If The Mean Of The Total Population Is Unknown:

So we would choose two samples from the community and compare between the two arithmetic means of these two samples, and here we have t-test for comparison between two sample means.

Example:
If we want to know whether English men are taller than Egyptian men? In this case we choose two samples.
Sample1: 100 English men
Sample 2: 100 Egyptian men

Then measure the height of all men and calculate the arithmetic mean and standard deviation for each sample. Then do t-test for comparison between these two arithmetic means.

Sample I: n_1 $\overline{X}_1$ **S_1**

Sample II: n_2 $\overline{X}_2$ **S_2**

Here we should calculate only one measure of dispersion estimated from the two samples and it is called pooled variance denoted (S^2_p).

$$S^2p = \frac{S^2{}_1(n_1-1)+S^2{}_2(n_2-1)}{n_1+n_2-2}$$

$$t = \frac{\overline{x}_1-\overline{x}_2}{\sqrt{\dfrac{S^2p}{n_1}+\dfrac{S^2p}{n_2}}}$$

H_0: No difference between the heights of the two groups.

H_1: English men are taller than Egyptian men.

$n_1 = 100$, $\overline{X}_1$ = 169 cm	S_1 = 12 cm	
$n_2 = 100$, $\overline{X}_2$ = 165 cm	S_2 = 10 cm	

$$S^2p = \frac{S^2{}_1(n_1 - 1) + S^2{}_2(n_2 - 1)}{n_1 + n_2 - 2}$$

$$= \frac{144 \times 99 + 100 \times 99}{100 + 100 - 2} = 122$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{S^2p}{n_1} + \dfrac{S^2p}{n_2}}}$$

$$= \frac{169 - 165}{\sqrt{\dfrac{122}{100} + \dfrac{122}{100}}} = 2.5607$$

The critical value (t^o) at 5% level of significance and d.f ($n_1 + n_2 - 2$) = 198 is 1.96. Since 2.5607 > 1.96 we accept H_1, i.e.: English men are significantly taller than Egyptian men.

Dependent *t*-test for paired samples

This test is used when the samples are dependent; that is, when there is only one sample that has been tested twice (repeated measures) or when there are two samples that have been matched or "paired". This is an example of a paired difference test.

$$t = \frac{\overline{X}_D - \mu_0}{s_D/\sqrt{n}}.$$

For this equation, the differences between all pairs must be calculated. The pairs are either one person's pre-test and post-test scores or between pairs of persons matched into meaningful groups (for instance drawn from the same family or age group). The average (X_D) and standard deviation (s_D) of those differences are used in the equation. The constant μ_0 is non-zero if you want to test whether the average of the difference is significantly different from μ_0. The degree of freedom used is $n - 1$.

Example: fluoride varnish study

In ten at-risk children, fluoride varnish is applied in randomly assigned half-mouths. The remaining half-mouths are left untreated. The children are followed for two years and the new dmfs and locations are recorded:

patient	varnish	untreated	difference
1	2	3	-1
2	1	2	-1
3	0	1	-1
4	2	0	2
5	0	0	0
6	0	2	-2
7	2	5	-3
8	1	1	0
9	3	7	-4
10	5	4	1
mean	1.6	2.5	-0.90
sd			1.79

To perform the paired t-test, compute a one-sample t-test on the last column where H_0: $\mu = 0$.

$$T = \frac{-.90 - 0}{1.79/\sqrt{10}} = -1.59$$

For a two-tailed test compare $|-1.59|=1.59$ to $t_{9, .975} = 2.262$. We do not reject since $1.59 < 2.262$. P-value is

$$P(|t_9| > |-1.59|) = 2 \times P(t_9 > 1.59) = 0.15.$$

Chi-square (X²) test

It is used when we want to study whether there is a relation between certain condition and certain characteristic.

Examples:
1. Relation between smoking and lung cancer.
2. Relation between consanguinity and congenital anomalies.
3. Relation between occurrence of breast cancer and presence of family history.
4. Relation between diabetes and prolonged healing of wounds.
5. Relation between the technique of vaccination (scratch or multiple pressure) and the success of vaccination.

So to do the test, first we should make the table, and in this case. the type of table is a contingency table or an association table or two by two table.
N.B.:
Chi-square test must be calculated from the absolute observed and expected numbers and not from percentages of other proportions.

Let us take an example which is the technique of vaccination and successful of vaccination and the following table illustrates the example:

Result	Technique		Total
	Scratch	Multiple pressure	
Successful	(1) 66	(3) 85	151
Unsuccessful	(2) 34	(4) 15	49
Total	100	100	200

The problem here is to determine whether or not there is any significant difference in the success rate of the two methods of vaccination. The success rate is 66% (66 children out of 100) for the scratch technique and 85% (85 children out of 100) for the multiple pressure technique. Is this difference just due to chance

or is the success rate significantly higher when the multiple pressure technique is used? (critical value = 3.84)

Null hypothesis (H_0): No difference between the success rates of the two techniques.

Alternative hypothesis (H_1): There is a significant difference.

First we have to calculate the expected frequencies (E) for each cell. These are the frequencies to be expected if there were no differences between the two techniques and E for each cell

$$E = \frac{\text{Total of column} \times \text{total of row of that cell}}{\text{Grand total}}$$

The actual or observed (O) frequencies and the expected (E) frequencies in each cell are then compared by a measure called X^2 (chi square) and this is called the calculated X^2. If O and E of each cell are equal; the value of X^2 will be zero. The greater the difference between O and E, the greater the value of X^2. The value on this calculated X^2 is compared with the critical value of X^2 using a statistical table of the X^2 distribution and our comparison must depend on:

One)Degree of freedom = (c - 1) $\times$ (r - 1) and here in this example
 = (2 - 1) $\times$ (2 - 1) = 1 $\times$ 1 = 1.

Two)Level of significance and usually = 5% (critical value of X^2 by d.f = 1 at 5% level of significance = 3.84. If the calculated X^2 > critical value of X^2, so we accept H_1 and reject H_0, if calculated X^2 < critical value of X^2 we accept H_0 and reject H_1.

$$E_1 = \frac{100 \times 151}{200} = 75.5$$

$$E_2 = \frac{100 \times 49}{200} = 24.5$$

$$E_3 = \frac{100 \times 151}{200} = 75.5$$

$$E_4 = \frac{100 \times 49}{200} = 24.5$$

$$X^2 = \sum \frac{(O - E)^2}{E}$$

$$= \frac{(O_1 - E_1)^2}{E_1} + \frac{(O_2 - E_2)^2}{E_2} + \frac{(O_3 - E_3)^2}{E_3} + \frac{(O_4 - E_4)^2}{E_4}$$

$$= \frac{(66-75.5)^2}{75.5} + \frac{(34-24.5)^2}{24.5} + \frac{(85-75.2)^2}{75.5} + \frac{(15-24.5)^2}{24.5}$$

$$= \frac{(9.5)^2}{75.5} + \frac{(9.5)^2}{24.5} + \frac{(9.5)^2}{75.5} + \frac{(9.5)^2}{24.5}$$

= 9.7581

Since calculated X^2 (9.7581) > critical value (3.84) so success rate of the multiple pressure method is significantly greater than the scratch method.

Practice questions: Using the data in the following table to test if there is a relation between consanguinity and congenital anomalies.

Consanguinity	Congenital anomalies		Total
	Yes	No	
Yes	30	10	40
No	20	40	60
Total	50	50	100

Critical value at 5% level of significance and 1 d.f = 3.84.
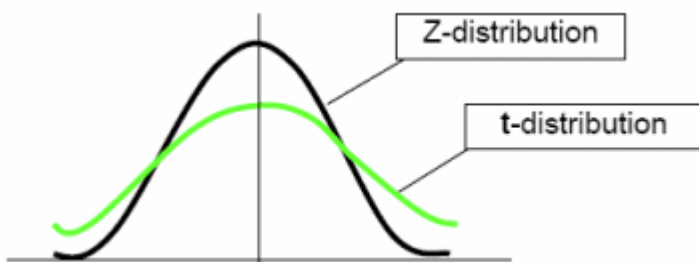
Again, **the t-distribution approaches the normal distribution as n approaches infinity.**

2. Statistical tests for Continuous data:

2.4. Testing continuous, non-normal data

1.   All of the above tests assume that your data are normally or approximately normally distributed, or your sample size is large enough to apply the properties of the central limit theorem. But sometimes your data are not normal and your sample size is relatively small. You can try to mathematically transform the data into a normal

distribution (for example by taking the square root, or the logarithm of all the values). If you can make them normal, you can use the t-tests or ANOVA.

2.   If the data are still not normally distributed, we use a different class of tests known as "non-parametric" tests, i.e. the Mann Whitney U test. These tests are based on the ranking or ordering of the data, rather than their numerical values.

2.5. Statistical Test for Nominal Data:

Categorical or nominal data is usually tested with the Chi-square test statistic. Here's an example: **Null hypothesis**: Cigarette use does not affect the risk of lung cancer in men; or Proportion of smokers who get lung Ca = Proportion of nonsmokers who get lung Ca. **Alternative hypothesis**: The two proportions are not equal (two-sided test). **Set alpha** = 0.05. **Study Design**: 20-year cohort study of 210 men, ages 30-50 living. After 20 years, we OBSERVE:

	Lung Cancer	**No Cancer**	total
Smokers	25 **(A)**	75 **(B)**	100
Nonsmokers	17 **(C)**	93 **(D)**	110
total	42	168	210

•   Smokers and nonsmokers are the two groups being compared. The data of interest is the rate of lung cancer, which is a categorical variable (yes/no). This is a 2x2 table; it has 4 cells; each is arbitrarily named A-D. For categorical data, use a Chi-square test statistic:

$$X^2 = \sum \frac{(Observed-Expected)^2}{Expected}$$

•   We calculate **EXPECTED** values under the null hypothesis of no difference between the two groups using the following

(Column total * Row total)

•   **Expected for each cell = _____**

Grand total

	Lung Cancer	**No Cancer**	
Smokers	100 x 20% = 20	100 x 80% = 80	100
Nonsmokers	110 x 20% = 22	110 x 80% = 88	110
	42	168	210

Then, we can calculate the chi-square test statistic:

Cell	Observed	Expected	O-E	$(O-E)^2$	$(O-E)^2/E$
A	25	20	5	25	25/20 = 1.25
B	75	80	-5	25	25/80 = 0.31
C	17	22	-5	25	25/22 = 1.14
D	93	88	5	25	25/88 = 0.28
				X^2= 2.98	

- **Getting a p-value:** alculate the "degrees of freedom" (df) = (# rows - 1) * (# columns - 1), For example, a 2x2 table always has: (2 - 1) * (2 - 1)= 1*1 = 1 df. The probability has been calculated for seeing any particular chi-square value with any number of degrees of freedom by chance alone, under the chi-square distribution. These probabilities can be found in X^2 tables or computer programs. So, we look up the probability of getting this value of 2.98 (or one more extreme) with 1 degree of freedom by chance alone…**p=0.09**. **P>alpha, so we cannot reject our null hypothesis. Conclude**: The difference we observed between cigarette smokers and non-smokers in the rate of lung cancer could have occurred by chance alone.
Note: If there are too few data in a single cell of an r x c table (less than 5 observations per cell), the chi-square test is not accurate. You then need to use a special test, called the Fisher's Exact test.
3.   SUMMARY OF STATISTICAL TESTS:

CATEGORICAL DATA	Enough data	Too little data (<5 in a cell)
Any r x c table	Chi-square	Fisher's Exact
CONTINUOUS DATA	Normal (even if transformed to normal) or large n	Not normal: (non-parametric tests)
One (group) sample	1-sample t-test	Kolmogorov-Smirnov
Two samples	2-sample t-test	Mann-Whitney U or Rank Sum
Paired data	1-sample t-test on paired differences (paired t-test)	Wilcoxon Signed-Rank
Three or more samples	Analysis of variance (ANOVA)	Kruskal-Wallis

Practice questions: Using the data in the following table to test if there is a relation between consanguinity and congenital anomalies.

Consanguinity	Congenital anomalies		Total
	Yes	No	
Yes	30	10	40
No	20	40	60
Total	50	50	100

Critical value at 5% level of significance and 1 d.f = 3.84.