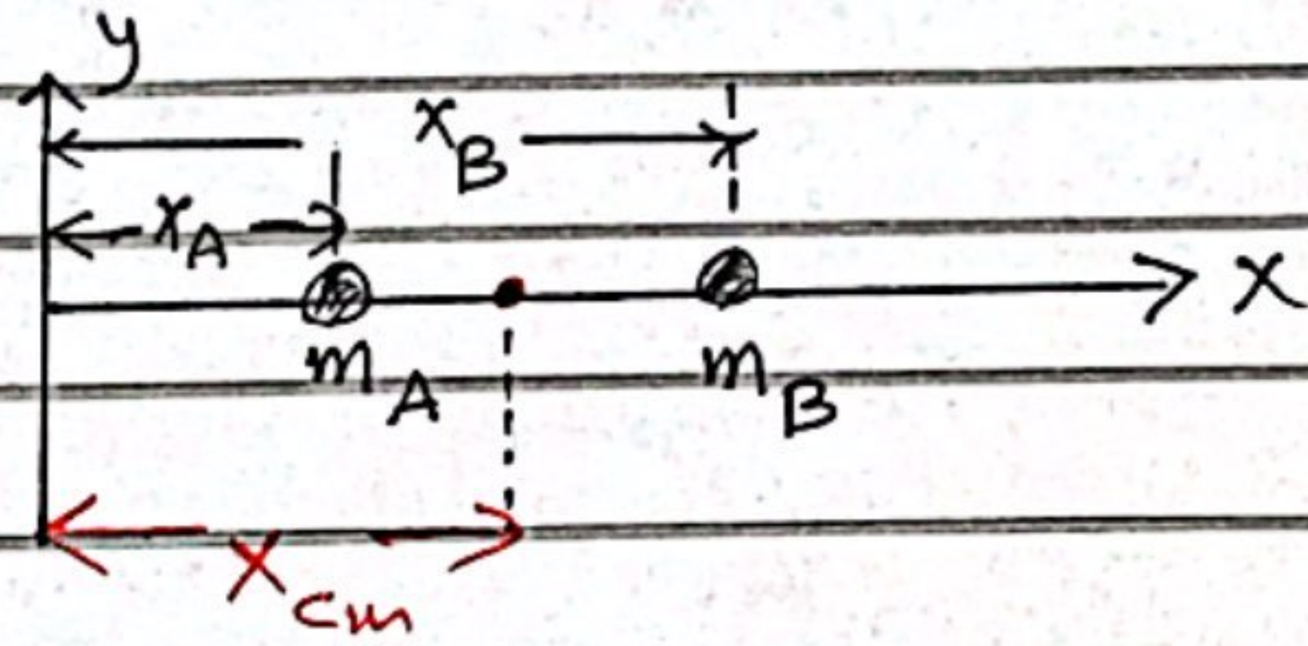
Chapter 7
Linear Momentum

7-8 Center of Mass (CM)

consider a system of two particles lie on the x-axis at positions x_A and x_B. The center of mass of this system is defined to be at the position x_{cm}, given by

$$x_{cm} = \frac{m_A x_A + m_B x_B}{m_A + m_B}$$



if there are more than two particles along a line, there will be addition terms

$$x_{cm} = \frac{m_A x_A + m_B x_B + m_c x_c + \cdots}{m_A + m_B + m_c + \cdots} = \frac{m_A x_A + m_B x_B + m_c x_c + \cdots}{M}$$

where $M = m_A + m_B + m_c + \cdots$ (the total mass of all particles)

Example 7-12
consider three persons have the same mass sit along the x-axis at positions $x_1 = 1m$, $x_2 = 5m$, $x_3 = 6m$. Find the position of the center of mass

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$= \frac{m(x_1 + x_2 + x_3)}{3m} = \frac{1}{3}(1+5+6) = 4 \text{ m}$$

If the particles are spread out in two or three dimensions, then we have in addition to x_{cm}

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + \cdots}{M}$$

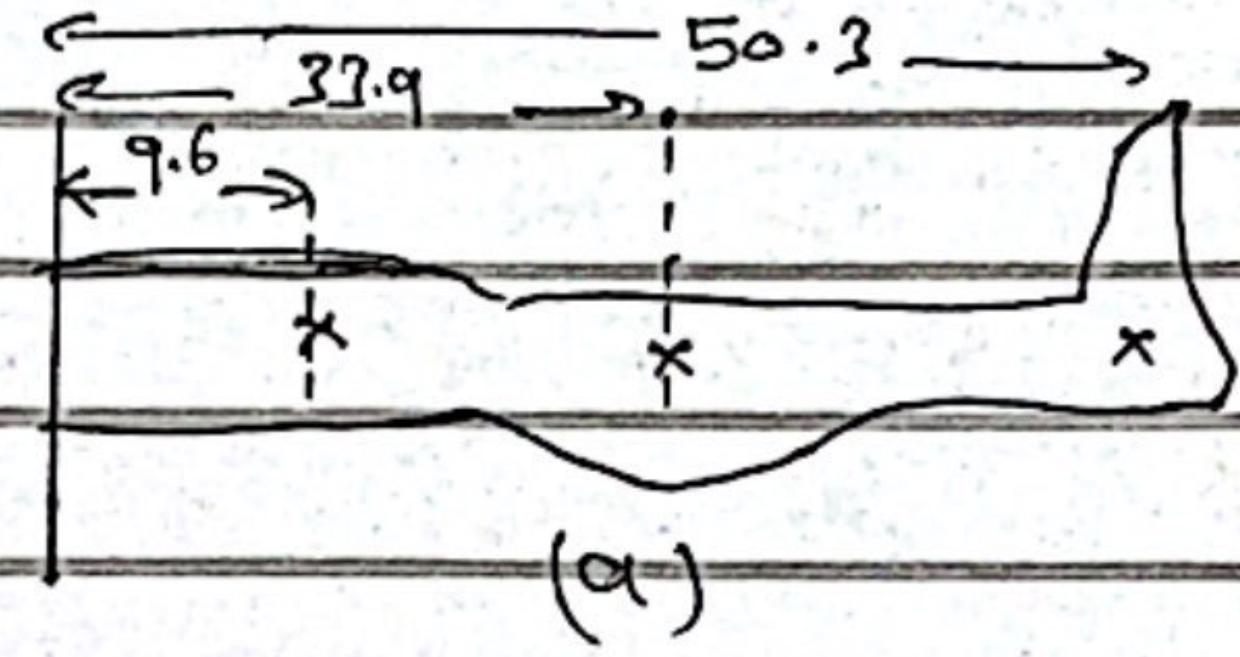
$$z_{cm} = \frac{m_1 z_1 + m_2 z_2 + \cdots}{M}$$

Example 7-13 (A Leg's cm)

Determine the position of the cm of a whole leg

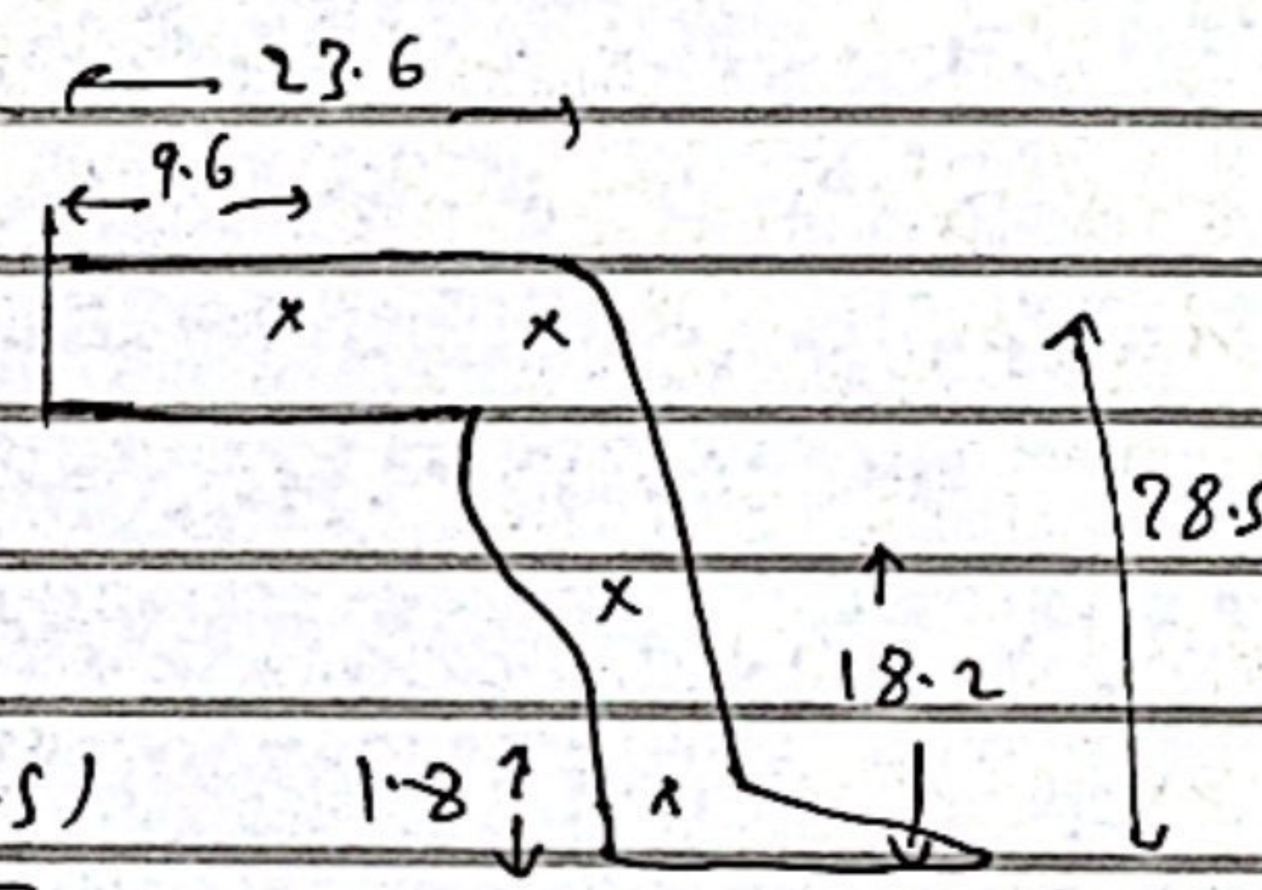
a) when stretched out

b) when bent at 90°



a) $$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$= \frac{(21.5)(9.6) + (9.6)(33.9) + (3.4)(50.3)}{21.5 + 9.6 + 3.4} = 20.4 \text{ units}$$



b) $$x_{cm} = \frac{(21.5)(9.6) + (9.6)(18.2) + (21.5)(28.5)}{3.4 + 9.6 + 21.5}$$

$$= 14.9 \text{ units}$$

$$y_{cm} = \frac{(3.4)(1.8) + (9.6)(18.2) + (21.5)(28.5)}{3.4 + 9.6 + 21.5}$$

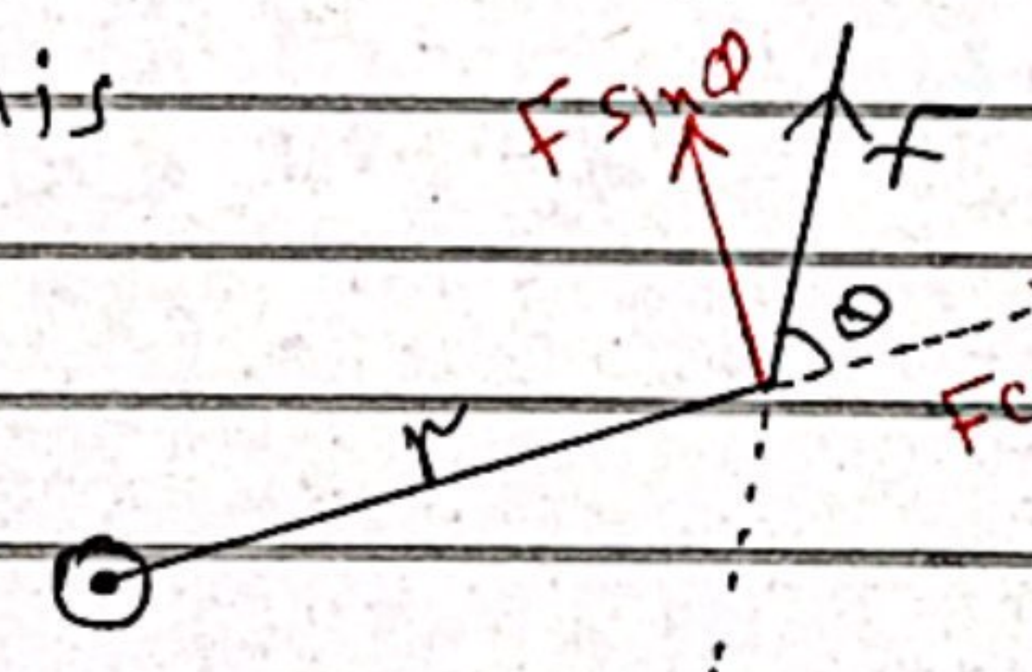
$$= 23 \text{ units}$$

* 1 unit = 1.7% m (meter)

Chapter 8
Rotational Motion

8-4 Torque

Torque is a measure of the force can cause an object to rotate about an axis.
Consider an object located at position r relative to its axis of rotation. When a force F is applied to the object, only the perpendicular component of F produces a torque (τ), this torque is given by

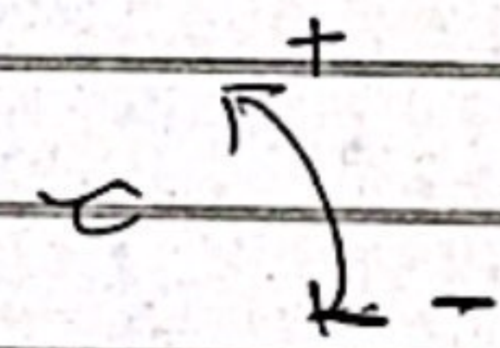


$$\vec{\tau} = \vec{r} \times \vec{F}$$

and it has the magnitude

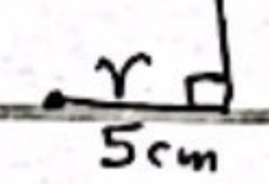
$$\tau = rF \sin\theta$$

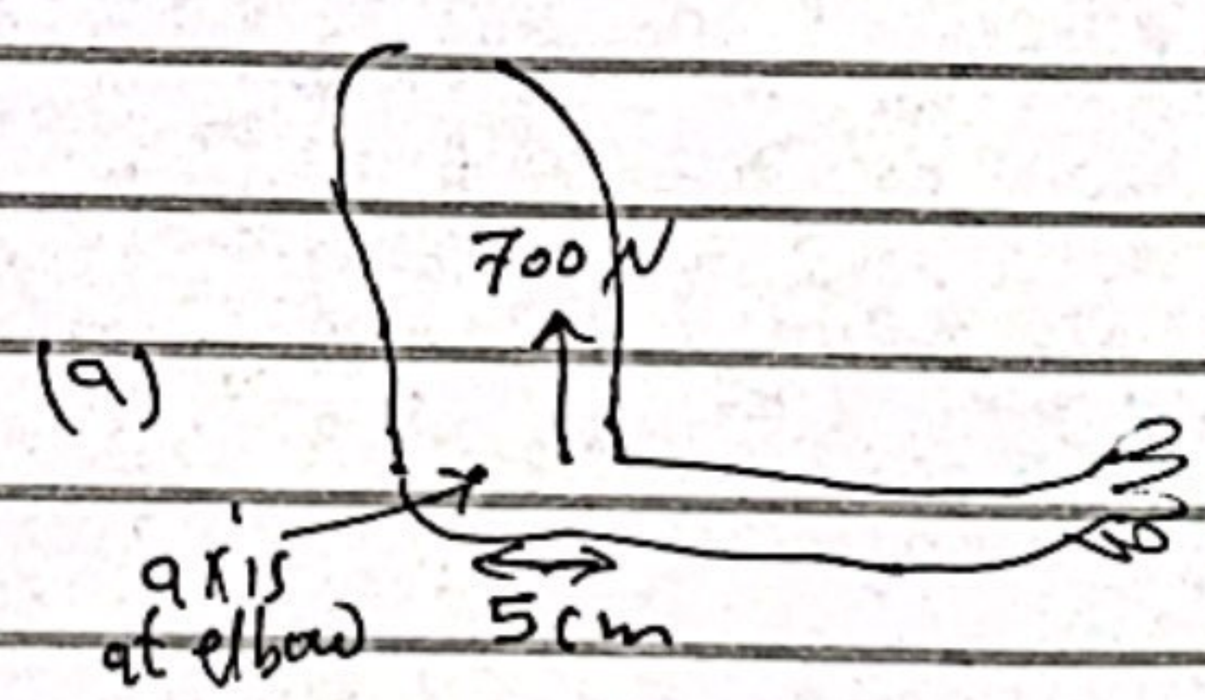
the torque has positive sign if counter clockwise and has minus sign if clockwise



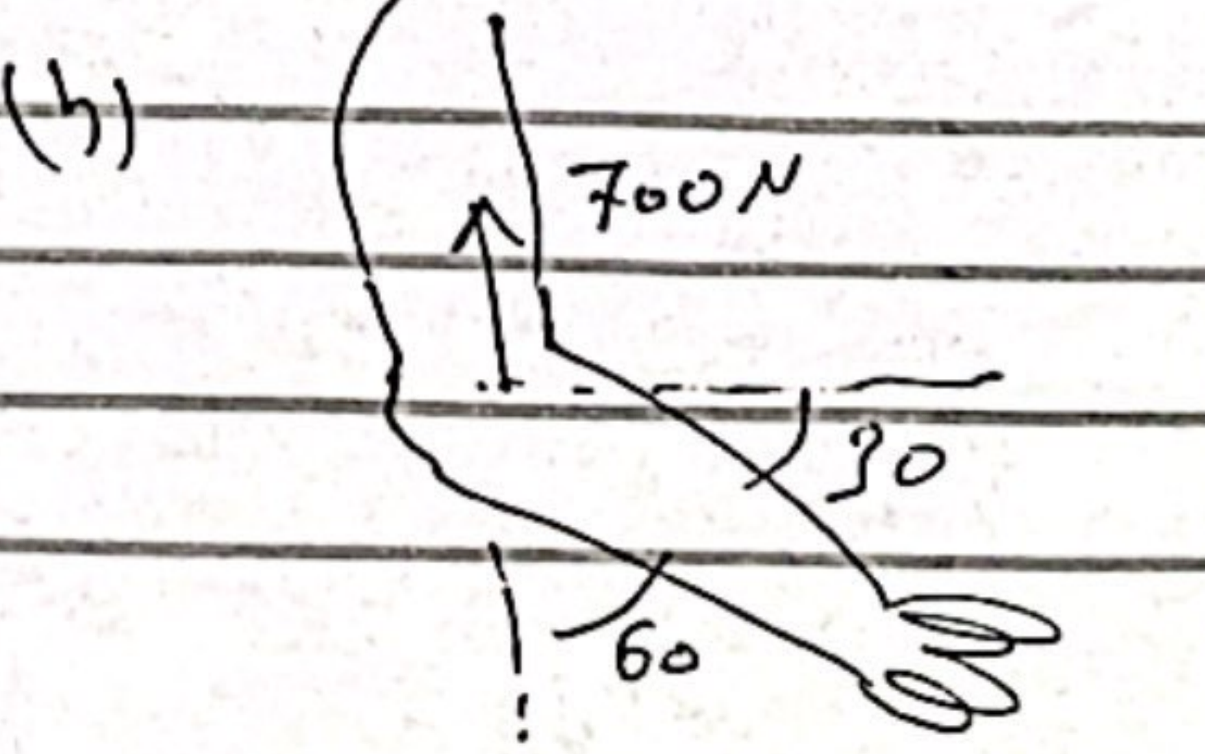
Example 8-8 (Biceps torque)
The biceps (~~dies~~) muscle exerts a vertical force on the lower arm, bent as shown in (a) and (b). For each case, calculate the torque about the axis of rotation through the elbow joint, assuming the muscle is attached 5 cm from the ~~~~ elbow as shown.

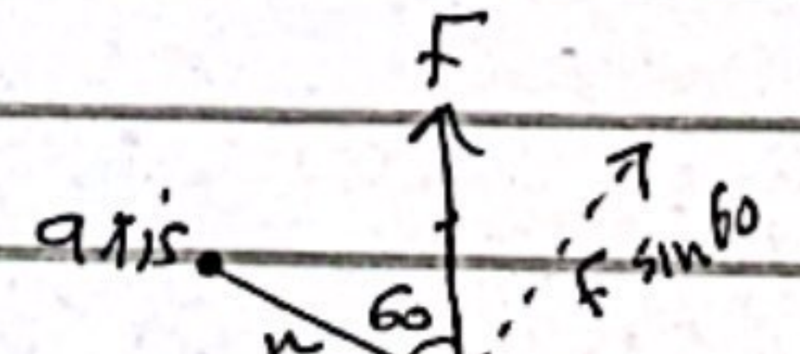
(a) $\tau = rF \sin 90$
$= (0.05)(700) = 35$ N.m





(b) $\tau = rF \sin\theta$
$= (0.05)(700)(\sin 60)$
$= 30$ N.m



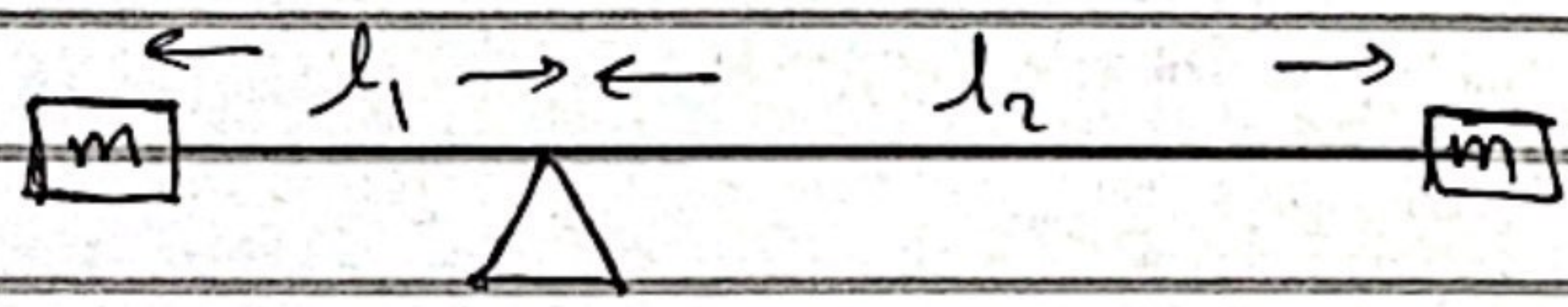


Exercise

Two blocks, each of mass m, are attached to the ends of a massless rod which pivots as shown. Initially the rod is held in the horizontal position and then released. Calculate the magnitude and direction of the torque on this system when it is first released.

$\tau_1 = F_1 l_1 = mg l_1 \ (+)$

$\tau_2 = F_2 l_2 = mg l_2 \ (-)$



$\tau = \tau_1 + \tau_2 = mgl_1 - mgl_2 = mg(l_1 - l_2)$, $l_2 > l_1$

clockwise

Exercise

In the figure shown, find the net torque about point O

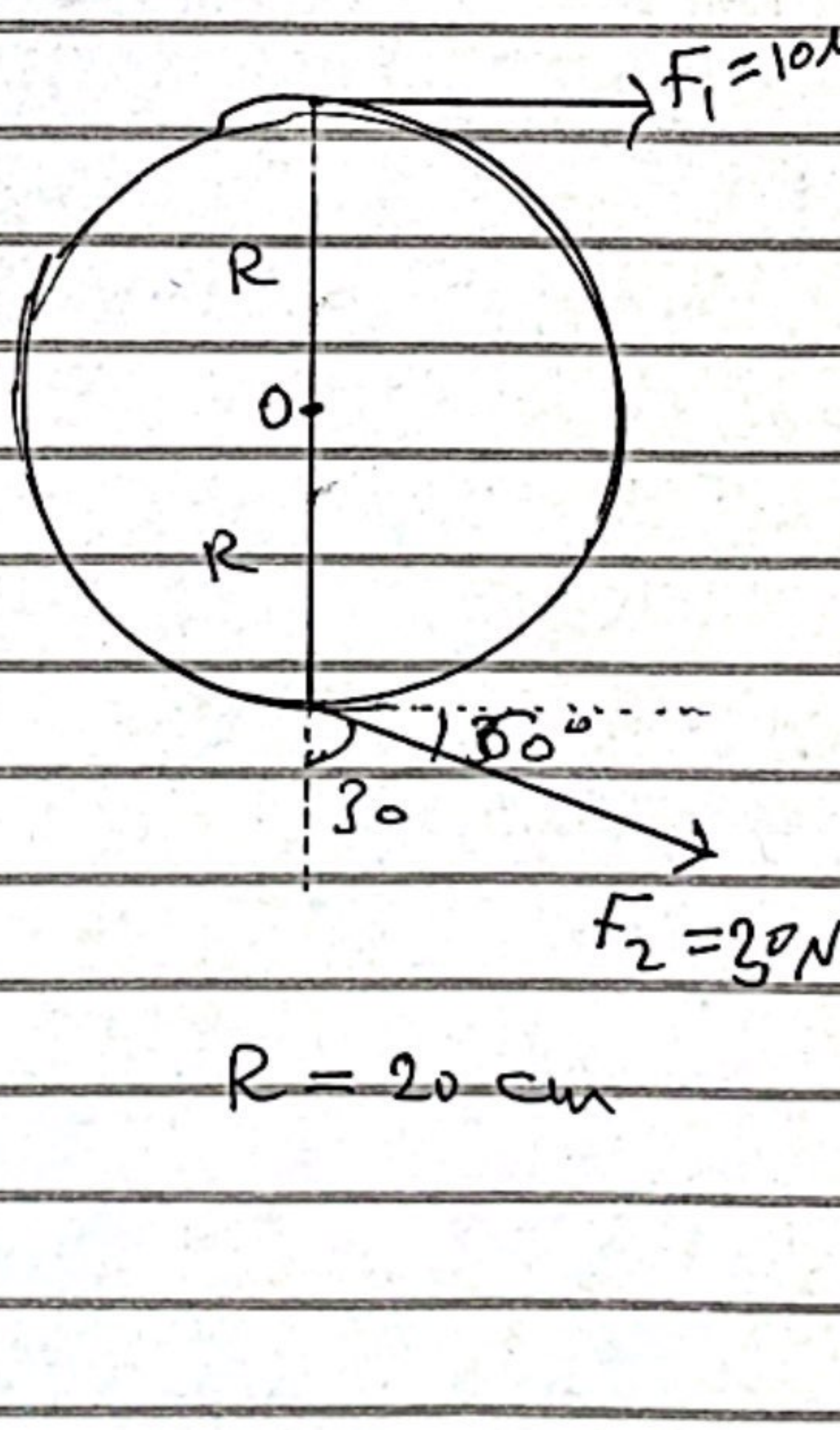
$\tau_1 = F_1 r_1 = -10(0.2) = -2 \ N.m$

$\tau_2 = F_2 r_2 \sin\theta = +30(0.2)(\sin 30)$

$= +3 \ N.m$

$\tau_{net} = \tau_1 + \tau_2 = -2 + 3 = 1 \ N.m$

Counter clockwise



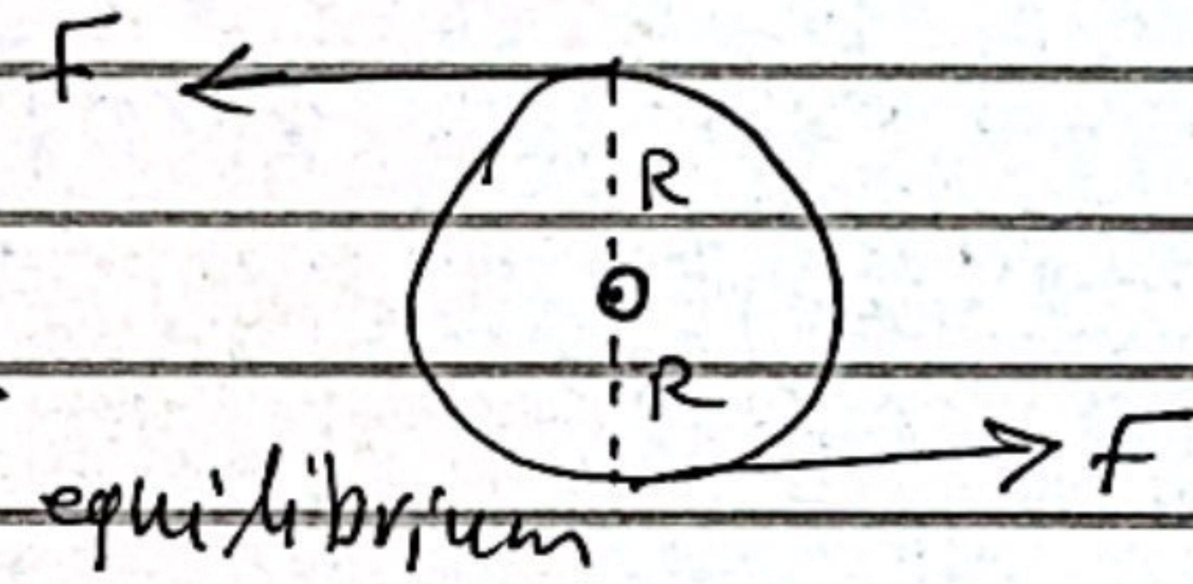
$R = 20$ cm

Chapter 9

Static Equilibrium, Elasticity and Fracture

9-1 The conditions for equilibrium

$$\sum \vec{F} = 0 \quad , \quad \sum \vec{\tau} = 0$$



here $\sum \vec{F} = F - F = 0$

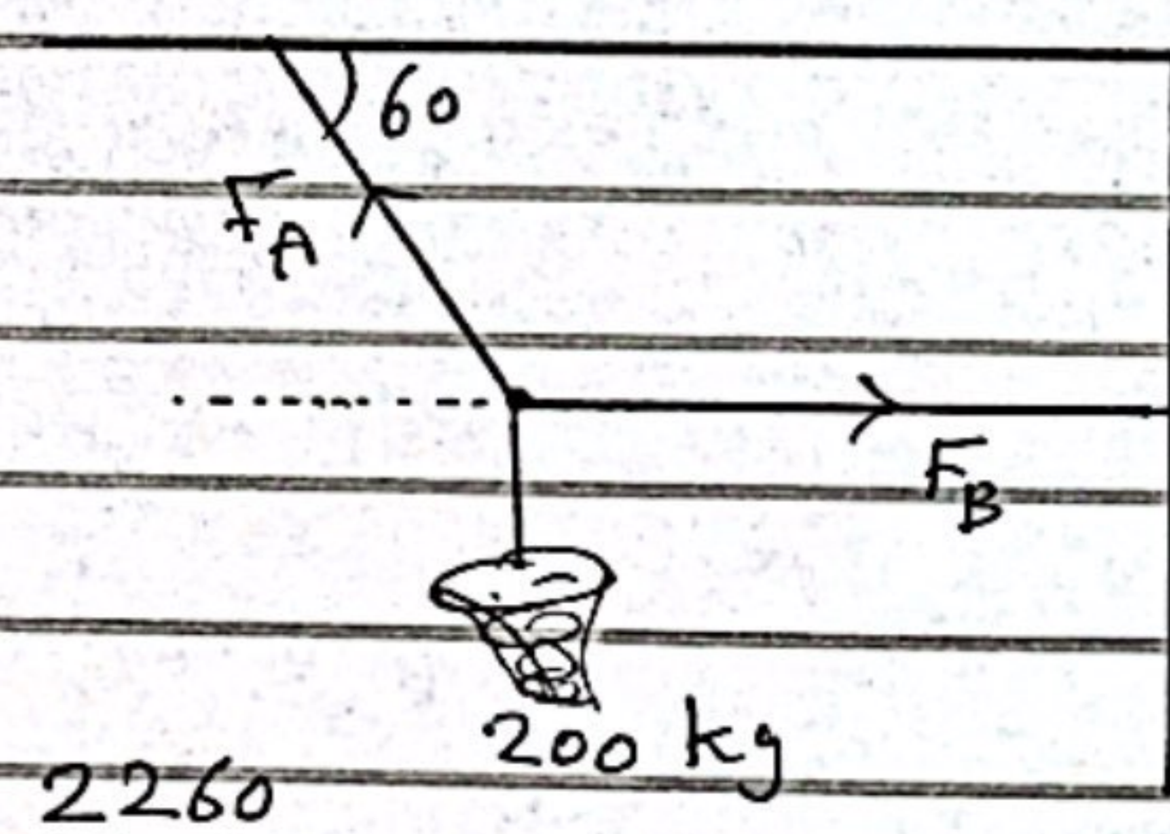
but $\tau = FR + FR = 2FR$

the wheel is not in equilibrium

in equilibrium

Example 9-2

Calculate the tensions $\vec{F}_A$ and $\vec{F}_B$ in the two cords that are connected to the vertical cord supporting the 200-kg chandelier (ثريا).



$\sum F_x = 0$

$F_B - F_A \cos 60 = 0$

$\sum F_y = 0$

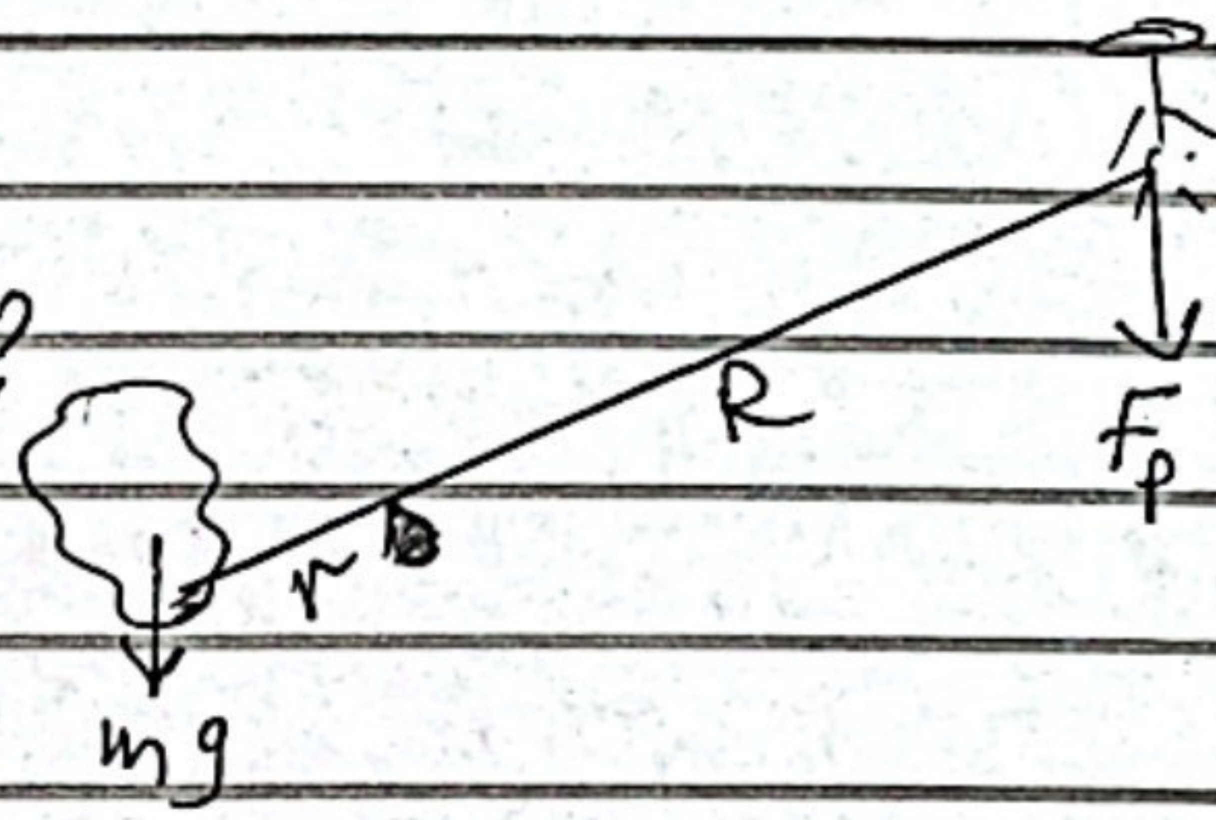
$F_A \sin 60 - mg = 0$

$$F_A = \frac{mg}{\sin 60} = \frac{200(9.8)}{\sin 60} = \cancel{2309}\ 2260\ N$$

$F_B = F_A \cos 60 = 2309(0.5) = 1130\ N$

Example 9-3 (A lever)

The bar is being used as a lever to pry up a large rock. The small rock acts as a pivot point. For what ratio (r/R) to get a balance?



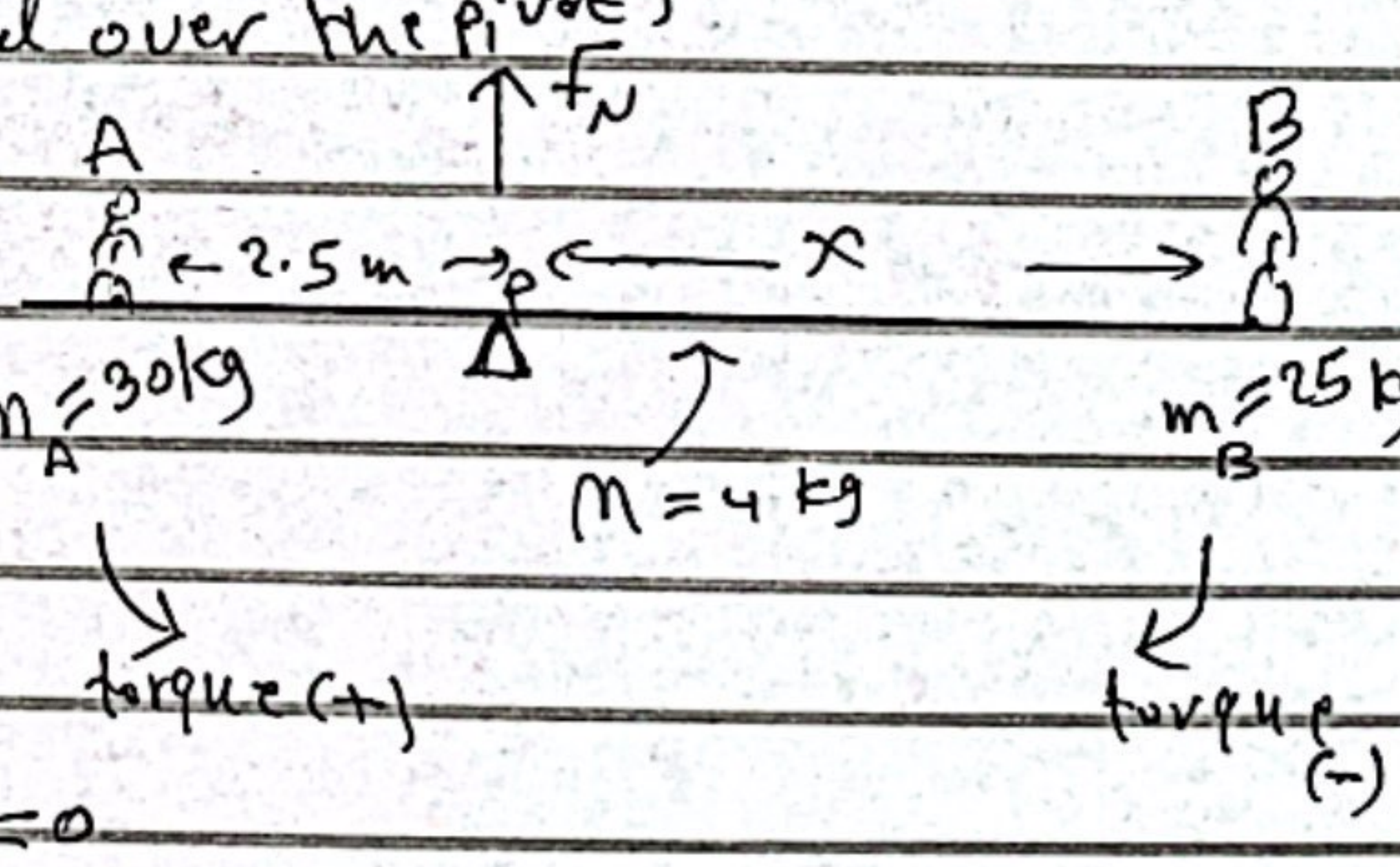
$\sum \tau = 0$

$$mgr - F_p R = 0$$

$$\frac{r}{R} = \frac{F_p}{mg}$$

Example 9-4 (Balancing a seesaw)

A board of mass $M = 4$ kg serves as a seesaw for two children. child A has a mass of 30 kg and sits 2.5 m from the pivot point, P. At what distance x from the pivot must child B, of mass 25 kg, place herself to balance the seesaw? (Assume the board centered over the pivot)



$\sum F_y = 0$

$$F_N - m_A g - m_B g - Mg = 0$$

$\sum \tau = 0$

assumption

$$m_A g r_A - m_B g x + Mg(0) + F_N(0) = 0$$

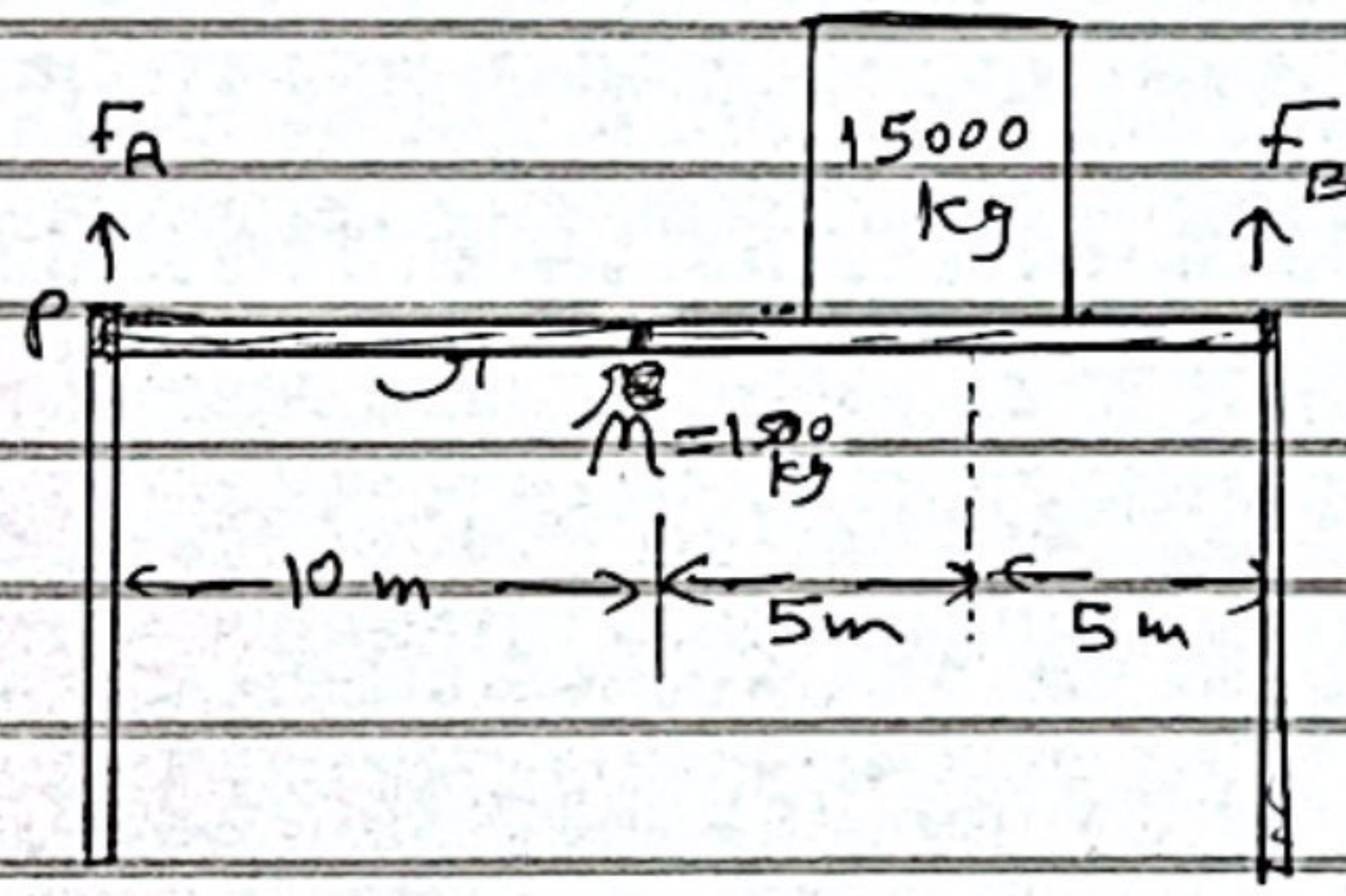
$$m_A g (2.5) - m_B g x = 0$$

$$x = \frac{m_A}{m_B}(2.5) = \frac{30}{25}(2.5) = 3 \text{ m}$$

and you can get $F_N = m_A g + m_B g + Mg$

Example 9-5

A uniform 1500-kg beam ([illegible]), 20 m long, supports a 15000-kg printing press ([illegible]) 5 m from the right support column. Calculate the force on each of the vertical support columns.



the net torque about the point P is zero

$\sum \tau = 0$ (static equilibrium)

$-(10)(1500) - (15)(15000) + 20F_B = 0$

$\Rightarrow F_B = 118000$ N

$\sum F_y = 0$ (static equilibrium)

$F_A + F_B - 1500\,g - 15000\,g = 0$

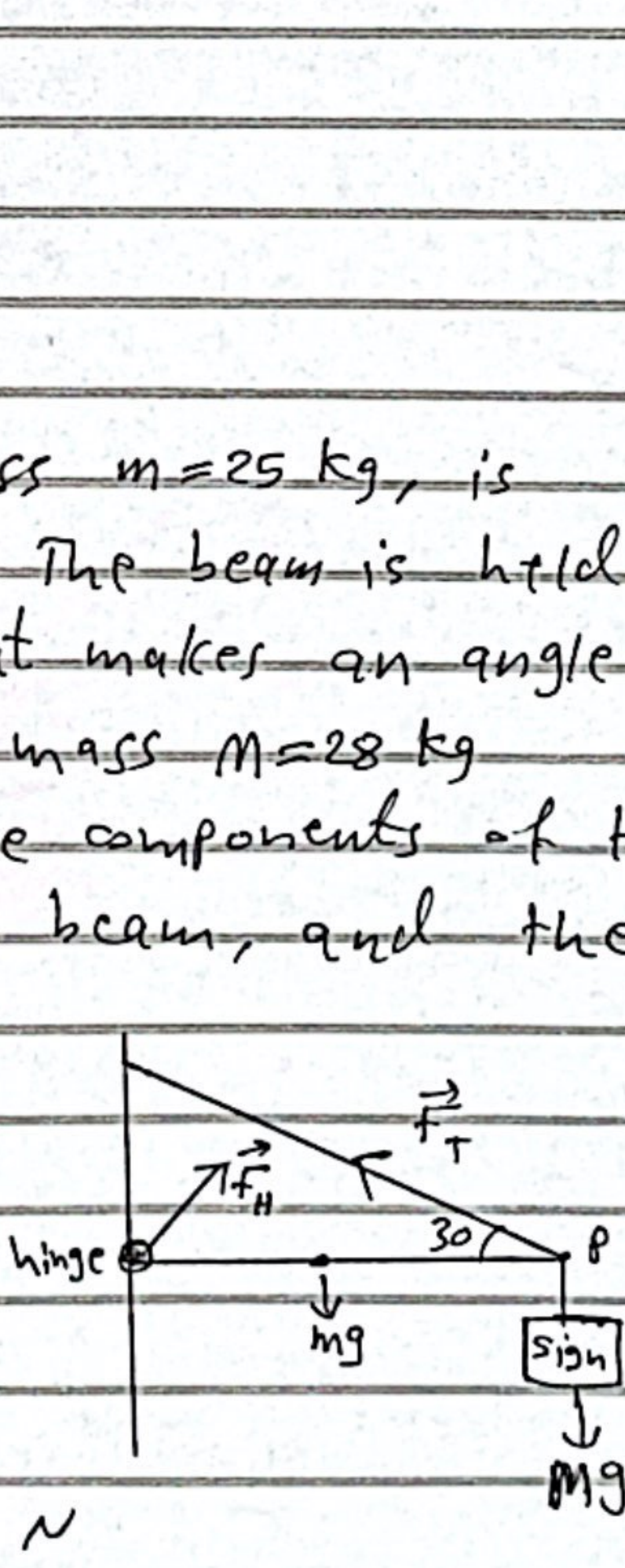
$F_A = (1500 + 15000)(9.8) - F_B$

$= (16500)(9.8) - 118000$

$F_A = 44100$ N

Example 9-6 (Hinged beam and table)

A uniform beam, 2.2 m long with mass $m = 25$ kg, is mounted by a small hinge([illegible]) on a wall. The beam is held in a horizontal position by a cable that makes an angle $\theta = 30°$. The beam supports a sign of mass $M = 28$ kg suspended from its end. Determine the components of the force F_H that the hinge exerts on the beam, and the tension F_T in the supporting cable.



$\sum F_x = 0 \Rightarrow F_{Hx} - F_{Tx} = 0 \Rightarrow F_{TX} = F_{HX}$

$\sum F_y = 0 \Rightarrow F_{Hy} + F_{Ty} - mg - Mg = 0$

$\sum \tau = 0$ (about P)

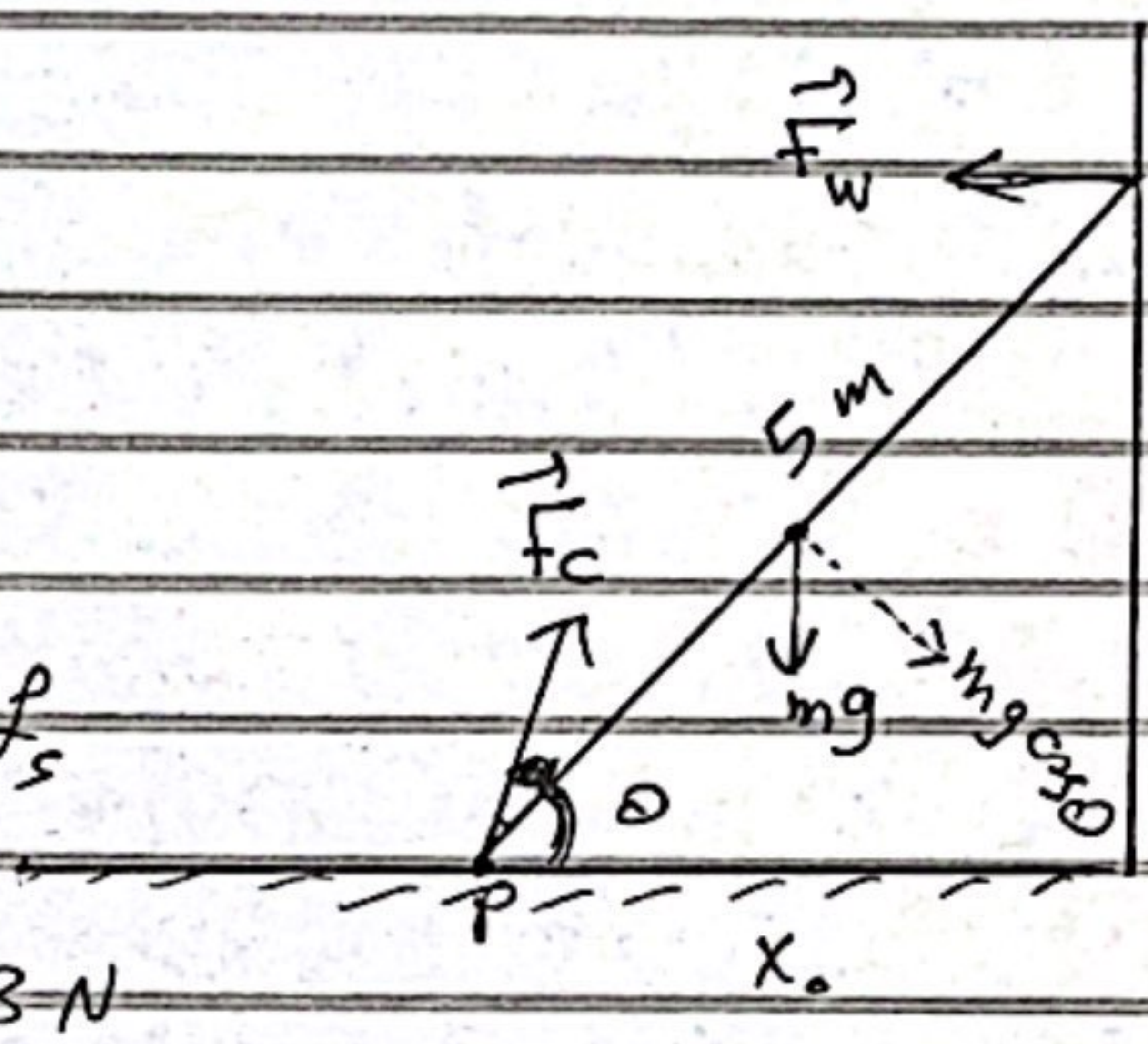
$F_{Hy}(2.2) - mg(1.1) = 0 \Rightarrow F_{Hy} = 123$ N

$\Rightarrow F_{Ty} = \cancel{} \; 396$ N $\Rightarrow F_T = 89$ N $\qquad F_{TX} = 686$ N

$F_H = 7 \qquad F_{HX} = 686$ N

Example 9-7 (Ladder)

A 5-m long ladder leans ( ) against a wall at point 4-m above the floor. The ladder is uniform and has mass $m = 12\ kg$. Assume the wall is frictionless, but the floor is not, determine the forces exerted on the ladder by the floor and by the wall.



use the equilibrium conditions

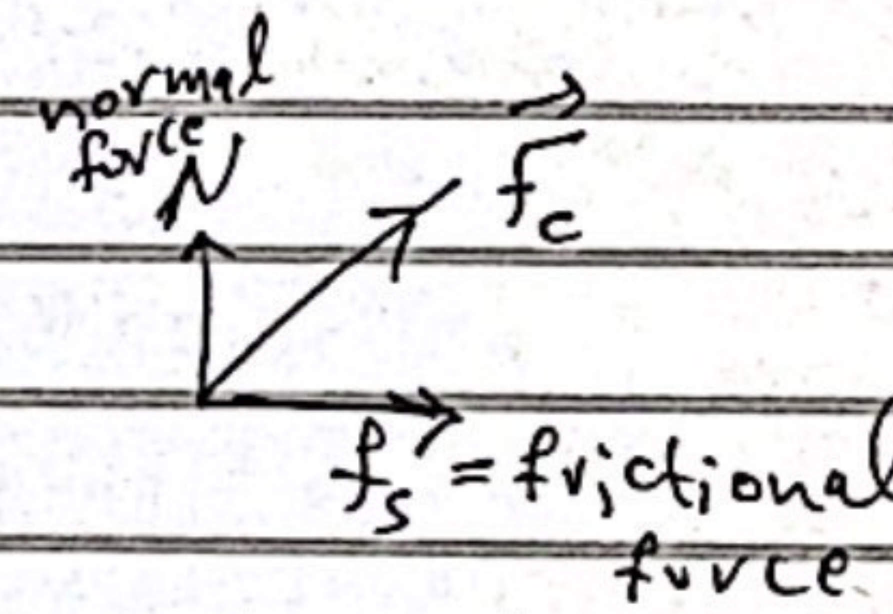
$\sum F_x = 0, \sum F_y = 0, \sum \tau = 0$

now

$\sum F_x = 0 \Rightarrow F_{cx} - F_W = 0 \Rightarrow F_W = F_{cx} = f_s$

$\sum F_y = 0 \Rightarrow F_{cy} - mg = 0 \Rightarrow F_{cy} = mg = 118\ N$

$x_o^2 + 4^2 = 5^2 \Rightarrow x_o = 3m$



$\sum \tau = 0 \Rightarrow 4(F_W) - (2.5)(mg\cos\theta) = 0$
about P

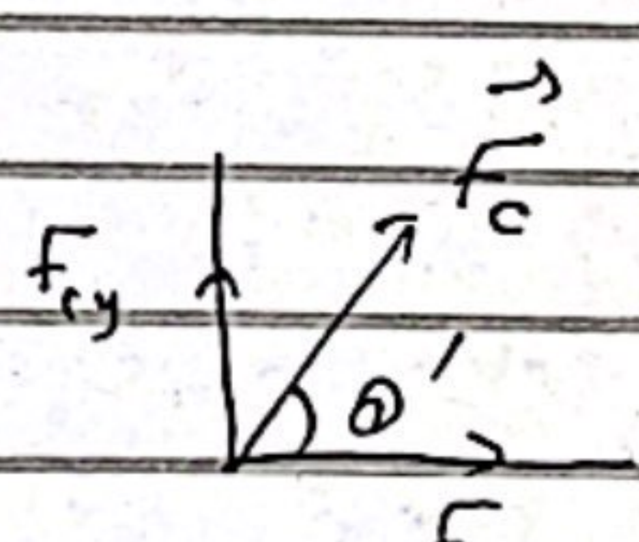
$\Rightarrow F_W = 44\ N = F_{cx}$

$F_{cx} = f_s$

$F_{cy} = N$

and $F_c = \sqrt{F_{cx}^2 + F_{cy}^2} = \sqrt{44^2 + 118^2} = 126\ N$

$\theta' = \tan^{-1}\frac{F_{cy}}{F_{cx}} = \tan^{-1}\frac{118}{44} = 70^\circ$



note $f_{s\,max} = F_{cx} = \mu_s N$

$44 = \mu_s\ mg$

$\mu_s = \frac{44}{12(9.8)} = 0.37$

* $(2.5)(mg\cos\theta) = 2.5\ mg\ \frac{3}{5} = 1.5\ mg$
* F_c is not along the direction of the ladder.

9-3 Applications to muscles and Joints

Example 9-8

How much force must the biceps muscle exert when a 5-kg ball is held in the hand

a) with the arm horizontal

b) when the arm is at a 45° angle

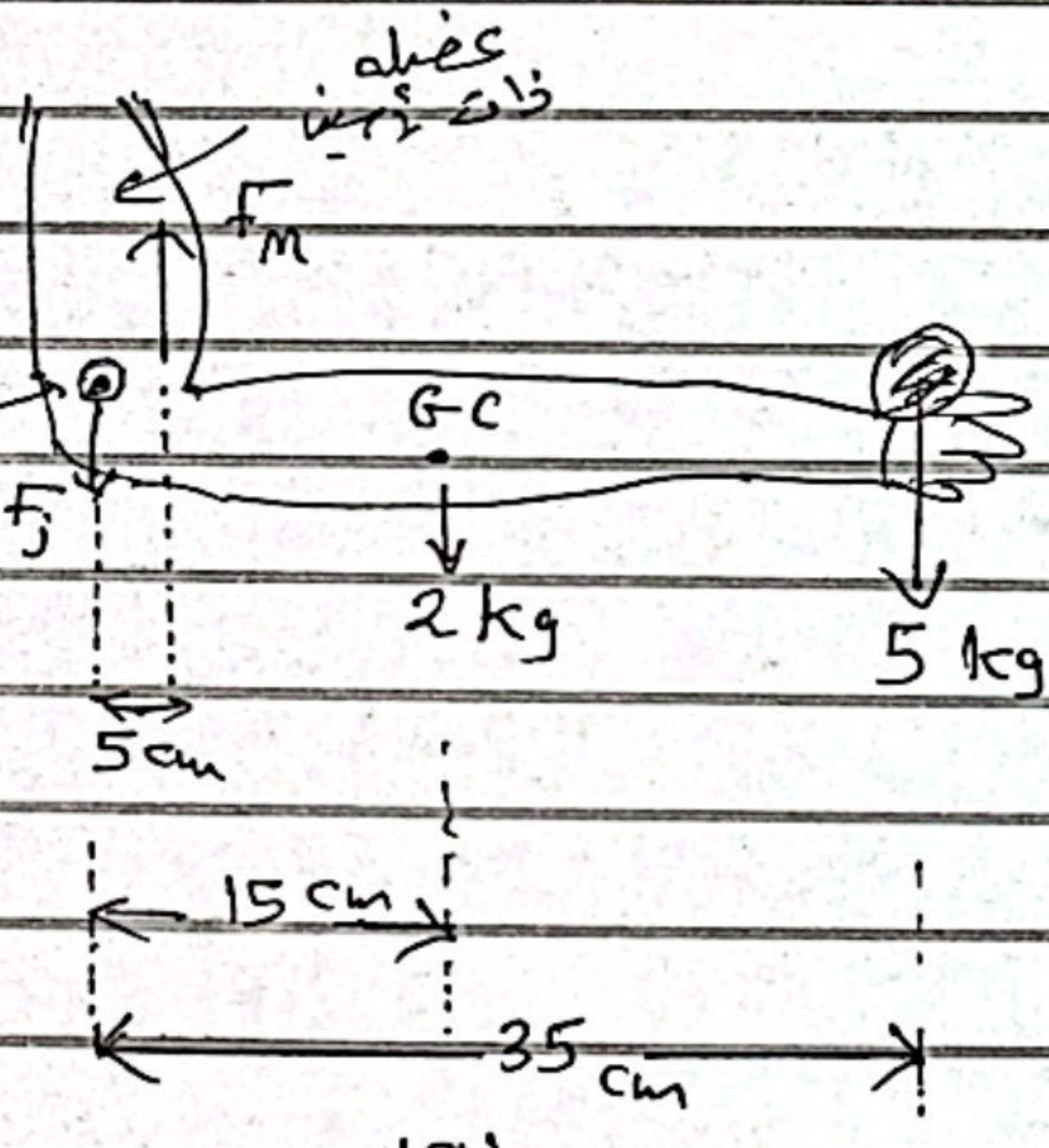
* The biceps muscle is connected to the forearm by a tendon attached 5 cm from the below joint. Assume that the mass of forearm and hand together is 2 kg and their CG is as shown.

a)

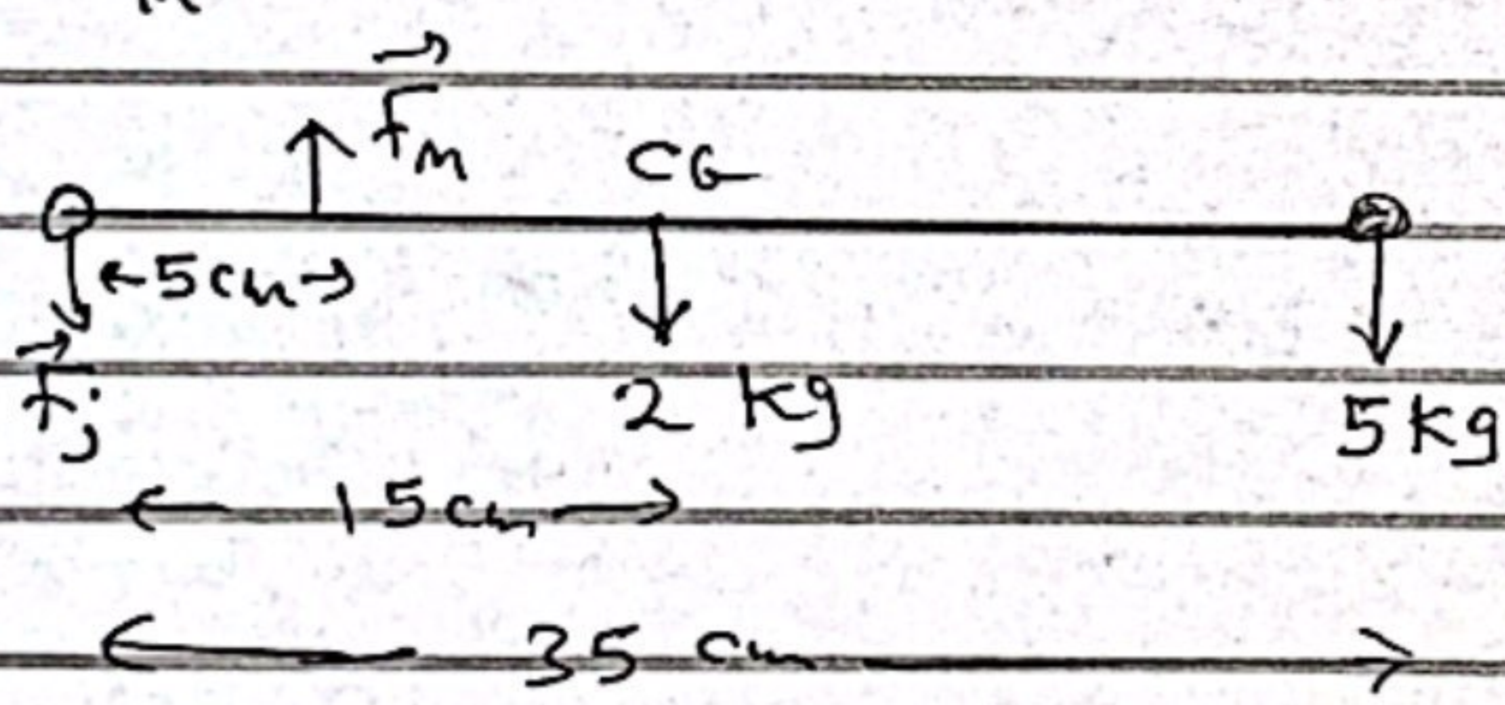
$\sum \tau = 0$ (about the joint)

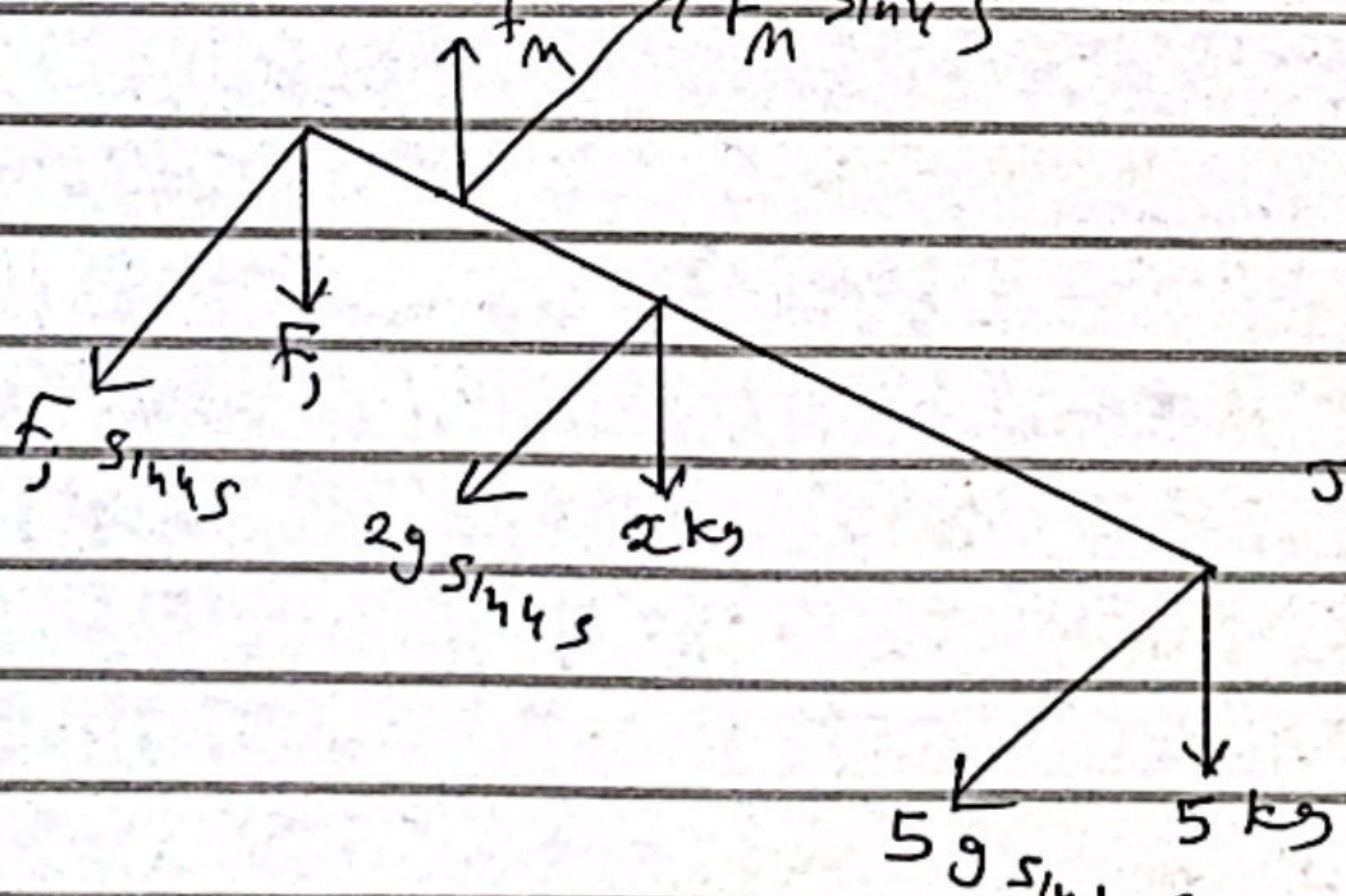
$0.05 F_M - 0.15(2)(9.8) - 0.35(5)(9.8) = 0$

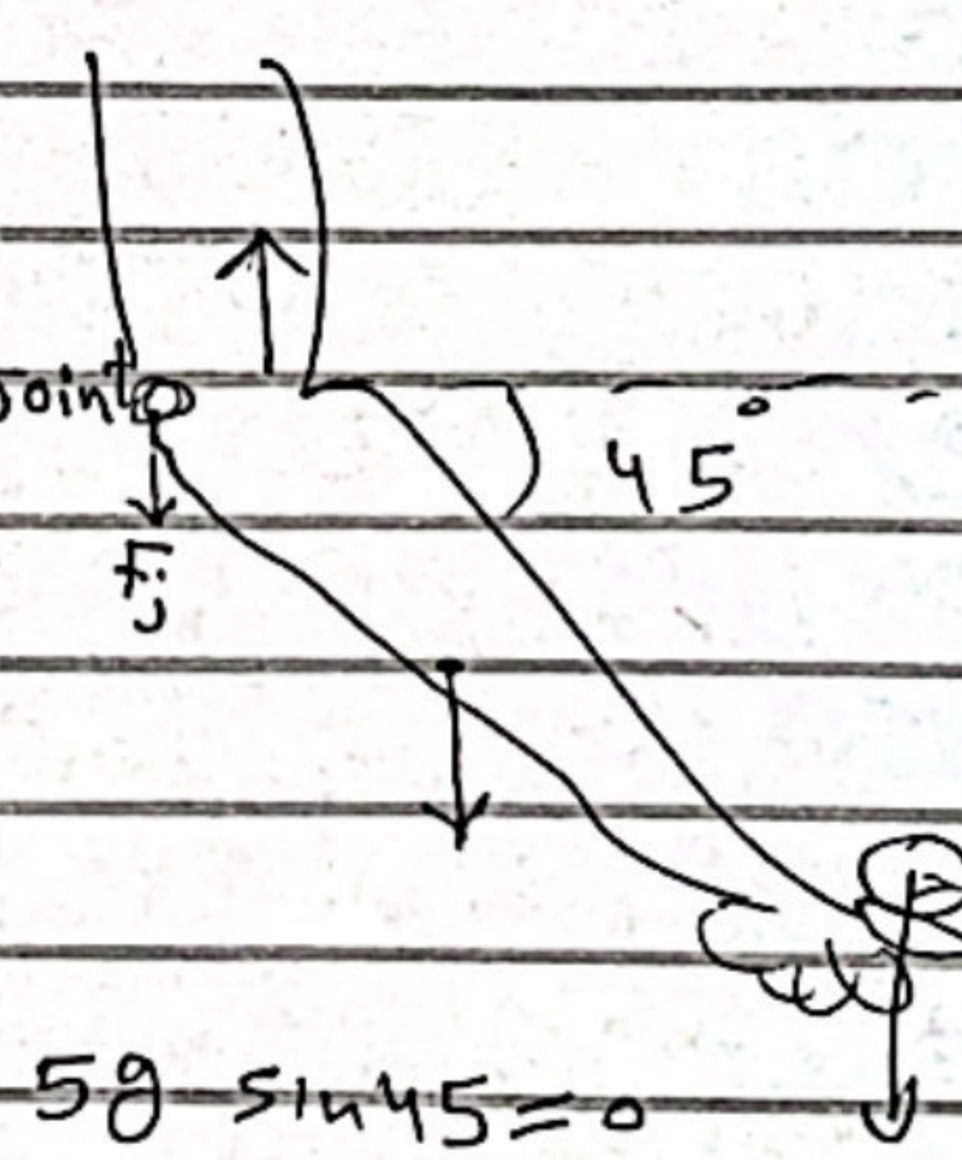
$\Rightarrow F_M = 400\ N$



(a)







$\sum \tau = 0$

$0.05 F_M \sin 45 - 2g(0.15)\sin 45 - (0.35)5g \sin 45 = 0$

$\Rightarrow F_M = 400 N$ (the same)

Example 9-9 (Force on your back)

Calculate the magnitude and direction of the force $\vec{F}_v$ acting on the fifth Lumbar vertebra as represented in fig.

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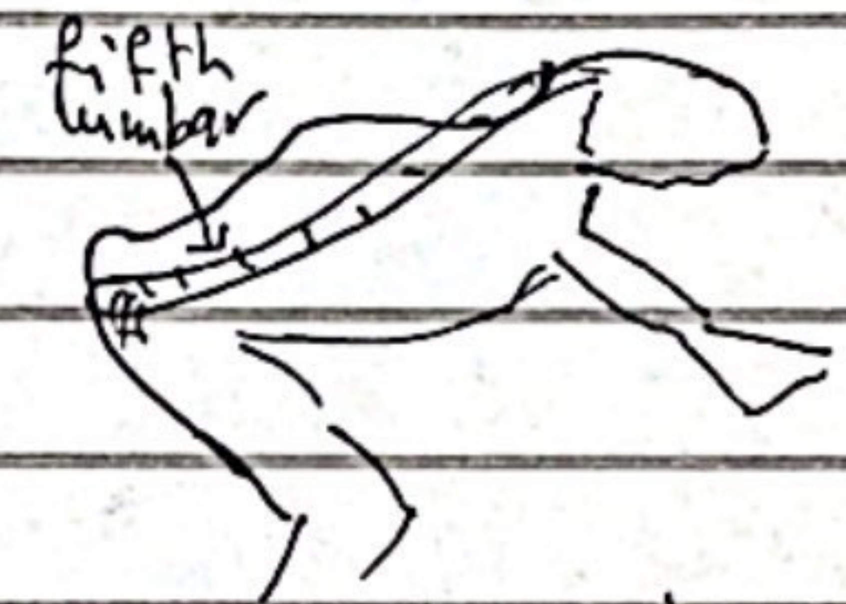
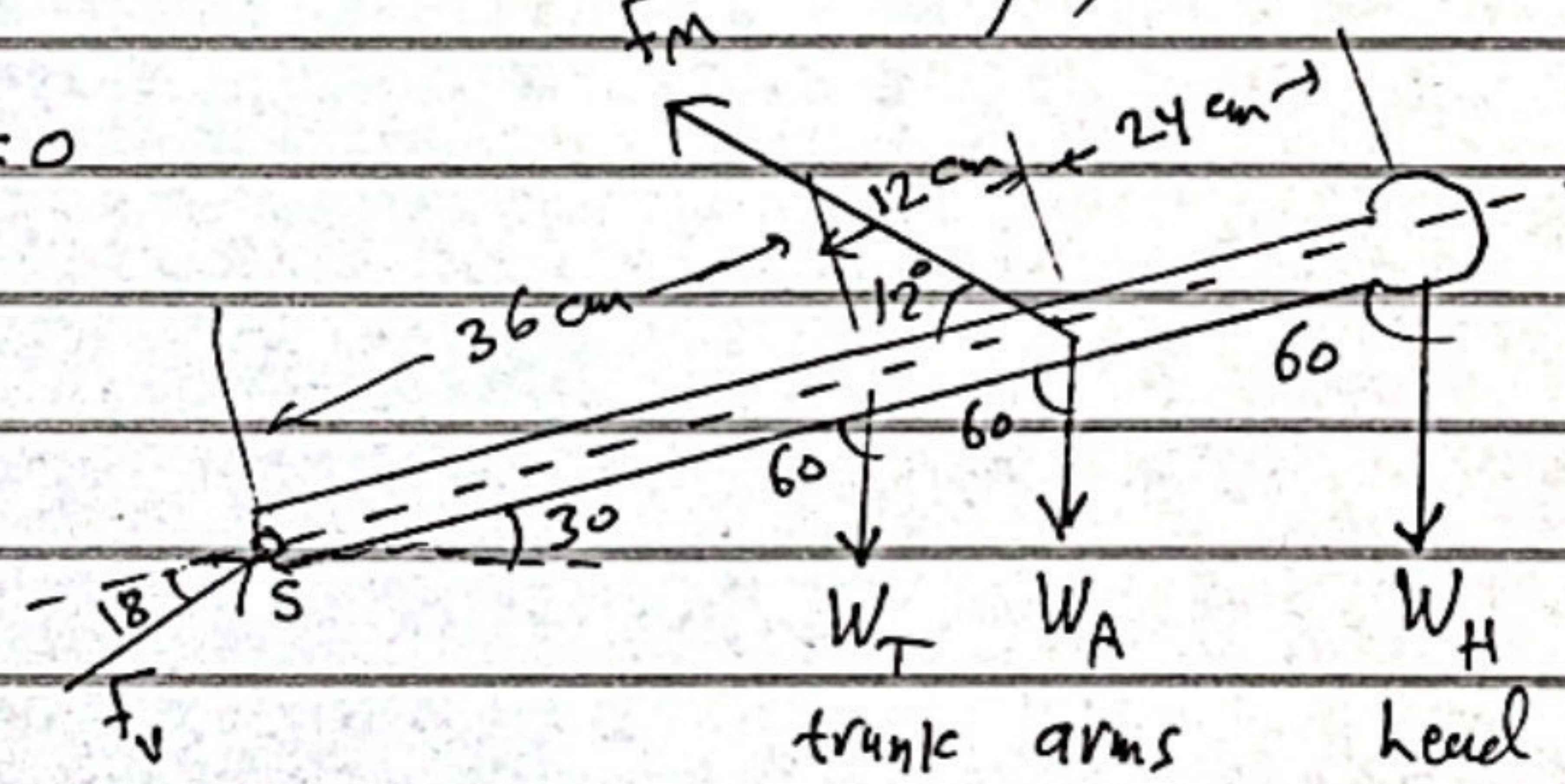


$$\sum \tau = 0$$

$$(0.48)(\sin 12) F_M - (0.72)(\sin 60) W_H - (0.48)(\sin 60) W_A - (0.36)(\sin 60) W_T = 0$$

$$\Rightarrow F_M = 2.37\,W \simeq 2.4\,W$$



$$\sum F_y = 0$$

$$F_{vy} - F_M \sin 18 - W_H - W_A - W_T = 0$$

$$\Rightarrow F_{vy} = 1.38\,W \simeq 1.4\,W$$

$W_H = 0.07\,W$ (head)

$W_A = 0.12\,W$ (2 arms)

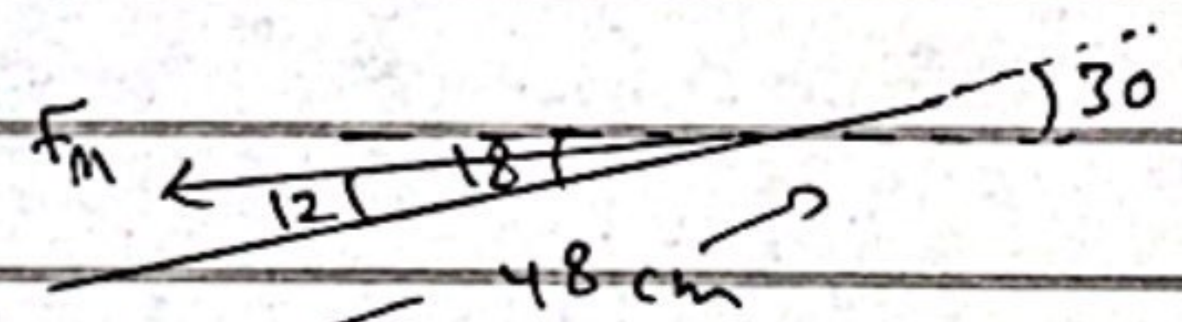
$W_T = 0.46\,W$ (trunk)

$W =$ total weight

and $\sum F_x = 0$

$$F_{vx} - F_M \cos 18 = 0$$

$$\Rightarrow F_{vx} = 2.25\,W \simeq 2.3\,W$$



$$F_v = \sqrt{F_{vx}^2 + F_{vy}^2} = 2.6\,W$$

$$\theta = \tan^{-1} \frac{F_{vy}}{F_{vx}} = 32^\circ$$

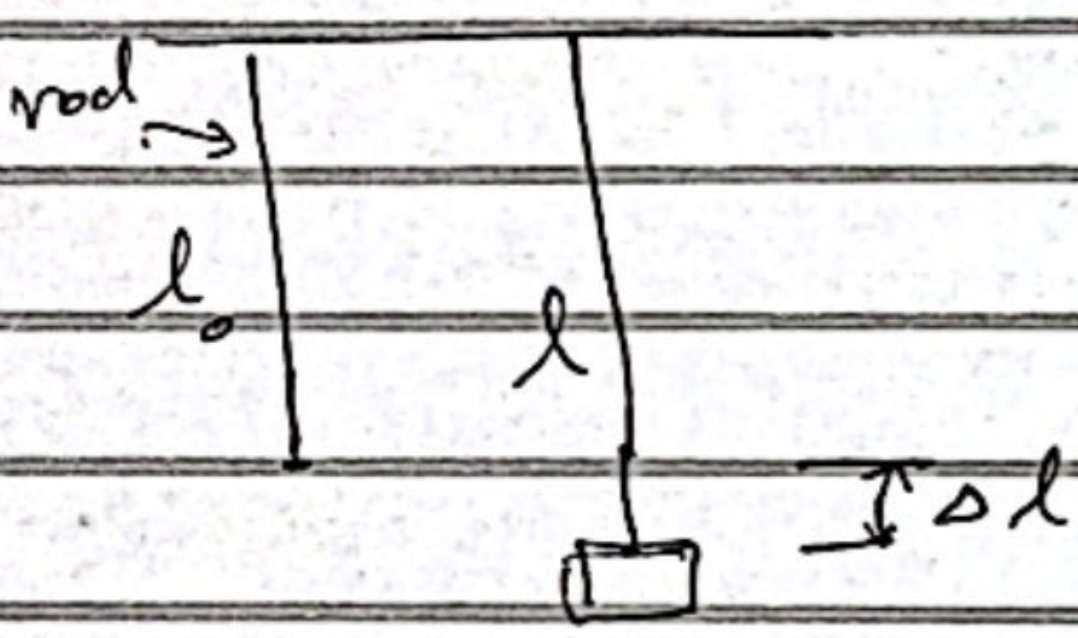
9-5 Elasticity; stress and strain

Stress and strain

Stress: is defined as the force per unit area that acts on a material ~~by~~ that aries from externally applied forces

$$\text{stress} = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$$

Strain: strain is the amount of deformation experienced by the object in the direction of force applied, divided by the initial dimensions of the object



$$\text{strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta l}{l_0}$$

$$\Delta l = l - l_0$$

stress is applied ~~by~~ to the material by external agents, whereas strain is the material's responce to the stress

Elasticity and Hooke's law

Hooke's law states that the force applied on an object is directly proportional to the displacement from its equilibrium position

$$F = K \Delta l \quad , K: \text{proportional constant}$$

Young's Modulus (E)

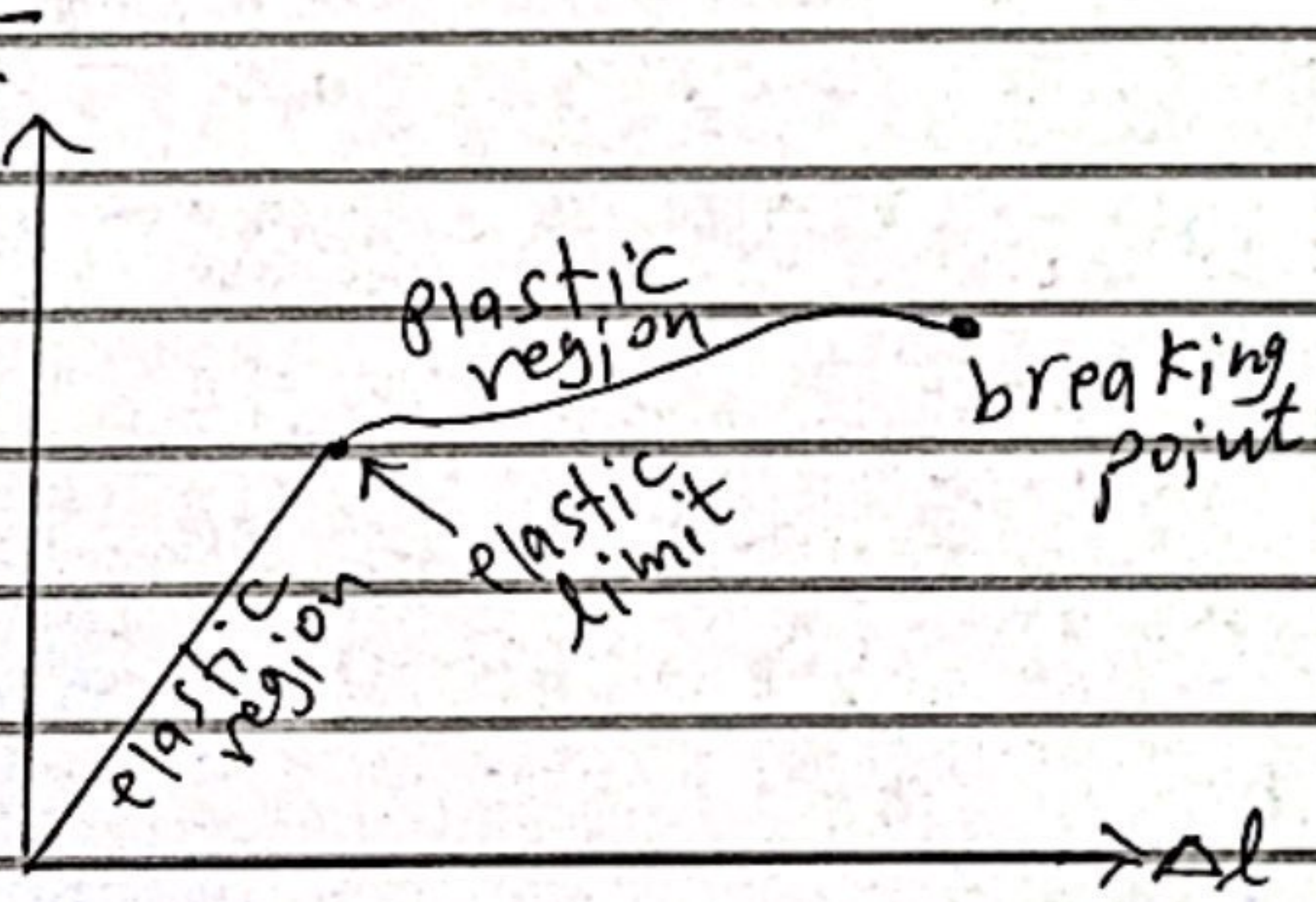
$$E = \frac{F/A}{\Delta l / l_0} = \frac{\text{stress}}{\text{strain}}$$

The proportionality in Hooke's law holds until the force reaches the proportional limit. Beyond that, the object will still return to its original shape up to the elastic limit. Beyond the elastic limit, the material is permanently deformed, and it breaks at the breaking point.

□ Young's modulus

$$E = \frac{\text{stress}}{\text{strain}} = \frac{\text{longitudinal stress}}{\text{longitudinal strain}}$$

$$= \frac{F/A}{\Delta l / l_0}$$



Example 9-10

A 1.6-m long steel piano wire has a diameter of 0.2 cm. How great is the tension in the wire if it reaches 0.25 cm when tightened?

$E = 2 \times 10^{11} \ N/m^2$

$r = 0.1 \ cm$

$$F = E \frac{\Delta l}{l_0} A$$

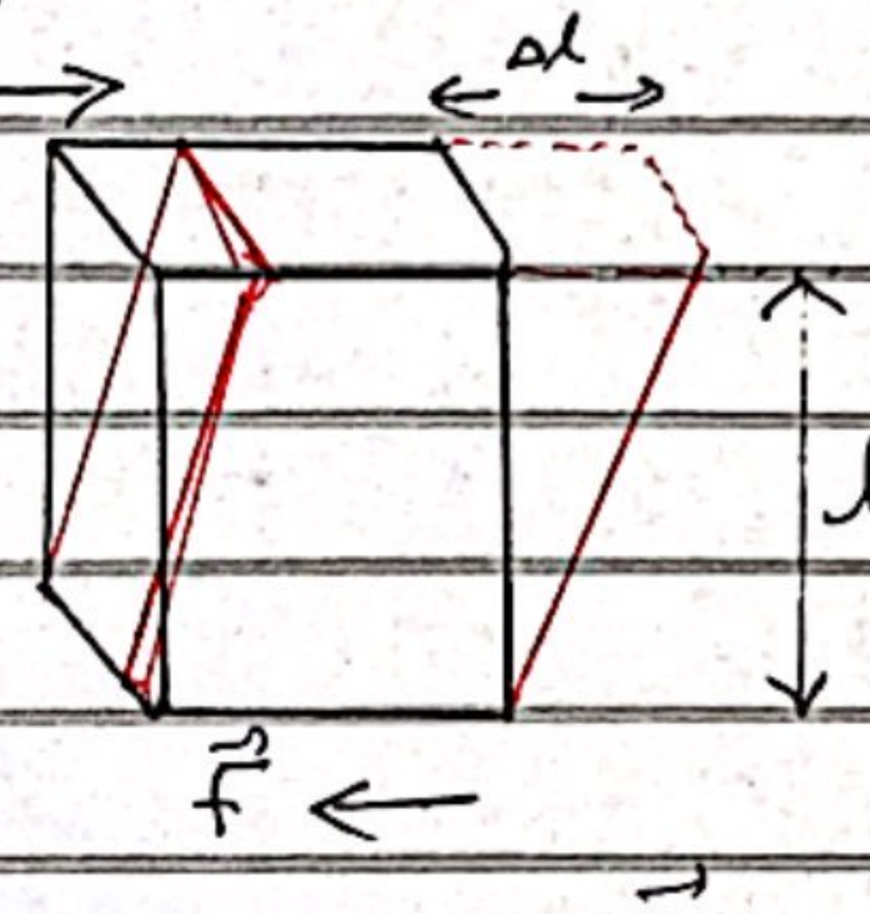
$$= (2 \times 10^{11}) \left(\frac{0.0025}{1.6} \right) (r^2 \pi) = 980 \ N$$

9-6 Fracture

When stress on an object is large, the object may break.

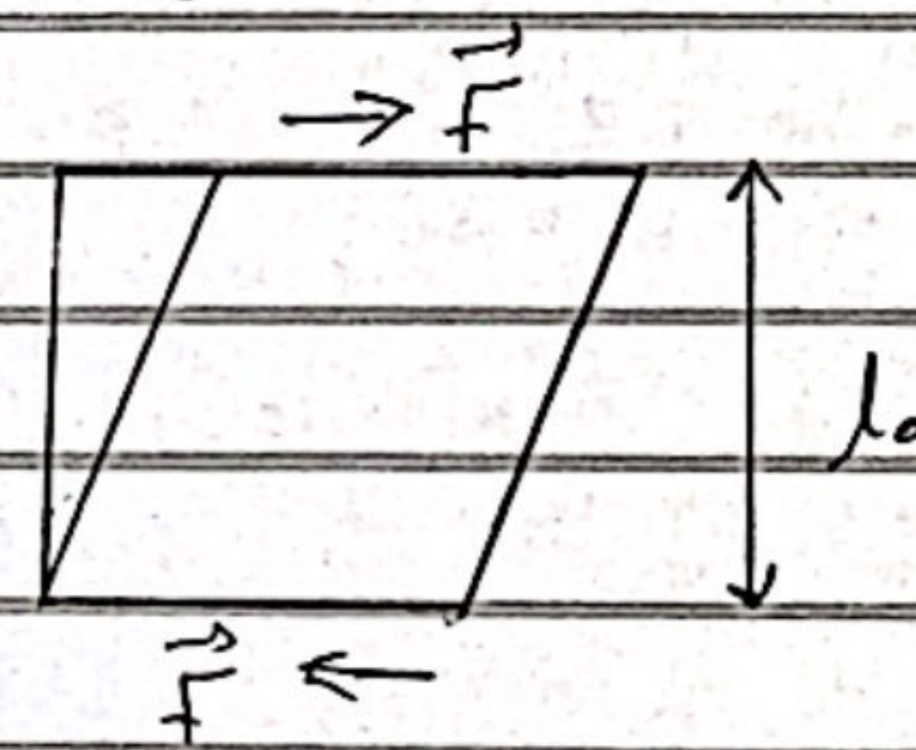
Shear modulus

In shear stress, the dimensions of the object don't change, but the shape changes

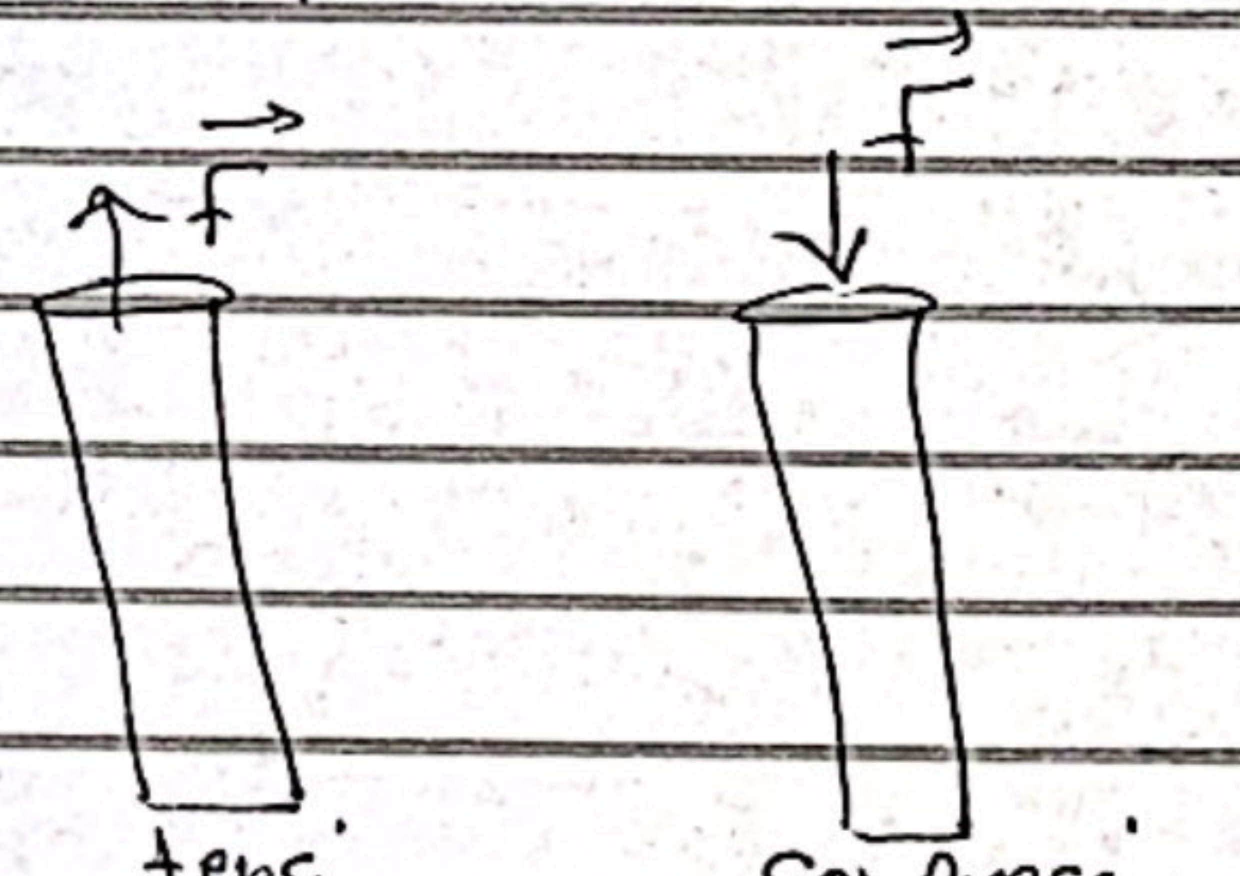


$$G = \frac{F/A}{\Delta l/l_0} \quad \text{(shear modulus)}$$

note that A is the area of the surface parallel to the applied Force (and not perpendicular as for tension or compression), and Δl is perpendicular to l_0.



* types of stress for rigid objects
tension, compression, shear



tension　　compression

Volume change — Bulk modulus
if an object is subjected to inward forces from all sides, its volume will decrease.

bulk modulus it is the ratio of pressure applied to the corresponding relative decrease in the volume of the material

$$B = \frac{-\Delta P}{\Delta V/V_0} \quad \text{(for liquids and gases)}$$

the minus sign means the volume decreases with an increase in pressure.