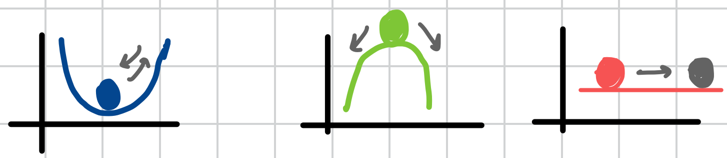
Chapter 9

Lecture 11.2
sections 9.4 - 9.5 - 9.6

• Stability and Balance



→ **stable equilibrium** : Object goes back to it's original position.

→ **unstable equilibrium** : Object moves further from it's original position.

→ **neutral equilibrium** : Object remains at it's new position.

• Elasticity

→ Applying External Force → deformation - change in size, shape

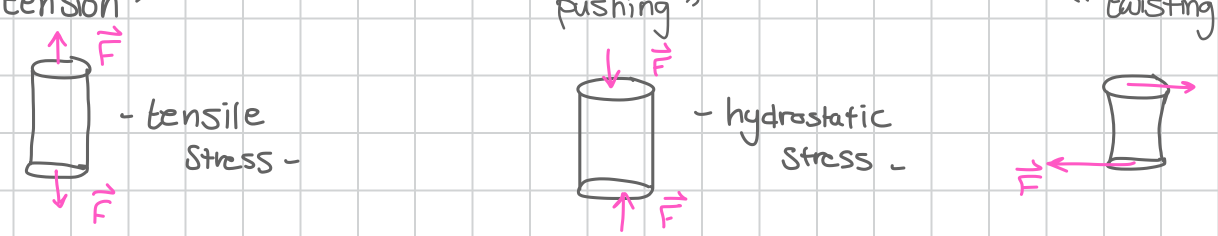
- ↳ small = After removing $\vec{F}$
  - • it return to it's original shape, size.
- ↳ past the **Elastic Limit**
  - • object remains permanently deformed
- ↳ way too beyond **Elastic Limit**.
  - • Break, fracture.

→ In Solids : 3 types of deformation.

Stretching — " tension " — tensile stress

Compression — " pushing " — hydrostatic stress

Shearing — " twisting " — sheering stress



→ All have in Common :

Stress —Produces→ **Strain**

Stress ↓ deformation $\vec{F}$ per unit Area

"Cause"

Strain ↓ unit deformation.

"Response" ⟶ newtons 2nd Law.

- **Stress = [Modulus of Elasticity] Strain.**
  - ↳ Applied as long as the Elastic Limit is not exceeded.
  - ↳ empirical Linear Relationship.

Modulus Of Elasticity

Young E

resistance For the change in **Length ΔL**

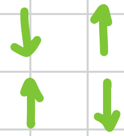
Bulk B

resistance For the change in **Volume ΔV**

Shear G

resistance to the **motion of planes** parallel to each other.

1. Changing The Length of a Solid

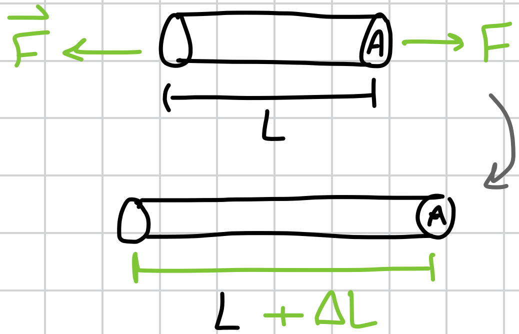


* Force is Applied to the End of an Object.

- Stretching

$$\frac{\vec{F}}{A} = E \, \frac{\Delta L}{L} \Rightarrow \Delta P = E \frac{\Delta L}{L}$$

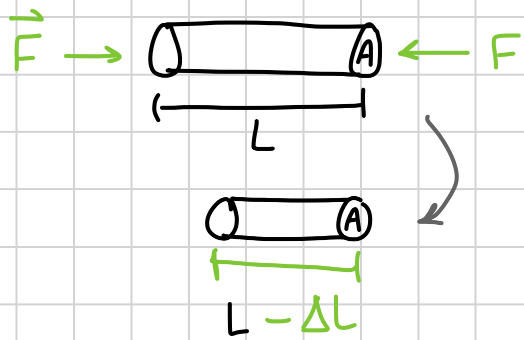
$\frac{\vec{F}}{A}$: Stress; E: Young's Modulus; $\frac{\Delta L}{L}$: Strain



- Compression

$$\frac{\vec{F}}{A} = - E \, \frac{\Delta L}{L} \Rightarrow \Delta P = - E \frac{\Delta L}{L}$$

$\frac{\vec{F}}{A}$: Stress; E: Young's Modulus. $\frac{\Delta L}{L}$: Strain



$$\Delta L = \frac{F \, L}{A \, E}$$

$F\,L$ → directly proportional $\Delta L \propto F, L$

$A\,E$ → inversely proportional $\Delta L \propto \frac{1}{E}, \frac{1}{A}$

E young's Modulus

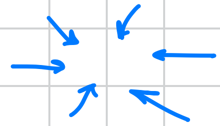
→ unit : $\frac{N}{m^2}$ → where $\frac{\Delta L}{L}$ Dimensionless.

→ material property → doesn't depend on size or shape

→ E values are large + slightly different for compression and stretching for some materials, Like human bones

$E_{stretching} > E_{compression}$ ⟿ $\Delta L_{Stretching} < \Delta L_{Compression}$

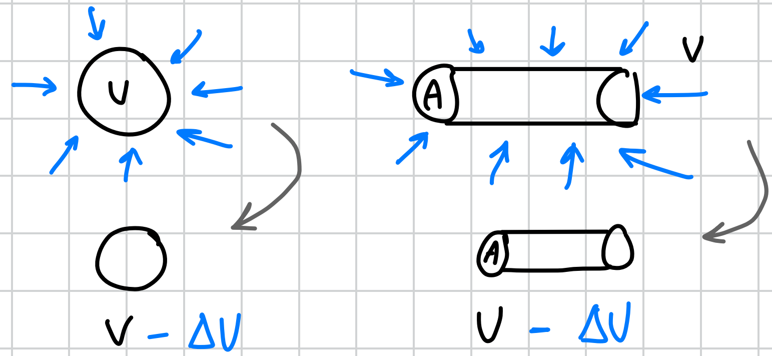
2. Changing the Volume of Solids



* Force is Applied to the Entire Surface Area of an object

$$\frac{F}{A} = -B \frac{\Delta V}{V} \Rightarrow \Delta P = -B \frac{\Delta V}{V}$$

- $\frac{F}{A}$: Stress
- B: Bulk Modulus
- $\frac{\Delta V}{V}$: Strain



* Volume decreases when pressure increases.

$$\Delta V = -\frac{F V}{A B}$$

- (-) = decrease in volume
- $F V$ → directly Proportional
- $A B$ → inversely proportional.

Bulks Modulus

→ unit : $\frac{N}{m^2}$ → $\frac{\Delta V}{V}$ dimensionless

→ Always (+)

→ Applies to Fluids & Solids → Fluids ⇒ Volume

→ $B_{gases} < B_{Liquids} < B_{solids}$. → gases are easily compressed

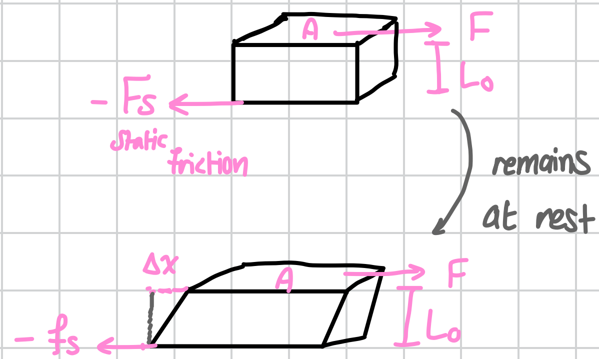
3. Changing the Shape of a Solid

* Force Applied to the top surface of an object

$$\frac{F}{A} = G \frac{\Delta x}{L_0} \Rightarrow \Delta P = G \frac{\Delta x}{L_0}$$

$\frac{F}{A}$ → Stress, G → Shear Modulus, $\frac{\Delta x}{L_0}$ → Strain

L_0 → thickness



* The object becomes Slanted by the amount Δx

$$\Delta x = \frac{L_0 F}{G A}$$

→ directly proportional (L_0, F)
→ inversely proportional (G, A)

· **G** Shear Modulus

→ unit: $\frac{N}{m^2}$ → $\frac{\Delta x}{L_0}$ dimensionless

→ Always +

→ G values are large → large Force causes Small Shear

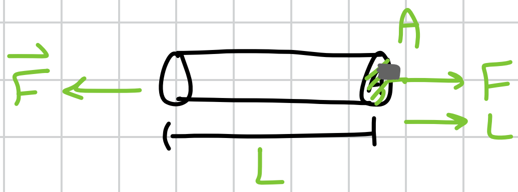
→ G < E For All solids → $G = \frac{1}{3} / \frac{1}{2} E$

Except the human bones → $G_{HB} > E_{HB}$

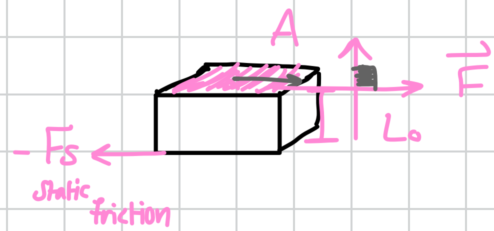
* resistance for fracture in shear is 5 times greater than in tension.

• Comparing Eq 1 & Eq. 3

$$\frac{F}{A} = E\frac{\Delta L}{L}$$



$$\frac{F}{A} = G\frac{\Delta x}{L_0}$$



• L ⇒ Length

→ measured in the direction of $\vec{F}$

• L_0 ⇒ thickness

→ measured in a direction Perpendicular to $\vec{F}$.

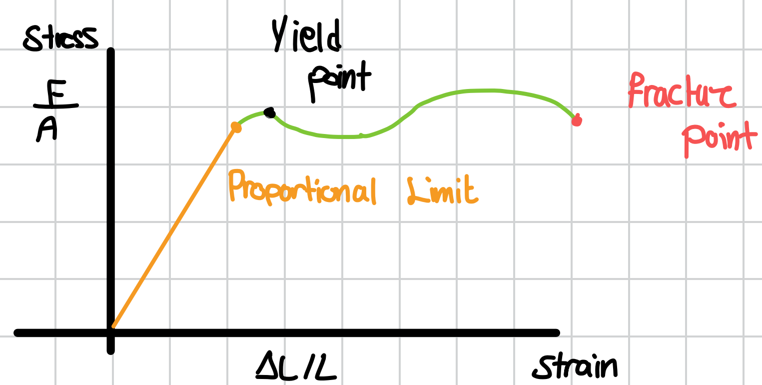
• A ⇒ cross sectional Area

→ is perpendicular to $\vec{F}$

• A ⇒ Area of the Surface

→ is parallel to $\vec{F}$

• Fracture



→ up to the proportional Limit

if the stress is removed the material returns to its original Length

deformation - respons vanishes.

- reversible deformation → elastic deformation

→ Yield point : stress causes sudden deformation without an increase in Force

→ from Proportional Limit to Fracture point

if the stress is removed the material will not return to its original Length

Permanently deformed and it enters Plastic region

→ Additional stress until the Fracture point

- Fracture Point : the material **breaks** or tears apart.

- Ultimate (breaking) stress : where the maximum elongation is reached

- Ultimate strength = tensile strength = maximum value the material Can with strand without breaking.

* it is necessary to maintain 3 to 10 Safety factor.

⇒ stresses should not exceed $\frac{1}{10}$ to $\frac{1}{3}$ of the ultimate Strengths.