | 5                              |
| Sum                       | 57                             |
| Mean (Average)            | 11.4                           |
| Median                    | 10                             |
| Mode                      | All values appeared just once. |
| Largest                   | 20                             |
| Smallest                  | 5                              |
| Range                     | 15                             |
| Sample Standard Deviation | 5.8137767414995                |
| Sample Variance           | 33.8                           |

- **Variable** is a condition or characteristic that can take on different values.

- A **value** is just a number, such as 4, - 81, or 367.12. A value can also be a category (word), such as male or female, or a psychological diagnosis (major depressive disorder, post-traumatic stress disorder, schizophrenia).

An **outlier** is an observation of data that does not fit the rest of the data. An outlier is sometimes called an *extreme value*. When you graph an outlier, it will appear not to fit the pattern of the graph. Some outliers are due to **mistakes** (for example, writing down 50 instead of 500) while others may indicate that **something unusual is happening**.

The **interquartile range (IQR)** is the range of the middle 50% of the scores in a distribution and is sometimes used to communicate where the bulk of the data in the distribution are located. It is computed as follows: IQR = 75th percentile - 25th percentile.

| <u>Sample size (n)</u> | <u>Sample average</u> | <u>Population average</u> | <u>Difference between Sample &amp; Population</u> |
|------------------------|-----------------------|---------------------------|---|
| 1                      | 12                    | 8                         | 4   |
| 2                      | 15                    | 8                         | 7   |
| 5                      | 9.8                   | 8                         | 1.8   |
| 25                     | 9.5                   | 8                         | 1.5   |
| 250                    | 8.3                   | 8                         | 0.3   |
| 2500                   | 7.9                   | 8                         | 0.1   |

The larger the sample size, the more closely it represents the population.

- If data describe a **sample** it is called a **statistic**.
- If data describe a **population** it is called a **parameter**.

→ Type of Quantitative data (Weight, Height, Temperature...)

- Mean is preferred when using **ratio level data** unless distribution includes outliers
- Median is the preferred when using ordinal data
- Median is preferred when data include outliers
- Mode is preferred when using nominal data

**Extreme Scores.** Range is affected most by extreme scores or outliers but standard deviation and variance are also affected by extremes because they are based on squared deviations. One extreme score can have a disproportionate effect on the overall statistic or parameter.

**Sample size.** Increased sample size is associated with an increase in range because of the potential to increase or decrease values in a set of data.

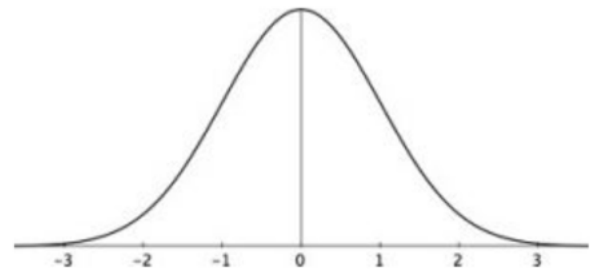
- 68% of all scores will fall between a Z score of -1.00 and +1.00
- 95% of all scores will fall between a Z score of -2.00 and +2.00
- 99.7% of all scores will fall between a Z score of -3.00 and +3.00
- 50% of all scores lie above/below a Z score of 0.00

Seven features of normal distributions are listed below.

- Normal distributions are symmetric around their mean.
- The mean, median, and mode of a normal distribution are equal.
- The area under the normal curve is equal to 1.0.
- Normal distributions are denser in the center and less dense in the tails.
- Normal distributions are defined by two parameters, the mean ( $\mu$ ) and the standard deviation ( $\sigma$ ).

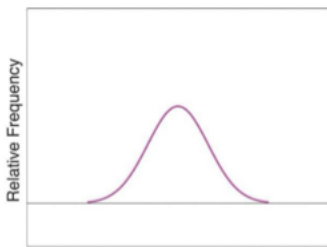
- 68% of the area of a normal distribution is within one standard deviation of the mean.
- Approximately 95% of the area of a normal distribution is within two standard deviations of the mean.

Z-scores & the standard normal distribution

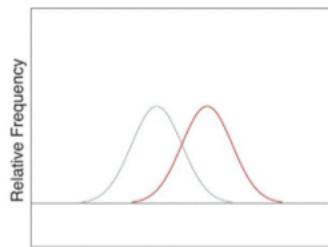


Z-score distribution

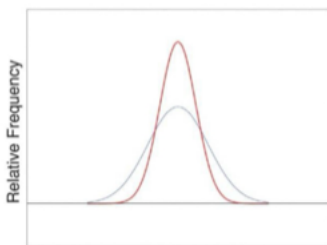
- If the Z score is negative, then the score falls **below** the mean
- If the Z score is 0, then the score falls **at** the mean
- If the Z score is positive, then the score falls **above** the mean



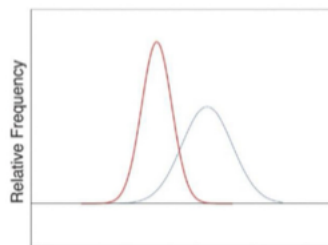
(a) Two Identical Sets



(b) Locations Differ



(c) Variabilities Differ



(d) Locations and Variabilities Differ

Given the standard normal distribution, find the area under the curve, above the  $z$ -axis between  $z = -\infty$  and  $z = 2$ .



$$P(Z < 2) = 0.9773$$

$$97.73\%$$

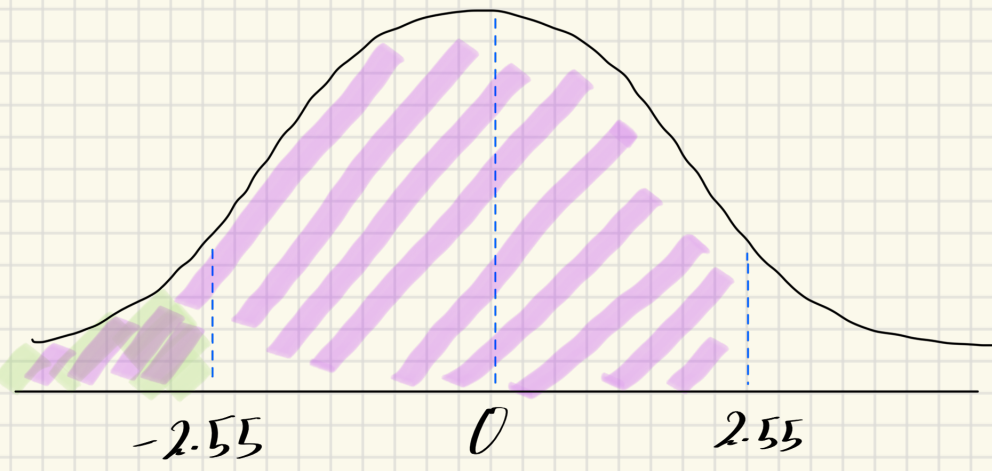
STANDARD STATISTICAL TABLES  
1. Areas under the Normal Distribution

The table gives the cumulative probability up to the standardized normal value  $z$ .  
i.e.  $P(Z \leq z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} \exp(-t^2/2) dt$



| z   | 0.00   | 0.01   | 0.02   | 0.03   | 0.04   | 0.05   | 0.06   | 0.07   | 0.08   | 0.09   |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7824 | 0.7854 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7968 | 0.7996 | 0.8025 | 0.8053 | 0.8081 | 0.8109 | 0.8137 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9773 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9865 | 0.9868 | 0.9871 | 0.9874 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9924 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9978 | 0.9979 | 0.9980 | 0.9981 | 0.9982 | 0.9983 |
| 2.9 | 0.9984 | 0.9985 | 0.9986 | 0.9987 | 0.9988 | 0.9989 | 0.9990 | 0.9991 | 0.9992 | 0.9993 |
| 3.0 | 0.9994 | 0.9995 | 0.9996 | 0.9997 | 0.9998 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 1.0000 |

What is the probability that a  $z$  picked at random from the population of  $z$ 's will have a value between  $-2.55$  and  $+2.55$ ?



$$\left. \begin{array}{l} P(Z < 2.55) \\ P(Z < -2.55) \end{array} \right\} P(-2.55 < Z < 2.55)$$

$$= ( ) - ( )$$

$$\rightarrow P(Z < 2.55) - (1 - P(Z < 2.55))$$

$$= 0.9946 - (1 - 0.9946)$$

$$= 0.9892$$

STANDARD STATISTICAL TABLES  
1. Areas under the Normal Distribution

The table gives the cumulative probability up to the standardized normal value  $z$ .  
i.e.  $P(Z \leq z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} \exp(-t^2/2) dt$



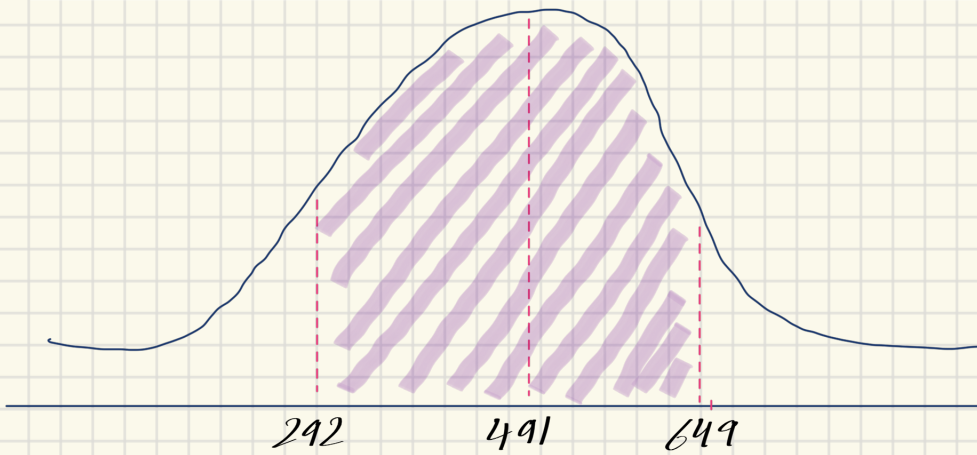
| z   | 0.00   | 0.01   | 0.02   | 0.03   | 0.04   | 0.05   | 0.06   | 0.07   | 0.08   | 0.09   |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7824 | 0.7854 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7968 | 0.7996 | 0.8025 | 0.8053 | 0.8081 | 0.8109 | 0.8137 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9773 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9865 | 0.9868 | 0.9871 | 0.9874 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9924 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9978 | 0.9979 | 0.9980 | 0.9981 | 0.9982 | 0.9983 |
| 2.9 | 0.9984 | 0.9985 | 0.9986 | 0.9987 | 0.9988 | 0.9989 | 0.9990 | 0.9991 | 0.9992 | 0.9993 |
| 3.0 | 0.9994 | 0.9995 | 0.9996 | 0.9997 | 0.9998 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 1.0000 |







Diskin et al. (A-11) studied common breath metabolites such as ammonia, acetone, isoprene, ethanol, and acetaldehyde in five subjects over a period of 30 days. Each day, breath samples were taken and analyzed in the early morning on arrival at the laboratory. For subject A, a 27-year-old female, the ammonia concentration in parts per billion (ppb) followed a normal distribution over 30 days with mean 491 and standard deviation 119. What is the probability that on a random day, the subject's ammonia concentration is between 292 and 649 ppb?



$$\frac{292 - 491}{119} = -1.67 \qquad \frac{649 - 491}{119} = 1.33 \qquad Z$$

$$P(292 < X < 649) = P(-1.67 < Z < 1.33)$$

$$P(Z < 1.33) - (1 - P(Z < 1.67))$$

$$0.9082 - (1 - 0.9525) = 0.8607$$

STANDARD STATISTICAL TABLES  
1. Areas under the Normal Distribution

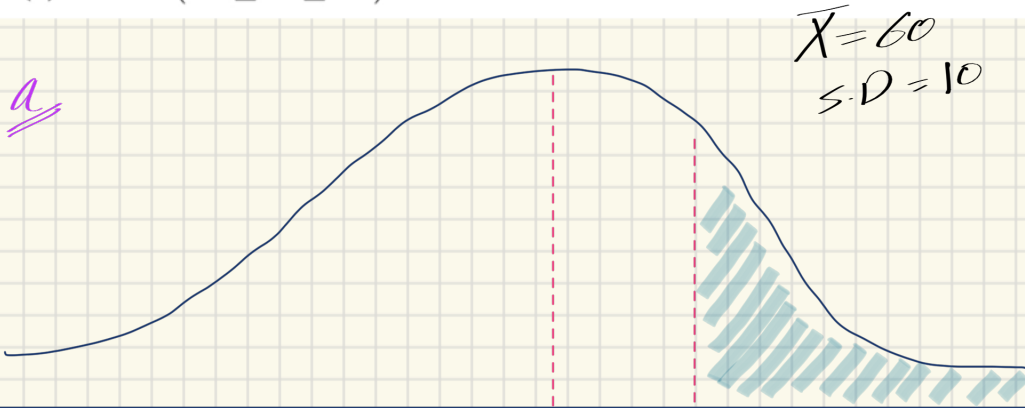
The table gives the cumulative probability up to the standardized normal value z.

$$P(Z \leq z) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^z \exp(-t^2/2) dt$$

| z   | 0.00   | 0.01   | 0.02   | 0.03   | 0.04   | 0.05   | 0.06   | 0.07   | 0.08   | 0.09   |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7421 | 0.7453 | 0.7484 | 0.7515 | 0.7546 |
| 0.7 | 0.7576 | 0.7607 | 0.7638 | 0.7668 | 0.7697 | 0.7726 | 0.7755 | 0.7784 | 0.7812 | 0.7841 |
| 0.8 | 0.7869 | 0.7896 | 0.7924 | 0.7952 | 0.7979 | 0.8006 | 0.8033 | 0.8059 | 0.8086 | 0.8112 |
| 0.9 | 0.8138 | 0.8164 | 0.8190 | 0.8216 | 0.8241 | 0.8267 | 0.8292 | 0.8317 | 0.8342 | 0.8367 |
| 1.0 | 0.8391 | 0.8413 | 0.8435 | 0.8456 | 0.8477 | 0.8497 | 0.8517 | 0.8537 | 0.8557 | 0.8576 |
| 1.1 | 0.8596 | 0.8615 | 0.8634 | 0.8653 | 0.8671 | 0.8689 | 0.8708 | 0.8726 | 0.8744 | 0.8762 |
| 1.2 | 0.8780 | 0.8798 | 0.8815 | 0.8832 | 0.8849 | 0.8866 | 0.8883 | 0.8899 | 0.8916 | 0.8932 |
| 1.3 | 0.8948 | 0.8964 | 0.8980 | 0.8995 | 0.9011 | 0.9026 | 0.9041 | 0.9056 | 0.9071 | 0.9086 |
| 1.4 | 0.9101 | 0.9115 | 0.9129 | 0.9143 | 0.9157 | 0.9171 | 0.9184 | 0.9197 | 0.9211 | 0.9224 |
| 1.5 | 0.9236 | 0.9249 | 0.9261 | 0.9274 | 0.9286 | 0.9298 | 0.9310 | 0.9321 | 0.9332 | 0.9344 |
| 1.6 | 0.9354 | 0.9365 | 0.9376 | 0.9387 | 0.9398 | 0.9408 | 0.9418 | 0.9428 | 0.9437 | 0.9447 |
| 1.7 | 0.9456 | 0.9465 | 0.9474 | 0.9483 | 0.9492 | 0.9501 | 0.9510 | 0.9519 | 0.9527 | 0.9535 |
| 1.8 | 0.9543 | 0.9551 | 0.9559 | 0.9567 | 0.9575 | 0.9583 | 0.9591 | 0.9599 | 0.9606 | 0.9613 |
| 1.9 | 0.9621 | 0.9628 | 0.9635 | 0.9643 | 0.9650 | 0.9657 | 0.9664 | 0.9671 | 0.9678 | 0.9685 |
| 2.0 | 0.9691 | 0.9698 | 0.9704 | 0.9711 | 0.9717 | 0.9724 | 0.9730 | 0.9736 | 0.9742 | 0.9747 |
| 2.1 | 0.9753 | 0.9759 | 0.9764 | 0.9770 | 0.9775 | 0.9780 | 0.9785 | 0.9790 | 0.9795 | 0.9799 |
| 2.2 | 0.9804 | 0.9808 | 0.9813 | 0.9817 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 |
| 2.3 | 0.9846 | 0.9850 | 0.9854 | 0.9857 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 |
| 2.4 | 0.9881 | 0.9884 | 0.9887 | 0.9890 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9903 | 0.9905 |
| 2.5 | 0.9907 | 0.9909 | 0.9911 | 0.9913 | 0.9915 | 0.9917 | 0.9918 | 0.9920 | 0.9921 | 0.9922 |
| 2.6 | 0.9923 | 0.9924 | 0.9925 | 0.9926 | 0.9927 | 0.9928 | 0.9929 | 0.9929 | 0.9930 | 0.9931 |
| 2.7 | 0.9931 | 0.9932 | 0.9932 | 0.9933 | 0.9933 | 0.9934 | 0.9934 | 0.9934 | 0.9935 | 0.9935 |
| 2.8 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 |
| 2.9 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 |
| 3.0 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 |

The IQs of individuals admitted to a state school for the mentally retarded are approximately normally distributed with a mean of 60 and a standard deviation of 10.

- (a) Find the proportion of individuals with IQs greater than 75.
- (b) What is the probability that an individual picked at random will have an IQ between 55 and 75?
- (c) Find  $P(50 \leq X \leq 70)$ .



STANDARD STATISTICAL TABLES  
1. Areas under the Normal Distribution

The table gives the cumulative probability up to the standardized normal value z.

$$P(Z \leq z) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^z \exp(-t^2/2) dt$$

| z   | 0.00   | 0.01   | 0.02   | 0.03   | 0.04   | 0.05   | 0.06   | 0.07   | 0.08   | 0.09   |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7421 | 0.7453 | 0.7484 | 0.7515 | 0.7546 |
| 0.7 | 0.7576 | 0.7607 | 0.7638 | 0.7668 | 0.7697 | 0.7726 | 0.7755 | 0.7784 | 0.7812 | 0.7841 |
| 0.8 | 0.7869 | 0.7896 | 0.7924 | 0.7952 | 0.7979 | 0.8006 | 0.8033 | 0.8059 | 0.8086 | 0.8112 |
| 0.9 | 0.8138 | 0.8164 | 0.8190 | 0.8216 | 0.8241 | 0.8267 | 0.8292 | 0.8317 | 0.8342 | 0.8367 |
| 1.0 | 0.8391 | 0.8413 | 0.8435 | 0.8456 | 0.8477 | 0.8497 | 0.8517 | 0.8537 | 0.8557 | 0.8576 |
| 1.1 | 0.8596 | 0.8615 | 0.8634 | 0.8653 | 0.8671 | 0.8689 | 0.8708 | 0.8726 | 0.8744 | 0.8762 |
| 1.2 | 0.8780 | 0.8798 | 0.8815 | 0.8832 | 0.8849 | 0.8866 | 0.8883 | 0.8899 | 0.8916 | 0.8932 |
| 1.3 | 0.8948 | 0.8964 | 0.8980 | 0.8995 | 0.9011 | 0.9026 | 0.9041 | 0.9056 | 0.9071 | 0.9086 |
| 1.4 | 0.9101 | 0.9115 | 0.9129 | 0.9143 | 0.9157 | 0.9171 | 0.9184 | 0.9197 | 0.9211 | 0.9224 |
| 1.5 | 0.9236 | 0.9249 | 0.9261 | 0.9274 | 0.9286 | 0.9298 | 0.9310 | 0.9321 | 0.9332 | 0.9344 |
| 1.6 | 0.9354 | 0.9365 | 0.9376 | 0.9387 | 0.9398 | 0.9408 | 0.9418 | 0.9428 | 0.9437 | 0.9447 |
| 1.7 | 0.9456 | 0.9465 | 0.9474 | 0.9483 | 0.9492 | 0.9501 | 0.9510 | 0.9519 | 0.9527 | 0.9535 |
| 1.8 | 0.9543 | 0.9551 | 0.9559 | 0.9567 | 0.9575 | 0.9583 | 0.9591 | 0.9599 | 0.9606 | 0.9613 |
| 1.9 | 0.9621 | 0.9628 | 0.9635 | 0.9643 | 0.9650 | 0.9657 | 0.9664 | 0.9671 | 0.9678 | 0.9685 |
| 2.0 | 0.9691 | 0.9698 | 0.9704 | 0.9711 | 0.9717 | 0.9724 | 0.9730 | 0.9736 | 0.9742 | 0.9747 |
| 2.1 | 0.9753 | 0.9759 | 0.9764 | 0.9770 | 0.9775 | 0.9780 | 0.9785 | 0.9790 | 0.9795 | 0.9799 |
| 2.2 | 0.9804 | 0.9808 | 0.9813 | 0.9817 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 |
| 2.3 | 0.9846 | 0.9850 | 0.9854 | 0.9857 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 |
| 2.4 | 0.9881 | 0.9884 | 0.9887 | 0.9890 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9903 | 0.9905 |
| 2.5 | 0.9907 | 0.9909 | 0.9911 | 0.9913 | 0.9915 | 0.9917 | 0.9918 | 0.9920 | 0.9921 | 0.9922 |
| 2.6 | 0.9923 | 0.9924 | 0.9925 | 0.9926 | 0.9927 | 0.9928 | 0.9929 | 0.9929 | 0.9930 | 0.9931 |
| 2.7 | 0.9931 | 0.9932 | 0.9932 | 0.9933 | 0.9933 | 0.9934 | 0.9934 | 0.9934 | 0.9935 | 0.9935 |
| 2.8 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 |
| 2.9 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 |
| 3.0 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 0.9935 |

$$Z = \frac{X - \bar{X}}{S.D}$$

$$Z = \frac{60 - 60}{10} = 0$$

$$Z = \frac{75 - 60}{10} = 1.5$$

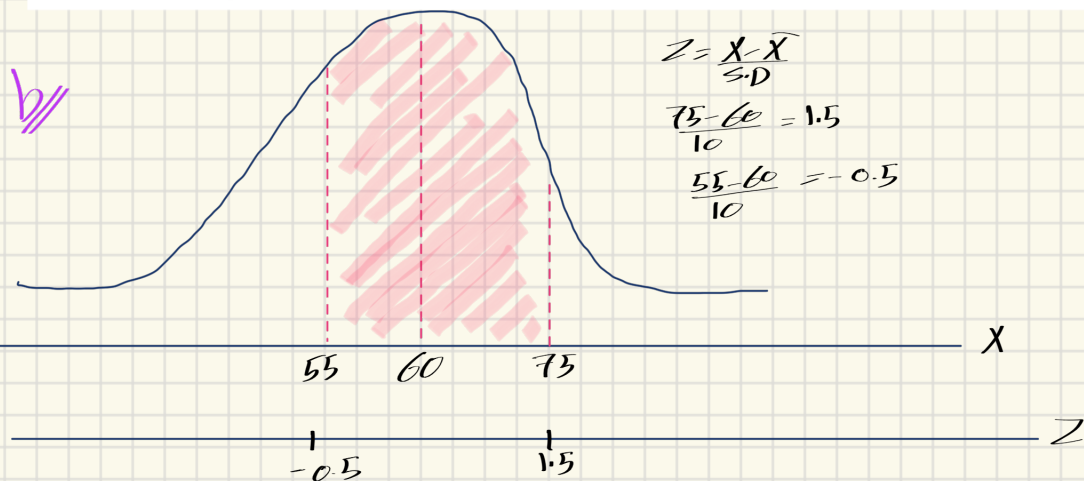
$$P(X > 75) = P(Z > 1.5)$$

$$\Rightarrow 1 - P(Z < 1.5)$$

$$1 - 0.9332 = 0.0668$$

The IQs of individuals admitted to a state school for the mentally retarded are approximately normally distributed with a mean of 60 and a standard deviation of 10.

- Find the proportion of individuals with IQs greater than 75.
- What is the probability that an individual picked at random will have an IQ between 55 and 75?
- Find  $P(50 \leq X \leq 70)$ .

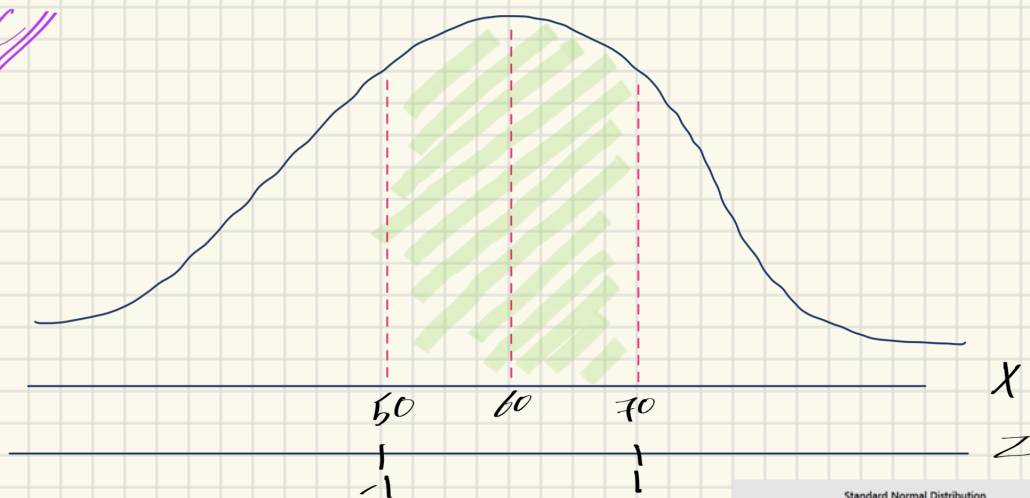


$$* P(55 < X < 75) \Rightarrow P(-0.5 < Z < 1.5)$$

$$P(Z < 1.5) - (1 - P(Z < 0.5))$$

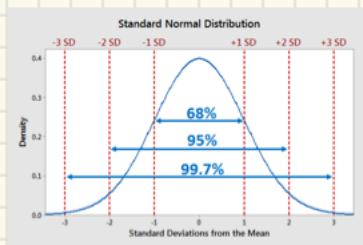
$$0.9332 - (1 - 0.6915)$$

$$= 0.6247$$



$$* P(-1 < Z < 1) = 68\%$$

↳ due to "Empirical Rule"  $\Rightarrow$



*or*

$$* P(50 \leq X \leq 70) = P(-1 \leq Z \leq 1)$$

$$P(Z \leq 1) - (1 - P(Z \leq 1))$$

$$0.8413 - (1 - 0.8413)$$

$$= 0.6826$$

Scores made on a certain aptitude test by nursing students are approximately normally distributed with a mean of 500 and a variance of 10,000.

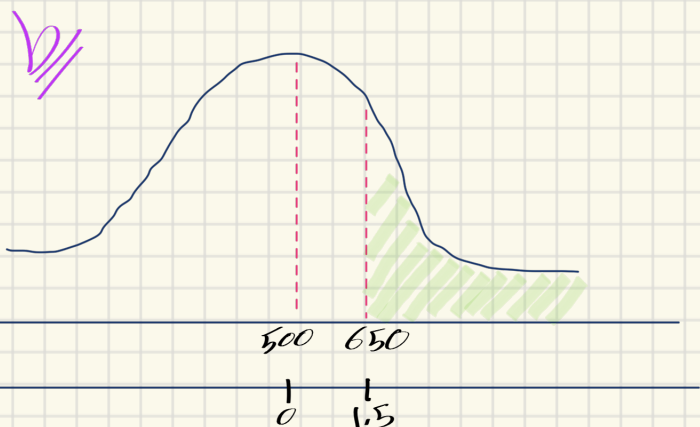
- (a) What proportion of those taking the test score below 200?
- (b) A person is about to take the test. What is the probability that he or she will make a score of 650 or more?
- (c) What proportion of scores fall between 350 and 675?



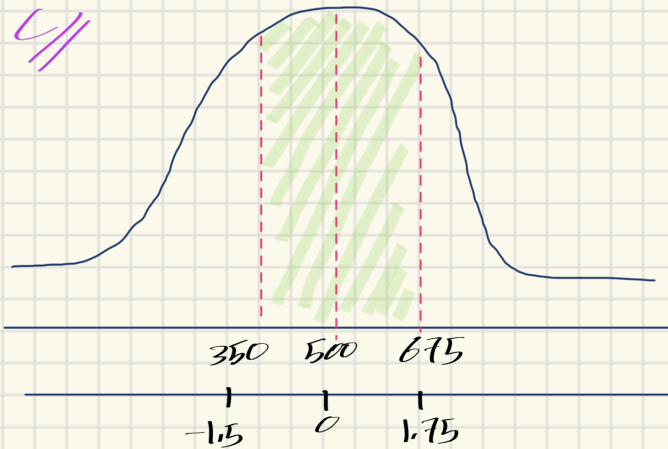
STANDARD STATISTICAL TABLES  
 1. Areas under the Normal Distribution  
 The table gives the cumulative probability  
 P(Z ≤ z) = ∫<sub>-∞</sub><sup>z</sup> (1/√2π) e<sup>-t<sup>2</sup>/2</sup> dt

| z   | 0.00   | 0.01   | 0.02   | 0.03   | 0.04   | 0.05   | 0.06   | 0.07   | 0.08   | 0.09   |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7020 | 0.7054 | 0.7088 | 0.7122 | 0.7156 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7421 | 0.7453 | 0.7484 | 0.7515 | 0.7546 |
| 0.7 | 0.7577 | 0.7607 | 0.7637 | 0.7667 | 0.7696 | 0.7725 | 0.7754 | 0.7782 | 0.7811 | 0.7839 |
| 0.8 | 0.7868 | 0.7896 | 0.7924 | 0.7952 | 0.7979 | 0.8006 | 0.8033 | 0.8060 | 0.8086 | 0.8113 |
| 0.9 | 0.8139 | 0.8166 | 0.8192 | 0.8218 | 0.8244 | 0.8269 | 0.8294 | 0.8319 | 0.8344 | 0.8369 |
| 1.0 | 0.8394 | 0.8419 | 0.8443 | 0.8468 | 0.8491 | 0.8514 | 0.8537 | 0.8559 | 0.8581 | 0.8603 |
| 1.1 | 0.8625 | 0.8646 | 0.8667 | 0.8688 | 0.8708 | 0.8728 | 0.8747 | 0.8767 | 0.8786 | 0.8805 |
| 1.2 | 0.8824 | 0.8843 | 0.8861 | 0.8879 | 0.8897 | 0.8915 | 0.8932 | 0.8949 | 0.8966 | 0.8983 |
| 1.3 | 0.8999 | 0.9015 | 0.9032 | 0.9049 | 0.9064 | 0.9079 | 0.9094 | 0.9109 | 0.9124 | 0.9139 |
| 1.4 | 0.9154 | 0.9169 | 0.9183 | 0.9197 | 0.9211 | 0.9225 | 0.9238 | 0.9251 | 0.9264 | 0.9277 |
| 1.5 | 0.9290 | 0.9302 | 0.9314 | 0.9326 | 0.9337 | 0.9348 | 0.9359 | 0.9369 | 0.9379 | 0.9389 |
| 1.6 | 0.9398 | 0.9407 | 0.9416 | 0.9425 | 0.9433 | 0.9441 | 0.9449 | 0.9456 | 0.9464 | 0.9471 |
| 1.7 | 0.9478 | 0.9484 | 0.9490 | 0.9496 | 0.9501 | 0.9506 | 0.9511 | 0.9516 | 0.9520 | 0.9525 |
| 1.8 | 0.9529 | 0.9533 | 0.9537 | 0.9541 | 0.9545 | 0.9549 | 0.9553 | 0.9556 | 0.9560 | 0.9564 |
| 1.9 | 0.9567 | 0.9570 | 0.9573 | 0.9576 | 0.9579 | 0.9582 | 0.9585 | 0.9588 | 0.9591 | 0.9594 |
| 2.0 | 0.9596 | 0.9599 | 0.9601 | 0.9604 | 0.9606 | 0.9608 | 0.9610 | 0.9612 | 0.9614 | 0.9615 |
| 2.1 | 0.9617 | 0.9618 | 0.9619 | 0.9621 | 0.9622 | 0.9623 | 0.9624 | 0.9625 | 0.9626 | 0.9627 |
| 2.2 | 0.9628 | 0.9628 | 0.9629 | 0.9629 | 0.9630 | 0.9630 | 0.9631 | 0.9631 | 0.9632 | 0.9632 |
| 2.3 | 0.9633 | 0.9633 | 0.9634 | 0.9634 | 0.9634 | 0.9635 | 0.9635 | 0.9635 | 0.9635 | 0.9636 |
| 2.4 | 0.9636 | 0.9636 | 0.9636 | 0.9636 | 0.9636 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 |
| 2.5 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 |
| 2.6 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 |
| 2.7 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 |
| 2.8 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 |
| 2.9 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 |
| 3.0 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 | 0.9637 |

$p(X < 200) \rightarrow p(Z < -3)$   
 $1 - p(Z < 3) \rightarrow 1 - 0.9986$   
 $= 0.0014$



$p(X \geq 650) = p(Z \geq 1.5)$   
 $1 - p(Z \leq 1.5)$   
 $1 - 0.9332 = 0.0668$



$p(350 < X < 675) = p(-1.5 < Z < 1.75)$   
 $p(Z < 1.75) - (1 - p(Z < 1.5))$   
 $0.9599 - (1 - 0.9332)$   
 $= 0.8931$

A nurse supervisor has found that staff nurses, on the average, complete a certain task in 10 minutes. If the times required to complete the task are approximately normally distributed with a standard deviation of 3 minutes, find:

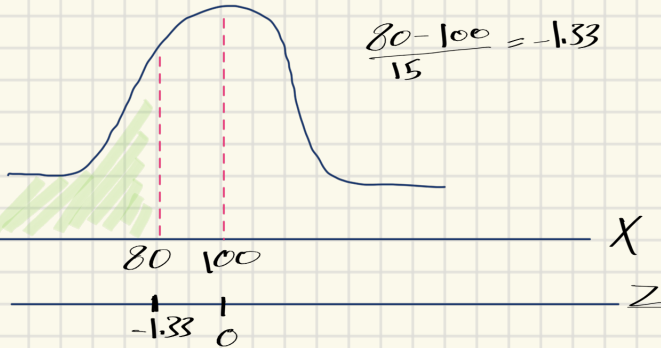
- (a) The proportion of nurses completing the task in less than 4 minutes
- (b) The proportion of nurses requiring more than 5 minutes to complete the task
- (c) The probability that a nurse who has just been assigned the task will complete it within 3 minutes

As shown previously!

Normally distributed IQ scores have a mean of 100 and a standard deviation of 15. Use the standard z-table to answer the following questions:  
 What is the probability of randomly selecting someone with an IQ score that is (a) less than 80? (b) greater than 136? (c) between 95 and 110?  
 (d) What IQ score corresponds to the 90th percentile? (e) The middle 30% of IQs fall between what two values?

$$\bar{X} = 100, s.p = 15$$

a/  $P(X < 80)$



$$P(X < 80) = P(Z < -1.33)$$

$$= 1 - P(Z < 1.33)$$

$$1 - 0.9082 = 0.0918$$

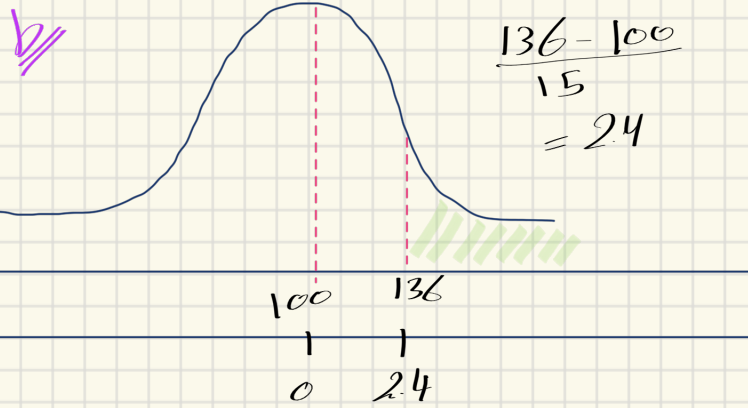
$$\Rightarrow 9.18\%$$

STANDARD STATISTICAL TABLES  
 1. Areas under the Normal Distribution

The table gives the cumulative probability up to the standardized normal value z

$$P(Z \leq z) = \int_{-\infty}^z \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) dx$$

| z    | 0.00   | 0.01   | 0.02   | 0.03   | 0.04   | 0.05   | 0.06   | 0.07   | 0.08   | 0.09   |
|------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0  | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5159 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1  | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2  | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3  | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4  | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5  | 0.6915 | 0.6950 | 0.6985 | 0.7020 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6  | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7  | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7854 |
| 0.8  | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9  | 0.8159 | 0.8186 | 0.8213 | 0.8238 | 0.8264 | 0.8289 | 0.8314 | 0.8339 | 0.8364 | 0.8389 |
| 1.0  | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1  | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2  | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3  | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4  | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5  | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6  | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9494 | 0.9504 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7  | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8  | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9685 | 0.9692 | 0.9699 | 0.9706 |
| 1.9  | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0  | 0.9773 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1  | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2  | 0.9861 | 0.9865 | 0.9868 | 0.9871 | 0.9874 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3  | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4  | 0.9918 | 0.9920 | 0.9922 | 0.9924 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5  | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6  | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9958 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 |
| 2.7  | 0.9964 | 0.9965 | 0.9966 | 0.9967 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 |
| 2.8  | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9980 | 0.9980 | 0.9981 |
| 2.9  | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0  | 0.9986 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.00 | 0.9986 | 0.9990 | 0.9993 | 0.9995 | 0.9997 | 0.9998 | 0.9998 | 0.9999 | 0.9999 | 1.0000 |

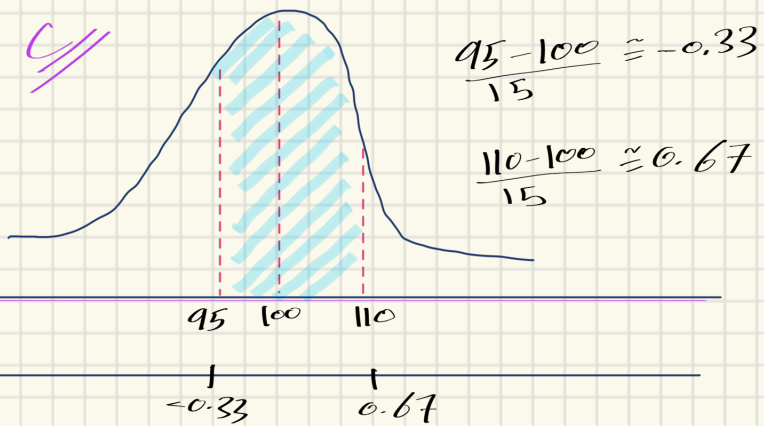


$$P(X > 136) = P(Z > 2.4)$$

$$1 - P(Z < 2.4)$$

$$1 - 0.9918 = 0.0082$$

$$\Rightarrow 0.82\%$$



$$P(95 < X < 110) = P(-0.33 < Z < 0.67)$$

$$P(Z < 0.67) - (1 - P(Z < 0.33))$$

$$0.7486 - (1 - 0.6293)$$

$$= 0.3779$$

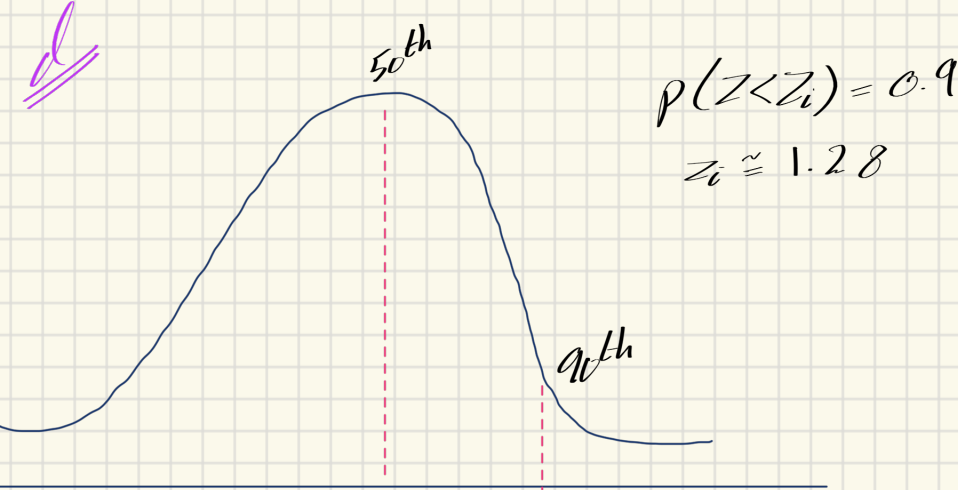
$$\Rightarrow 37.79\%$$

STANDARD STATISTICAL TABLES  
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$$P(Z \leq z) = \int_{-\infty}^z \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) dx$$

| z    | 0.00   | 0.01   | 0.02   | 0.03   | 0.04   | 0.05   | 0.06   | 0.07   | 0.08   | 0.09   |
|------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0  | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5159 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1  | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2  | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3  | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4  | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5  | 0.6915 | 0.6950 | 0.6985 | 0.7020 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6  | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7  | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7854 |
| 0.8  | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9  | 0.8159 | 0.8186 | 0.8213 | 0.8238 | 0.8264 | 0.8289 | 0.8314 | 0.8339 | 0.8364 | 0.8389 |
| 1.0  | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1  | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2  | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3  | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4  | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5  | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6  | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9494 | 0.9504 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7  | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8  | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9685 | 0.9692 | 0.9699 | 0.9706 |
| 1.9  | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0  | 0.9773 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1  | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2  | 0.9861 | 0.9865 | 0.9868 | 0.9871 | 0.9874 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3  | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4  | 0.9918 | 0.9920 | 0.9922 | 0.9924 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5  | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6  | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9958 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 |
| 2.7  | 0.9964 | 0.9965 | 0.9966 | 0.9967 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 |
| 2.8  | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9980 | 0.9980 | 0.9981 |
| 2.9  | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0  | 0.9986 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.00 | 0.9986 | 0.9990 | 0.9993 | 0.9995 | 0.9997 | 0.9998 | 0.9998 | 0.9999 | 0.9999 | 1.0000 |



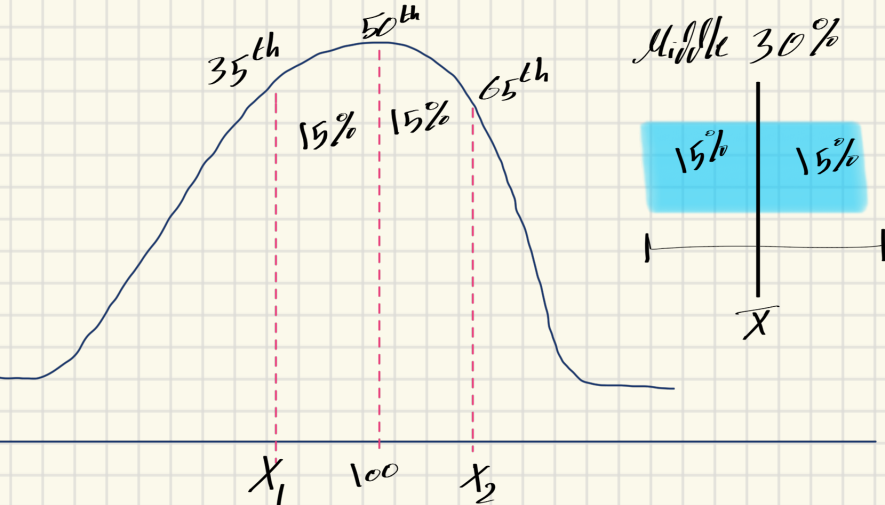
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| z   | 0.00   | 0.01   | 0.02   | 0.03   | 0.04   | 0.05   | 0.06   | 0.07   | 0.08   | 0.09   |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5159 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7421 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7824 | 0.7854 |
| 0.8 | 0.7883 | 0.7913 | 0.7942 | 0.7970 | 0.7998 | 0.8026 | 0.8054 | 0.8081 | 0.8109 | 0.8136 |
| 0.9 | 0.8163 | 0.8190 | 0.8217 | 0.8244 | 0.8271 | 0.8298 | 0.8324 | 0.8351 | 0.8377 | 0.8403 |
| 1.0 | 0.8438 | 0.8463 | 0.8488 | 0.8513 | 0.8538 | 0.8562 | 0.8587 | 0.8611 | 0.8635 | 0.8659 |
| 1.1 | 0.8683 | 0.8706 | 0.8729 | 0.8752 | 0.8774 | 0.8796 | 0.8818 | 0.8839 | 0.8860 | 0.8881 |
| 1.2 | 0.8900 | 0.8920 | 0.8939 | 0.8958 | 0.8976 | 0.8994 | 0.9012 | 0.9029 | 0.9046 | 0.9063 |
| 1.3 | 0.9079 | 0.9096 | 0.9112 | 0.9128 | 0.9144 | 0.9159 | 0.9174 | 0.9189 | 0.9204 | 0.9219 |
| 1.4 | 0.9232 | 0.9246 | 0.9259 | 0.9272 | 0.9284 | 0.9296 | 0.9308 | 0.9319 | 0.9330 | 0.9341 |
| 1.5 | 0.9352 | 0.9362 | 0.9372 | 0.9381 | 0.9390 | 0.9398 | 0.9406 | 0.9414 | 0.9422 | 0.9429 |
| 1.6 | 0.9436 | 0.9443 | 0.9450 | 0.9457 | 0.9463 | 0.9469 | 0.9475 | 0.9480 | 0.9485 | 0.9490 |
| 1.7 | 0.9495 | 0.9499 | 0.9504 | 0.9508 | 0.9512 | 0.9516 | 0.9520 | 0.9523 | 0.9527 | 0.9530 |
| 1.8 | 0.9534 | 0.9537 | 0.9540 | 0.9543 | 0.9546 | 0.9549 | 0.9552 | 0.9554 | 0.9557 | 0.9559 |
| 1.9 | 0.9561 | 0.9563 | 0.9565 | 0.9567 | 0.9569 | 0.9571 | 0.9572 | 0.9574 | 0.9575 | 0.9576 |
| 2.0 | 0.9577 | 0.9578 | 0.9579 | 0.9580 | 0.9581 | 0.9582 | 0.9583 | 0.9584 | 0.9584 | 0.9585 |
| 2.1 | 0.9585 | 0.9586 | 0.9586 | 0.9587 | 0.9587 | 0.9588 | 0.9588 | 0.9589 | 0.9589 | 0.9589 |
| 2.2 | 0.9590 | 0.9590 | 0.9590 | 0.9591 | 0.9591 | 0.9591 | 0.9592 | 0.9592 | 0.9592 | 0.9592 |
| 2.3 | 0.9593 | 0.9593 | 0.9593 | 0.9594 | 0.9594 | 0.9594 | 0.9594 | 0.9594 | 0.9594 | 0.9594 |
| 2.4 | 0.9594 | 0.9594 | 0.9594 | 0.9594 | 0.9594 | 0.9594 | 0.9594 | 0.9594 | 0.9594 | 0.9594 |
| 2.5 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 |
| 2.6 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 |
| 2.7 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 |
| 2.8 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 |
| 2.9 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 |
| 3.0 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 |

$z = 1.28$        $1.29$   
 $0.8997 < 0.9 < 0.9015$   
 $\Delta = 0.0003$        $\Delta = 0.0015$

$Z = \frac{X - \bar{X}}{s.D} \rightarrow 1.28 = \frac{X - 100}{15}$   
 $X = 119.2$   
 IQ score



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| z   | 0.00   | 0.01   | 0.02   | 0.03   | 0.04   | 0.05   | 0.06   | 0.07   | 0.08   | 0.09   |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5159 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7421 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7824 | 0.7854 |
| 0.8 | 0.7883 | 0.7913 | 0.7942 | 0.7970 | 0.7998 | 0.8026 | 0.8054 | 0.8081 | 0.8109 | 0.8136 |
| 0.9 | 0.8163 | 0.8190 | 0.8217 | 0.8244 | 0.8271 | 0.8298 | 0.8324 | 0.8351 | 0.8377 | 0.8403 |
| 1.0 | 0.8438 | 0.8463 | 0.8488 | 0.8513 | 0.8538 | 0.8562 | 0.8587 | 0.8611 | 0.8635 | 0.8659 |
| 1.1 | 0.8683 | 0.8706 | 0.8729 | 0.8752 | 0.8774 | 0.8796 | 0.8818 | 0.8839 | 0.8860 | 0.8881 |
| 1.2 | 0.8900 | 0.8920 | 0.8939 | 0.8958 | 0.8976 | 0.8994 | 0.9012 | 0.9029 | 0.9046 | 0.9063 |
| 1.3 | 0.9079 | 0.9096 | 0.9112 | 0.9128 | 0.9144 | 0.9159 | 0.9174 | 0.9189 | 0.9204 | 0.9219 |
| 1.4 | 0.9232 | 0.9246 | 0.9259 | 0.9272 | 0.9284 | 0.9296 | 0.9308 | 0.9319 | 0.9330 | 0.9341 |
| 1.5 | 0.9352 | 0.9362 | 0.9372 | 0.9381 | 0.9390 | 0.9398 | 0.9406 | 0.9414 | 0.9422 | 0.9429 |
| 1.6 | 0.9436 | 0.9443 | 0.9450 | 0.9457 | 0.9463 | 0.9469 | 0.9475 | 0.9480 | 0.9485 | 0.9490 |
| 1.7 | 0.9495 | 0.9499 | 0.9504 | 0.9508 | 0.9512 | 0.9516 | 0.9519 | 0.9523 | 0.9527 | 0.9530 |
| 1.8 | 0.9534 | 0.9537 | 0.9540 | 0.9543 | 0.9546 | 0.9549 | 0.9552 | 0.9554 | 0.9557 | 0.9559 |
| 1.9 | 0.9561 | 0.9563 | 0.9565 | 0.9567 | 0.9569 | 0.9571 | 0.9572 | 0.9574 | 0.9575 | 0.9576 |
| 2.0 | 0.9577 | 0.9578 | 0.9579 | 0.9580 | 0.9581 | 0.9582 | 0.9583 | 0.9584 | 0.9584 | 0.9585 |
| 2.1 | 0.9585 | 0.9586 | 0.9586 | 0.9587 | 0.9587 | 0.9588 | 0.9588 | 0.9589 | 0.9589 | 0.9589 |
| 2.2 | 0.9590 | 0.9590 | 0.9590 | 0.9591 | 0.9591 | 0.9591 | 0.9592 | 0.9592 | 0.9592 | 0.9592 |
| 2.3 | 0.9593 | 0.9593 | 0.9593 | 0.9594 | 0.9594 | 0.9594 | 0.9594 | 0.9594 | 0.9594 | 0.9594 |
| 2.4 | 0.9594 | 0.9594 | 0.9594 | 0.9594 | 0.9594 | 0.9594 | 0.9594 | 0.9594 | 0.9594 | 0.9594 |
| 2.5 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 |
| 2.6 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 |
| 2.7 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 |
| 2.8 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 |
| 2.9 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 |
| 3.0 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 | 0.9595 |

$z = 0.38$        $0.39$   
 $0.6480 < 0.65 < 0.6517$   
 $\Delta = 0.0020$        $\Delta = 0.0017$   
 $2 \times 10^{-3}$        $1.7 \times 10^{-3}$   
 almost same difference, so  
 average is taken  
 $0.38 \text{ --- } 0.39$   
 $= 0.385$

$X_2 = ?$   
 $p(Z < z_i) = 0.65$   
 $z_i = 0.385$

As  $\rightarrow$   
 $p(Z < z_i) + p(Z < -z_i) = 1$   
 $0.65 + 0.35 = 1$   
 $z_i = 0.385$   
 $-z_i = -0.385$

$Z = \frac{X - \bar{X}}{s.D}$   
 $0.385 = \frac{X - 100}{15}$   
 $X_2 = 105.775$

$X_1 = ?$   
 $-0.385 = \frac{X - 100}{15}$   
 $X = 94.225$

| Percentile | z-Score |
|------------|---------|
| 62         | 0.305   |
| 63         | 0.332   |
| 64         | 0.358   |
| 65         | 0.385   |

The type of data that categorizes humans as males or females is known as:

- A. Random data
- B. Ordinal data
- C. Nominal data
- D. Interval data

*c. Nominal data*

For a set of data classified as Strongly Agree, Agree, or Disagree, this is an example of which type of data?

- A. Ordinal
- B. Nominal
- C. Interval
- D. Continuous

*A. ordinal*

An exam scores for 10 students are recorded as 75, 82, 90, 92, 67, 95, 110, 80, 82, 86. Find the mean, median, and mode in order.

- A. 85.9, 84, 82
- B. 86, 86, 86
- C. 86, 84, —
- D. 85.9, 84, 84

*f*  
*Mean = 85.9*  
*Median = 84*  
*Mode = 82*

Which of the following represents continuous data?

- a. Height of children
- b. Number of languages a person speaks
- c. Number of cigarettes smoked per day by a person

*b. height of a children*

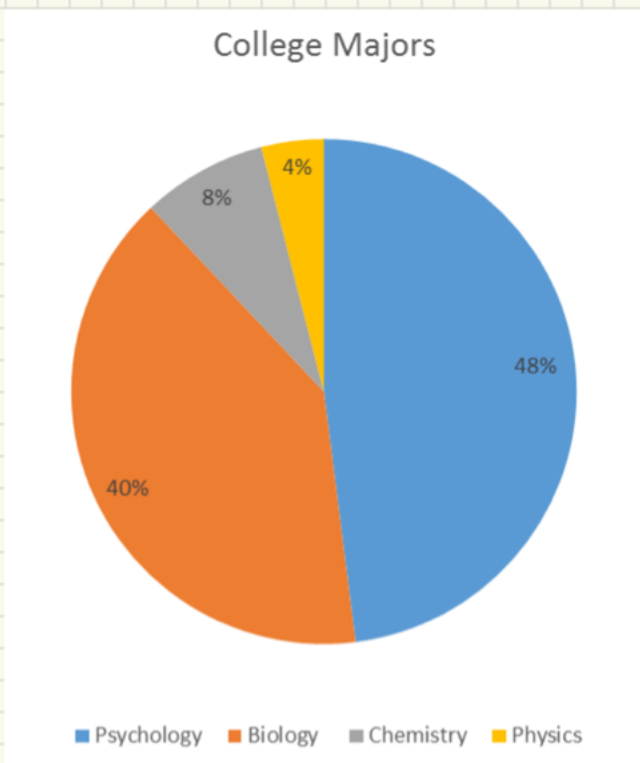
For each of the following, determine the level of measurement:

- 1. T-shirt size → *ordinal*
- 2. Time taken to run 100 meter race → *continuous*
- 3. First, second, and third place in 100 meter race → *ordinal*
- 4. Birthplace → *nominal*
- 5. Temperature in Celsius → *continuous*

Explain the differences between bar charts and histograms. When would each be used

In bar charts, the bars do not touch; in histograms, the bars do touch. Bar charts are appropriate for qualitative variables, whereas histograms are better for quantitative variables.

Based on the pie chart below, which was made from a sample of 300 students, construct a frequency table of college majors.



$$\frac{\text{frequency of each major}}{\text{total number}} \times 100\% = \text{each percentage}$$

$$\bullet \text{ psychology} \rightarrow \frac{X}{300} \times 100\% = 48\%$$

$$X = 144$$

$$\bullet \text{ biology} \rightarrow \frac{X}{300} \times 100\% = 40\%$$

$$X = 120$$

$$\bullet \text{ chemistry} \rightarrow \frac{X}{300} \times 100\% = 8\%$$

$$X = 24$$

$$\bullet \text{ physics} \rightarrow \frac{X}{300} \times 100\% = 4\%$$

$$X = 12$$

| Major      | Freq |
|------------|------|
| Psychology | 144  |
| Biology    | 120  |
| Chemistry  | 24   |
| Physics    | 12   |

If the mean time to respond to a stimulus is much *higher* than the median time to respond, what can you say about the shape of the distribution of response times?

If the mean is higher, that means it is farther out into the right-hand tail of the distribution. Therefore, we know this distribution is positively skewed



Your younger brother comes home one day after taking a science test. He says that some- one at school told him that "60% of the students in the class scored above the median test grade." What is wrong with this statement? What if he had said "60% of the students scored above the mean?"

The median is defined as the value with 50% of scores above it and 50% of scores below it; therefore, 60% of score cannot fall above the median. If 60% of scores fall above the mean, that would indicate that the mean has been pulled down below the value of the median, which means that the distribution is negatively skewed

Two normal distributions have exactly the same mean, but one has a standard deviation of 20 and the other has a standard deviation of 10. How would the shapes of the two distributions compare?

If both distributions are normal, then they are both symmetrical, and having the same mean causes them to overlap with one another. The distribution with the standard deviation of 10 will be narrower than the other distribution

Assume the following 5 scores represent a sample: 2, 3, 5, 5, 6. Transform these scores into z-scores.

$$N = 5, X = 2, 3, 5, 5, 6, \bar{X} = 4.2, S.D = 1.64$$

|      |                              |                              |                             |                            |
|------|------------------------------|------------------------------|-----------------------------|----------------------------|
| $X:$ | 2                            | 3                            | 5                           | 6                          |
| $Z:$ |                              |                              |                             |                            |
|      | $\frac{2-4.2}{1.64} = -1.34$ | $\frac{3-4.2}{1.64} = -0.73$ | $\frac{5-4.2}{1.64} = 0.49$ | $\frac{6-4.2}{1.64} = 1.1$ |

For a distribution with a standard deviation of 20, find z-scores that correspond to:

1. One-half of a standard deviation below the mean
2. 5 points above the mean
3. Three standard deviations above the mean
4. 22 points below the mean

$X \rightarrow$  Value in question

$\bar{X} \rightarrow$  Mean

$SD \rightarrow$  Standard Deviation

1.)



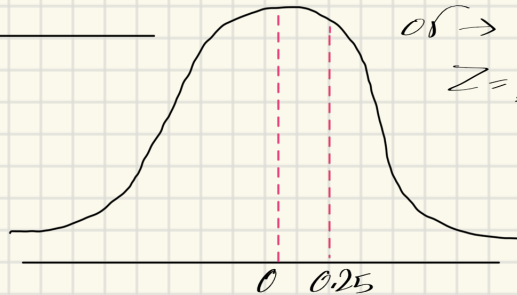
$\rightarrow -0.5$   
 one half of SD below the mean  
 one SD below the mean

or  $X = \bar{X} - 0.5SD$   
 $Z = \frac{X - \bar{X}}{SD} \rightarrow \frac{\bar{X} - 0.5SD - \bar{X}}{SD} = -0.5$

2.)  $X - \bar{X} = 5$  points

$$Z = \frac{X - \bar{X}}{SD} = \frac{5}{20} = 0.25$$

$$Z = 0.25$$



or  $X = \bar{X} + 5$

$$Z = \frac{X - \bar{X}}{SD} \rightarrow \frac{\bar{X} + 5 - \bar{X}}{SD} = \frac{5}{20} = 0.25$$

3.)

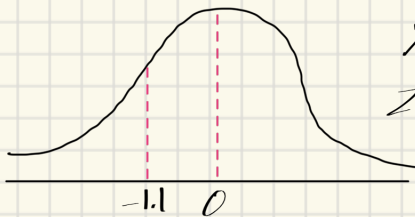


$$Z = 3$$

or  $X = \bar{X} + 3SD$

$$Z = \frac{X - \bar{X}}{SD} \rightarrow \frac{\bar{X} + 3SD - \bar{X}}{SD} = 3$$

4.)



$$X - \bar{X} = -22$$

$$Z = \frac{-22}{20} = -1.1$$

or

$$X = \bar{X} - 22$$

$$Z = \frac{X - \bar{X}}{SD} \rightarrow \frac{\bar{X} - 22 - \bar{X}}{20} = -1.1$$



A nutritional research team followed serum levels of vitamin B12 in 120 children for three years to determine the association between cyanocobalamin deficiency and the subsequent risk of developing Megaloblastic anemia. The results were as follows:

VITAMIN B12 LEVELS      Mean 260 pg/mL      Median 226 pg/mL      Mode 194 pg/mL

From the data, it can be concluded that this distribution is:

- a. Normal      b. Negatively      c. Positively skewed      d. Bimodal      e. Multimodal

**Key: True: c**

$$\bar{X} = 260, Md = 226, Mo = 194$$

$$\text{Mean} > \text{Median} > \text{Mode}$$



In a descriptive study the mean is 220 and the standard error is 10, the 95 confidence limits would be:

- a. 210 to 230      b. 215 to 225      c. 200 to 240      d. 220 to 230      e. 205 to 235

**Key: True: c**

$$CI = \bar{X} \pm (Z)(SE)$$

$$220 \pm 1.96(10)$$

200.4 - 239.6 → Rounding to the nearest whole numbers, the 95% confidence limits are approximately [200 - 240]

The birth weights in a hospital are to be presented in a graph. This is best done by a:

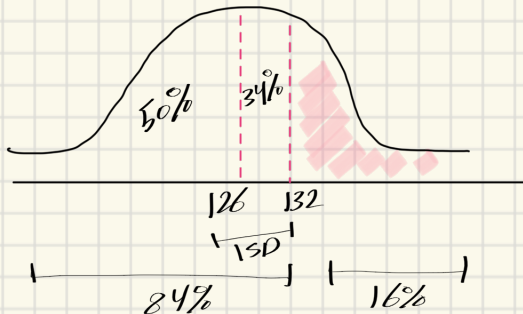
- a. Bar diagram      b. Pie chart      c. Histogram      d. Pictogram      e. Frequency chart

**Key: True: c**

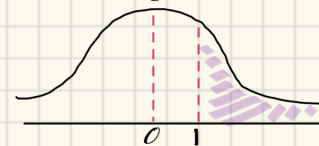
In a class of 134 medical students, the mean systolic blood pressure was found to be 126 mm Hg with a standard deviation of 6 mm Hg. If the blood pressures in this sample are normally distributed, what portion of the medical students will have systolic blood pressures above 132 mm Hg?

- a. 0.5%      b. 2.5%      c. 5%      d. 16%      e. 32%

**Key: True: d**



$$\text{or } Z = \frac{X - \bar{X}}{SD} \rightarrow \frac{132 - 126}{6} = 1$$



$$\begin{aligned} &\rightarrow 1 - p(Z < 1) \\ &= 1 - 0.8413 \\ &= 0.1587 \end{aligned}$$

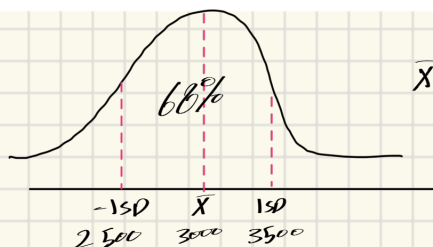
If, in one of the groups of premature infants, the maximum value for hexosaminidase A was substituted with a much higher value. The value which is unlikely to be affected by this higher value is:

- a. Variance      b. Range      c. Standard deviation      d. Median      e. Mean

**Key: True: d**

Birth rates of a population of infants at 40 weeks gestational age are approximately normally distributed, with a mean of 3000 grams. Roughly 68% of such infants weigh between 2500 and 3500 at birth. If a sample of 100 infants was studied, the standard error would be

- a. 50      b. 100      c. 200      d. 250      e. 500      **Key: True: a**



$$\bar{X} + 1SD = 3500$$

$$SD = 500$$

$$S.E = \frac{S.D}{\sqrt{n}} \rightarrow \frac{500}{\sqrt{100}} = 50$$

There are 50 individual in population and they have same hemoglobin level that is 14g/dL. As there is no variability, the standard deviation will be:

- a. 0      b. 1, -1      c. 0, 1      d. +2      e. -2      **Key: True: a**

At endocrinology unit, serum calcium levels of 200 patients of hyperparathyroidism were checked. The mean serum calcium level was 14mg/dl with a variance of 0.5. The 95% confidence interval would be:

- a) 13.2-15.2      b) 13.5-14.5      c) 13.5-15.5      d) 13.7-14.3      e) 13.9-14.1      **Key: e**

$n = 200, \bar{X} = 14, \sigma^2 = 0.5, \sigma = 0.71$   
 $95\% \rightarrow Z = 1.96$   
 $C.I = \bar{X} \pm (Z) \left( \frac{\sigma}{\sqrt{n}} \right)$   
 $= 14 \pm (1.96) \left( \frac{0.71}{\sqrt{200}} \right) \rightarrow 13.9 - 14.1$

Which of the following can have more than one value?

- a. The mean      b. The range      c. The mode      d. The median      e. Standard deviation.

**Key: True: c**

A study was conducted to assess the heights of 30 students. By chance all of the students were found to be of the same height. The standard deviation of this study sample is:

- a. Zero      b. 0 -- -1      c. 0 -- +1      d. 0 -- +2      e. 0 -- -2      **Key: True: a**

A normal distribution curve is based mainly on:

- a. Mean and sample size      b. Mean and standard deviation  
 c. Range and sample size      d. Range and standard deviation  
 e. Mean and range

**Key: True: b**

A study was conducted to assess the height of students of 4th year in 10 Medical colleges the values of heights ranged between 5.5 – 5.10 feet. A histogram has been selected by the researcher to present these results as it is a:

- a. Nominal data      b. Categorical data      c. Both qualitative and quantitative data  
 d. Continuous data      e. Discrete numerical data

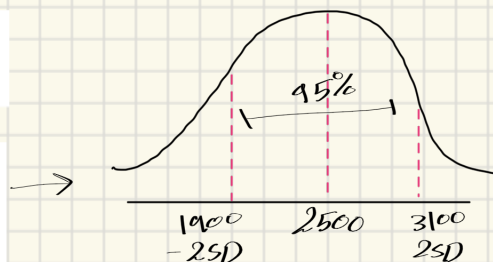
**Key: True: d**

After arranging the data in ascending or descending order of magnitude, the value of middle observation:

- a. Mean      b. Mode      c. Median      d. Geometric mean      e. Mean deviation      **Key: True: c**

Birth rates of a population of infants at 36 weeks gestational age are approximately normally distributed, with a mean of 2500 grams. Roughly 95% of such infants weigh between 1900 and 3100 grams at birth. If a sample of 225 infants was studied the standard error would be:

- a. 10      b. 20      c. 30      d. 40      e. 50      **Key: True: b**



$$2500 + 2SD = 3100$$

$$SD = \frac{600}{2} = 300$$

$$S.E = \frac{S.D}{\sqrt{n}} = \frac{300}{\sqrt{225}} = 20$$

The distribution of height of the girls in a University was plotted. The most frequent value was five feet and two inches, while mean height was five feet and eight inches. This show:

- a. Negative skewness      b. Positive skewness      c. Normal distribution  
 d. Large standard deviation      e. Multimodal distribution

**Key: True: b**

When a relationship between the heart rate and valsalva's ratio is studied, mean is useful but dispersion of the data is also very useful. Which method of spread will be more useful in this?

- a. Range      b. Standard deviation      c. Coefficient of variance      d. Percentage  
 e. Inter quartile range

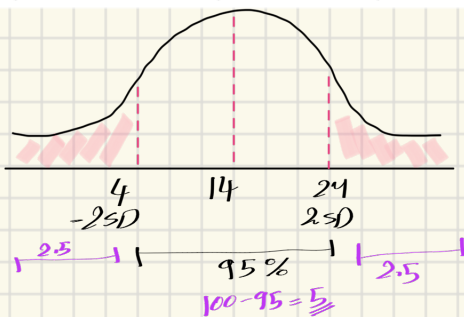
**Key: True: c**

$$\left( \frac{SD}{\bar{X}} \right) \times 100\%$$

In a study on 100 pregnant ladies to determine the average weight gain in pregnancy the mean of the sample was 14kg with a SD of 5kg. If the data is normally distributed the %age of the women that will have weight beyond the range of 4-24kg

- a) 68%      b) 34%      c) 27%      d) 13.5%      e) 5%

Key: e



$$\bar{x} = 14, \quad SD = 5$$

Since the Z-scores for the range 4-24 kg are  $\pm 2$ , approximately 95% of the data falls within this range. Therefore, the percentage of data beyond this range is:

$$100\% - 95\% = 5\%$$

The cardiologist expresses the amount of edema in CCF patients in terms of absent, mild, moderate and severe. The statistician will describe this data as:

- a) Nominal      b) Ordinal      c) Dichotomous      d) Continuous      e) Discrete

key: b

When the data reveals a mean less than the median less than the mode it can be described as being?

- Normally distributed (2%)  
Skewed to the right (20%)

- Skewed to the left (74%)  
Positively Skewed (4%)

**Correct Answer:** Skewed to the left 74% of people

**Comment:**

- Being "skewed to the left" is the same as saying the data is "negatively skewed". To clarify further, when they say skewed to the left, this means the mean is less than the median and/or the tail of the curve is on the left side of the graph.
- Thus, the opposite is true, if the data is skewed to the right the data is called positively skewed. Here the mean is now greater than the median (acting as an outlier) and thus skews the data to the right (note the tail of the curve is on the right of the graph)
- You should be able to quickly look at a graph like this and be able to interpret it any of the above explanations (or listed answer choices).

Which of the following best describes properties of confidence intervals?

- A. Describe the variability in the sample.       $CI = \bar{x} \pm (z) \left( \frac{SD}{\sqrt{n}} \right)$   
 B. Cannot be used in hypothesis testing.  
 C. Provide the same information as p values.       $\therefore \frac{SD}{\sqrt{n}} = SE$   
 D. Are calculated based on the standard error of the mean.

**Answer is D**

Which measure(s) of central tendency is/are sensitive to outliers?

- (A) Mean      (B) Median      (C) Mode

**Answer is A**

- > Mean is the correct answer because it is affected by outliers. Median and mode are incorrect because they are not affected by outliers.

Answer true or false for the following statements:

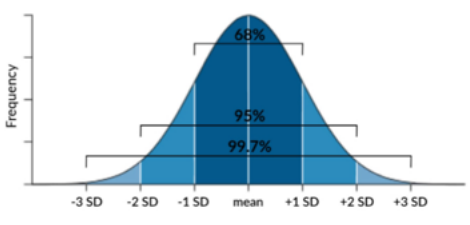
The 95% confidence interval for the mean:

- a. Contains the sample mean with 95% certainty.
- b. Is less likely to contain the population mean than the 99% confidence interval.
- c. Contains 95% of the observations in the population.
- d. Is approximately equal to the sample mean plus and minus two standard deviations
- e. Can be used to give an indication of whether the sample mean is a precise estimate of the population mean.

Answers:

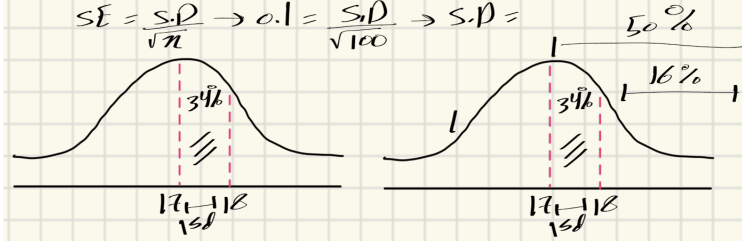
- a. False: it contains the population mean with 95% certainty. It always contains the sample mean.
- b. True
- c. False: In repeated samples, around 95% of the 95% confidence intervals (CI) will contain the population mean. Another way to think about 95% CI is if the same study were repeated 100 times then the mean of 95 of these 100 studies would lie somewhere within the 95% CI.
- d. False: is approximately equal to  $\bar{x}$  two standard errors about the sample mean.
- e. True (Narrow confidence intervals indicate the sample mean is a precise estimate.)

An investigator is studying the frequency of polycythemia in a population of a remote, mountainous region. A representative sample of 100 men shows a normal distribution of hemoglobin concentration with a mean concentration of 17 g/dL and a standard error of 0.1 g/dL. Which of the following best represents the probability that a subject will have a hemoglobin concentration greater than 18 g/dL?

|   | Answer | Image   |
|---|--------|---|
| A | 30%    |   |
| B | 15%    |  |
| C | 95%    |   |
| D | 99%    |   |
| E | 5%     |   |
| F | 70%    |   |

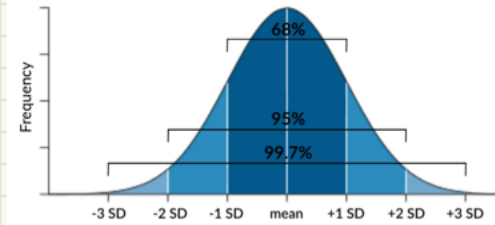
$$n = 100, \bar{X} = 17, SE = 0.1$$

$$SE = \frac{SD}{\sqrt{n}} \rightarrow 0.1 = \frac{SD}{\sqrt{100}} \rightarrow SD = 1$$



B - 15%

Image



Explanation Why

In a normal distribution, one standard deviation above and below the mean covers approximately 68% of the sample. With the standard deviation for this population at 1 g/dL, 68% of the population will fall between 16 g/dL and 18 g/dL. The remaining 32% of the population will fall above or below one standard deviation of the mean: half (16%) would fall under 16 g/dL, while the other half (16%) would be greater than 18 g/dL. 15% is the closest approximation to the probability that a subject has a hemoglobin concentration greater than 18 g/dL.

A pulmonologist is analyzing the vital signs of patients with chronic obstructive pulmonary disease (COPD) who presented to an emergency room with respiratory distress and subsequently required intubation. The respiratory rates of 7 patients with COPD during their initial visit to the emergency room are shown:

|           |                       |
|-----------|-----------------------|
| Patient 1 | 22 breaths per minute |
| Patient 2 | 32 breaths per minute |
| Patient 3 | 23 breaths per minute |
| Patient 4 | 30 breaths per minute |
| Patient 5 | 32 breaths per minute |
| Patient 6 | 32 breaths per minute |
| Patient 7 | 23 breaths per minute |

Which of the following is the mode of these respiratory rates?


|   | Answer                  | Image |
|---|-------------------------|-------|
| A | 30 breaths per minute   |       |
| B | 32 breaths per minute   |       |
| C | 10 breaths per minute   |       |
| D | 27.7 breaths per minute |       |

B - 32 breaths per minute


Explanation Why

32 breaths per minute is the mode (i.e., the most common value) of this set of values because it appears three times, whereas the other values appear only once or twice. The mode is most resistant to outliers, the mean is least resistant to them, and the median lies somewhere in between.




 Nationality is an example of what level of measurement?

(A) ordinal (B) nominal (C) ratio (D) interval

Use the following frequency distribution to answer questions 


|              |       |       |       |       |       |       |
|--------------|-------|-------|-------|-------|-------|-------|
| Class Limits | 40-49 | 50-59 | 60-69 | 70-79 | 80-89 | Total |
| Frequency    | 3     | 4     | 6     | 5     | 2     | 20    |

 The number of classes is .....

(A) 3 (B) 6 (C) 4 (D) 5

 What is the width of the class 60 -69?

(A) 15 (B) 10 (C) 20 (D) 5

 The value of the range is .....

(A) 50 (B) 45 (C) 40 (D) 55

Q8. When data are categorized as Saudi, Egyptian, Syrian, and Sudanese, the most appropriate measure of central tendency is the .....

(A) mean (B) median (C) mode (D) none

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References;

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