Chapter 10
Fluids

10-1 Phases of Matter

the three common Phases of matter are solid, liquid, and gas

A solid has a definite shape and size.

A liquid has a fixed volume but can be any shape.

A gas can be any shape and also can be easily compressed

Liquids and gases both flow, and are called Fluids.

10-2 Density and specific Gravity

the density of a substance is its mass per unit volume

$$\rho = \frac{m}{V}$$

the SI unit of density is kg/m^3.

Water at $4°C$ has a density of $1\ g/cm^3 \equiv 1000\ kg/m^3$

The specific gravity of a substance is the ratio of its density to that of water.

Example 10-1

What is the mass of a solid iron wrecking ball of radius 18 cm?

$$m = \rho V = \rho \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (7800)(0.18)^3 = 190\ kg$$

10-3 Pressure in Fluids

Pressure is defined as Force F per unit area, where F is acting perpendicular to the surface area A.

$$P = \frac{F}{A}$$

the SI unit of pressure is pascal (Pa), where

$$1\ Pa = 1\ N/m^2$$

Example 10-2

A 60-kg person's two feet cover an area of $500\ cm^2$.

a) determine the pressure exerted by the two feet on the ground

b) if the person stands on one foot, what will be the pressure under that foot ?

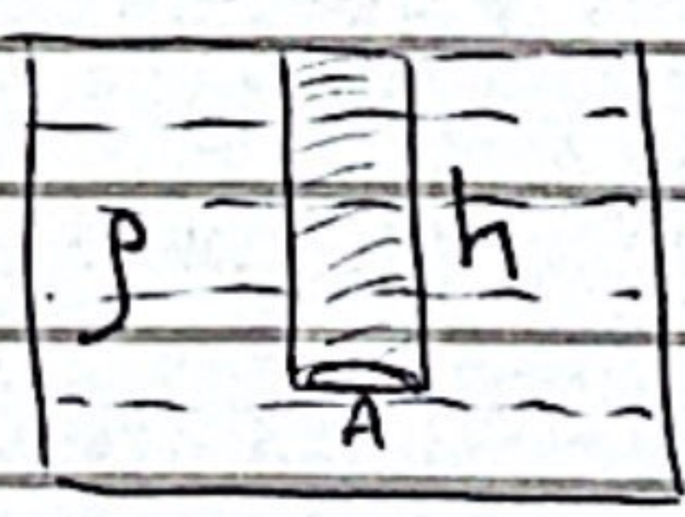
solution

a) $P = \frac{F}{A} = \frac{mg}{A} = \frac{60(9.8)}{0.05} = 12\times10^3\ N/m^2$

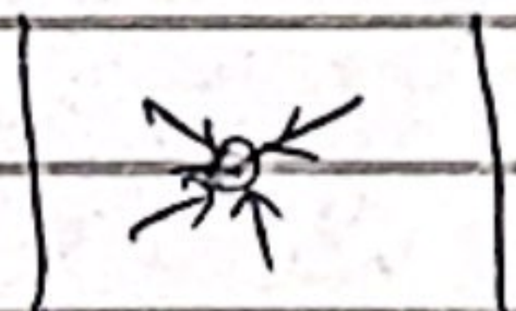
b) $P = \frac{F}{A} = \frac{mg}{A/2} = \frac{2mg}{A} = 24\times10^3\ N/m^2$

The pressure at a depth h below the surface of the liquid is due to the weight of the liquid above it

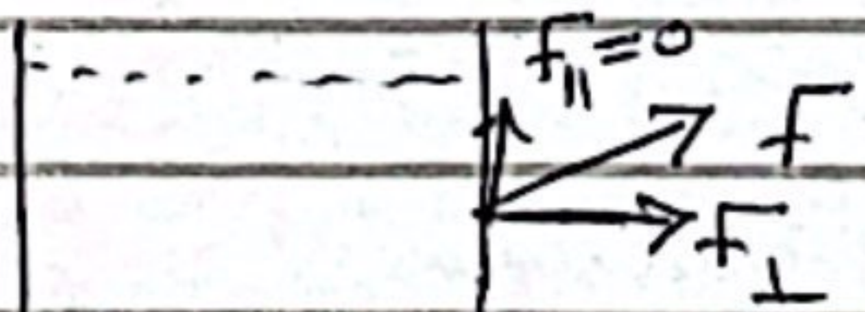
$$P = \frac{F}{A} = \frac{mg}{A} = \frac{\rho V g}{A} = \frac{\rho A h g}{A}$$



$$P = \rho g h$$

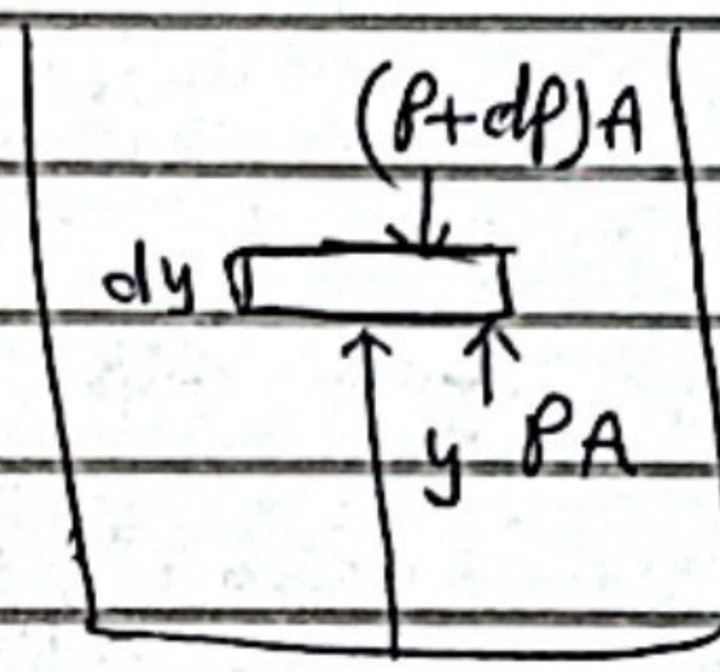


Pressure is the same in every direction



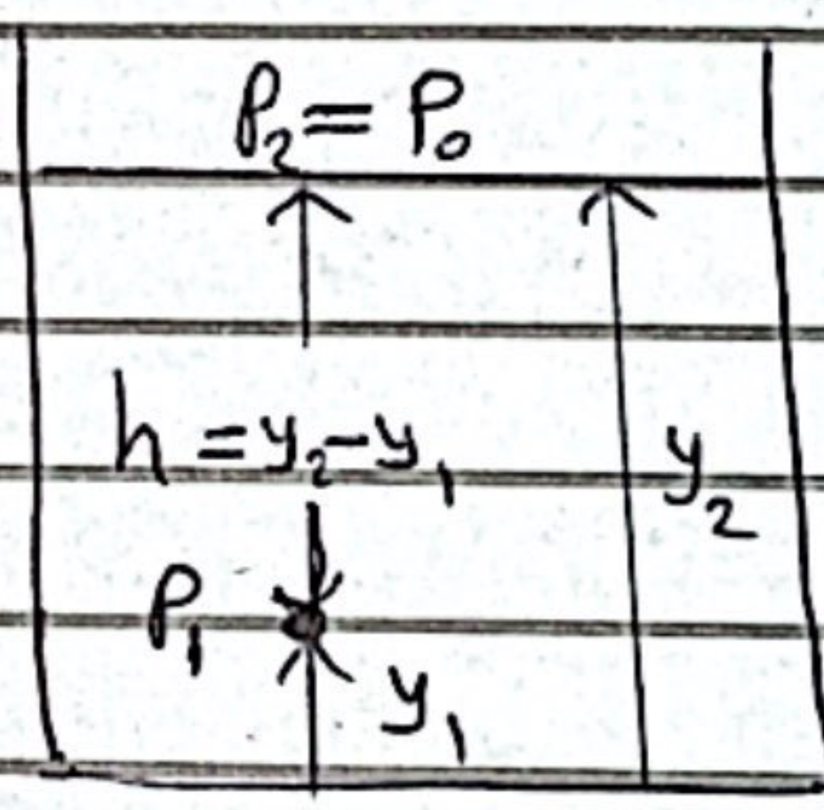
fluid at rest so force perpendicular to the surface if there is external pressure in addition to the weight of the fluid ~~it~~ itself, or if the density of the fluid is not constant, we calculate the pressure at a height y in the fluid

$$\frac{dP}{dy} = -\rho g$$



the negative sign indicates that the pressure decreases with height (increases with depth)

$$\int_{P_1}^{P_2} dP = -\int_{y_1}^{y_2} \rho g \, dy$$



$$P_2 - P_1 = -\rho g (y_2 - y_1)$$

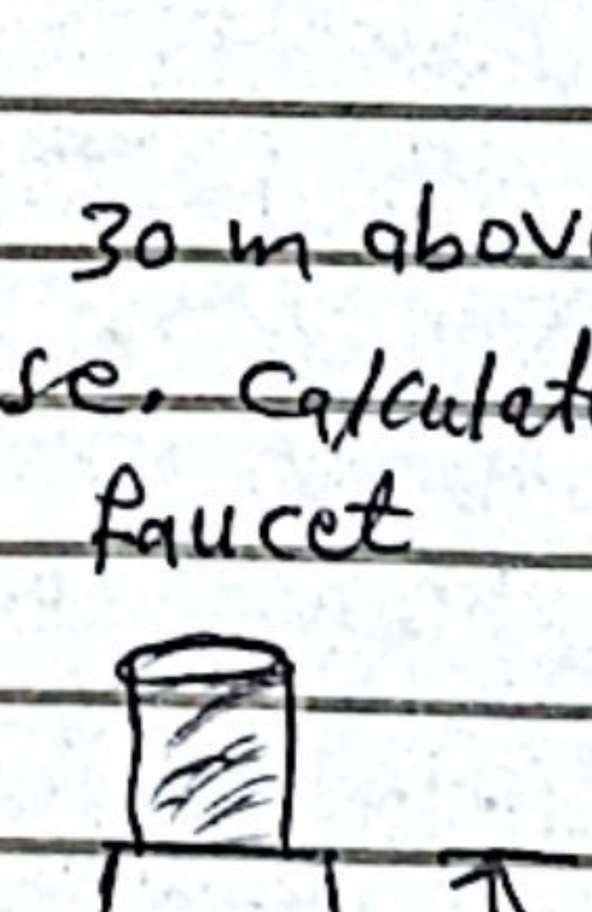
$$P_1 = P_2 + \rho g h$$

$$P = P_0 + \rho g h \quad \rightarrow P_1 = P, \; P_2 = P_0$$

where P_0 is the external pressure at the liquid's top surface.

Example 10-3

The surface of the water in a storage tank is 30 m above a water faucet in the kitchen of ~~the~~ a house. Calculate the difference in water pressure between the faucet and the surface of the water in the tank.



$$P = P_0 + \rho g h$$

$$\Delta P = P - P_0 = \rho g h$$

$$= (1000)(9.8)(30)$$

$$\Delta P = 2.9 \times 10^5 \ Pa$$

10-4 Atmospheric Pressure and Gauge Pressure

At sea level the atmospheric pressure is about 1.013×10^5 Pa; this is called 1 atmosphere (atm).

$$1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

another unit of pressure is the bar

$$1 \text{ bar} = 1 \times 10^5 \text{ Pa}$$

Gauge pressure

it is important to note that tire gauges, and most other pressure gauges, register the pressure above and below atmospheric pressure. This is called gauge pressure P_G.
The absolute pressure is the sum of the atmospheric pressure and the gauge pressure.

$$P = P_0 + P_G$$

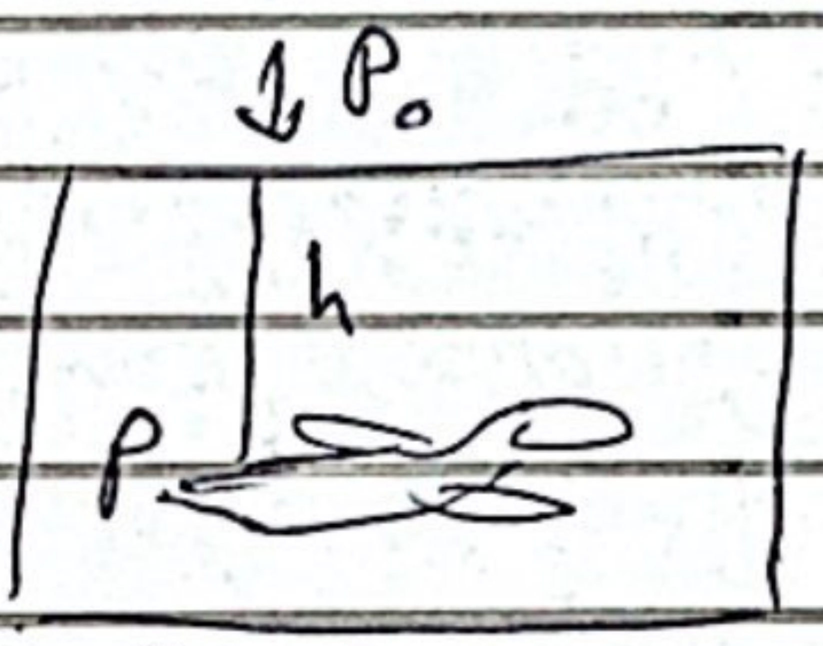
if a tire gauge registers 220 KPa the absolute pressure within the tire is

$$220 \text{ KPa} + 101 \text{ KPa} = 321 \text{ KPa} \simeq (3.2 \text{ atm})$$

Exercise: determine the gauge pressure at a house which is situated 100 m below the level of the storage dam.

Exercise

At what depth below the surface would the pressure on a scuba diver be 1 atm above the pressure at the surface?



$$P - P_0 = \rho g h \Rightarrow h = \frac{P - P_0}{\rho g}$$

$$h = \frac{1.013 \times 10^5}{(1025)(9.8)} = 10.08\ m$$

$\rho_{\text{sea water}} = 1025\ kg/m^3$

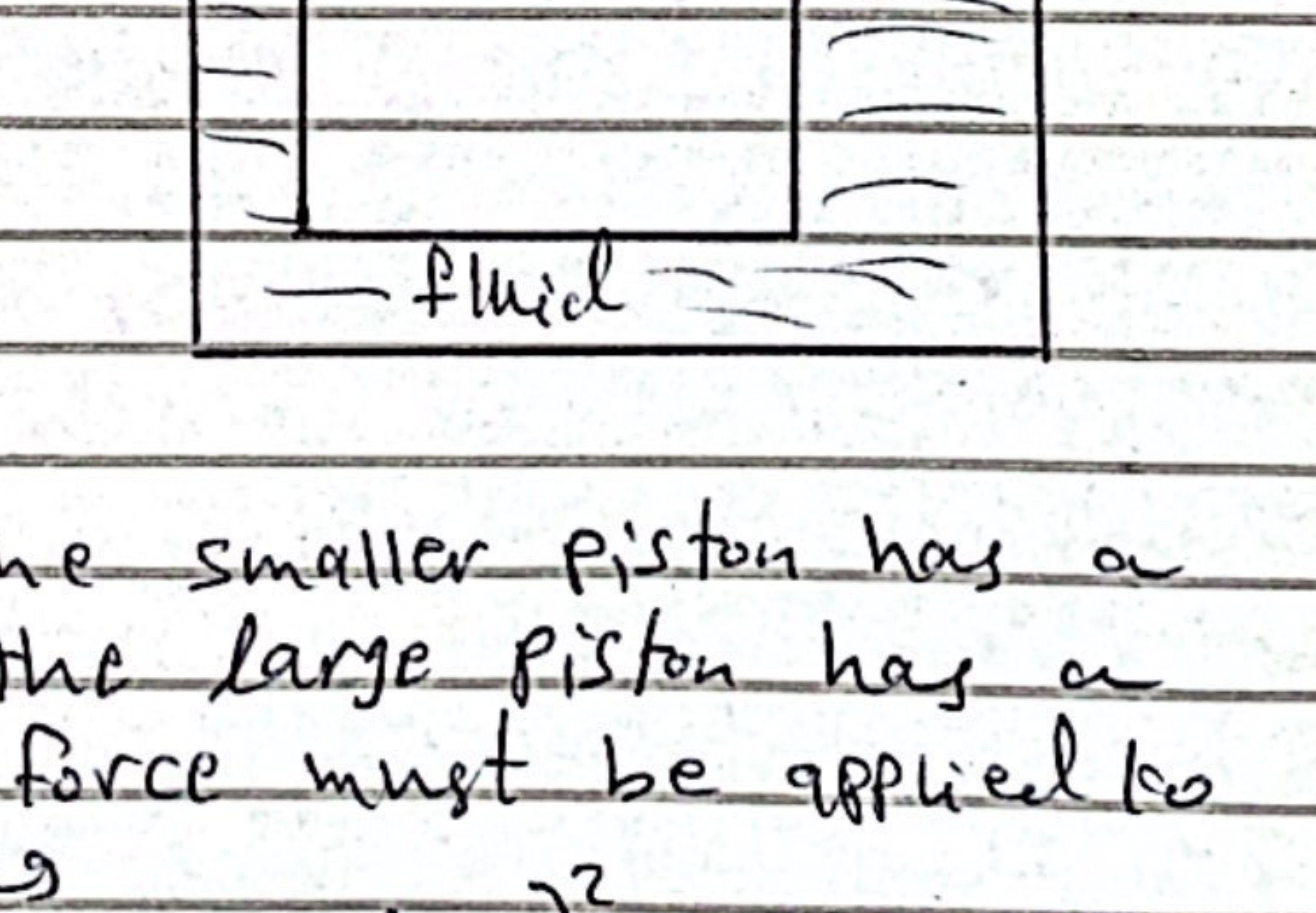
10-5 Pascal's Principle

if an external pressure is applied to a confined fluid, the pressure at every point within the fluid increases by that amount.

$$P_{in} = P_{out}$$

$$\frac{F_{in}}{A_{in}} = \frac{F_{out}}{A_{out}}$$

$$F_{out} = \frac{A_{out}}{A_{in}} F_{in}$$



Exercise

a) In a hydraulic press the smaller piston has a diameter of 40 mm and the large piston has a diameter of 500 m, What force must be applied to lift a mass of 2000 kg

$$F_{in} = \frac{A_{in}}{A_{out}} F_{out} = \frac{r_1^2 \pi}{r_2^2 \pi} f_2 = \frac{(0.04)^2}{(0.05)^2} (2000)(9.8)$$

$$= 125\ N$$

b) if it is required to lift the 200 kg mass 100 mm, how far must the small piston move?

the volume ~~E~~ must be the same on both sides

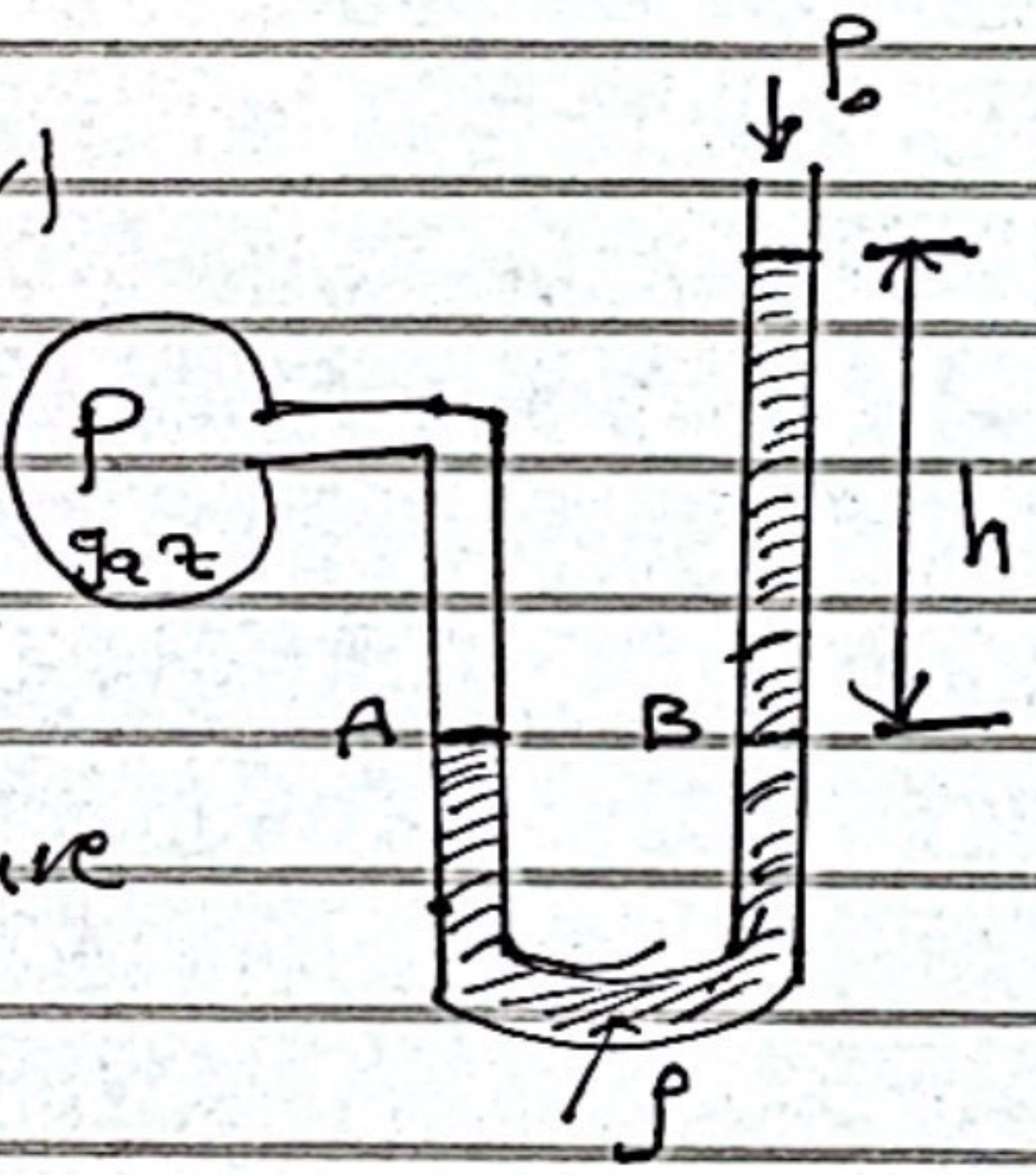
$$V_1 = V_2$$

$$r_1^2 \pi h_1 = r_2^2 \pi h_2$$

$$h_1 = \frac{r_2^2}{r_1^2} h_2 = \left(\frac{0.05}{0.04}\right)^2 (0.1) = 15.6\ m$$

10-6 measurement of pressure; Gauges and the Barometer

- Manometer (open tube manometer)



$$P_A = P_B$$

$$P = P_0 + \rho g h$$

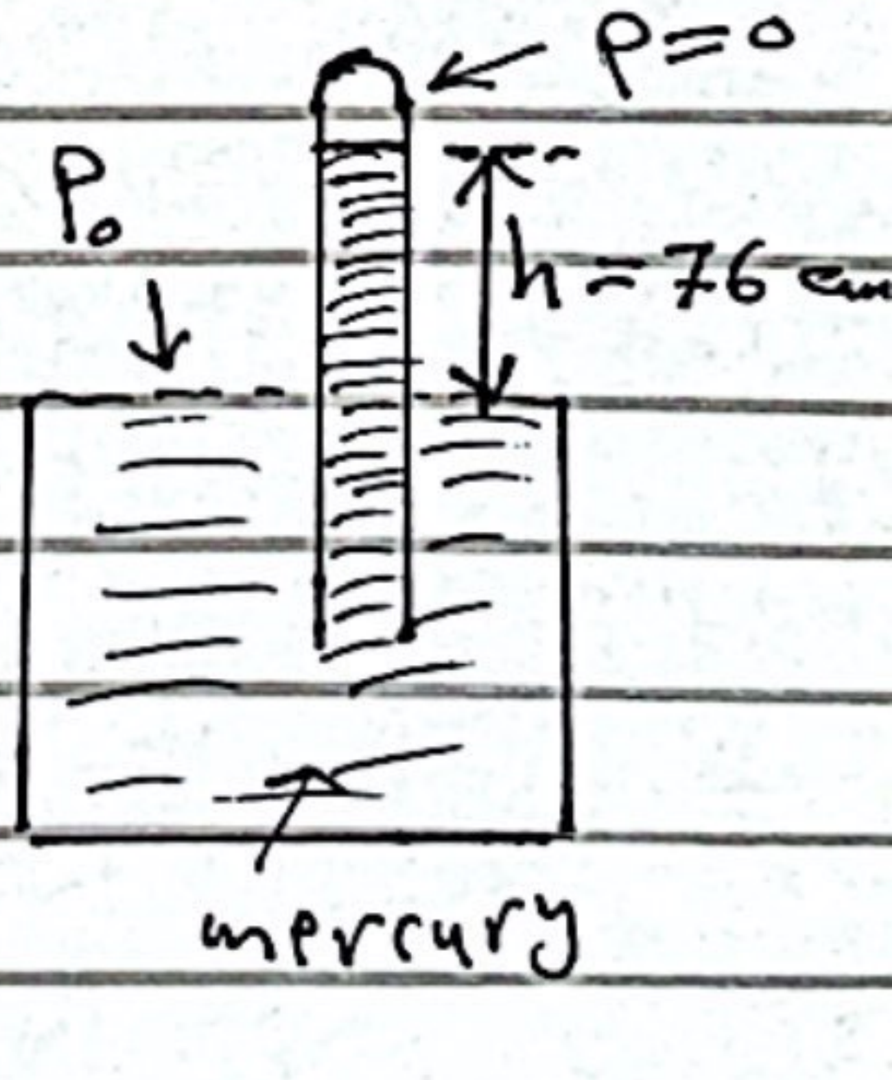
P is the absolute pressure and $P - P_0$ is called the gauge pressure

$$P_G = P - P_0 = \rho g h$$

- Mercury barometer : to measure atmospheric pressure

A long tube closed at one end is filled with mercury and then inverted into a dish of mercury. The closed end of the tube is nearly a vacuum. It follows that

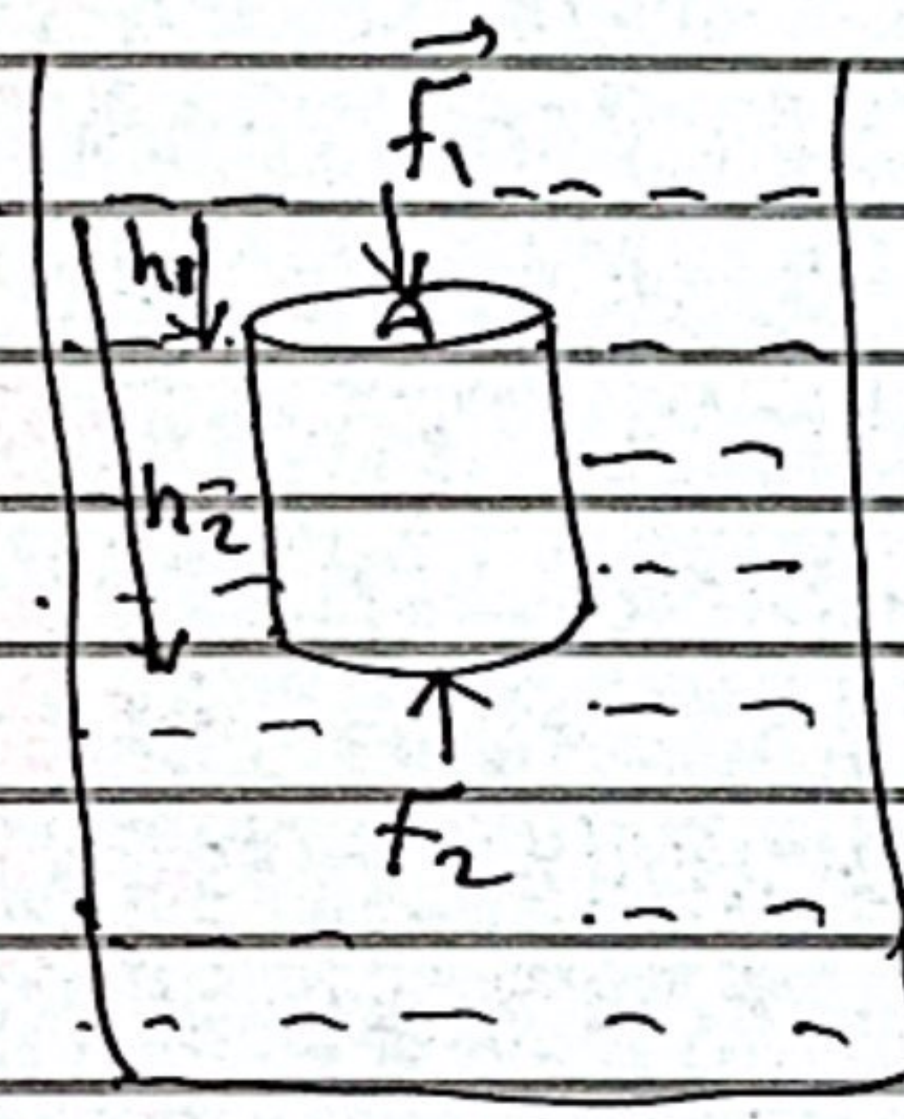
$$P_0 = \rho g h$$



10-7 Buoyancy and Archimedes' principle

consider an object submerged in a fluid. There is net force on the object because the pressure at the top and bottom of it are different.

Since $P = \frac{F}{A}$, we find



$$F_1 = P_1 A = \rho_F g h_1 A$$

$$F_2 = P_2 A = \rho_F g h_2 A$$

$$F_2 - F_1 = \rho_F g A (h_2 - h_1) \equiv F_B$$
$$= \rho_F g V$$

$\therefore F_B = \rho_F g V$ (buoyant force)

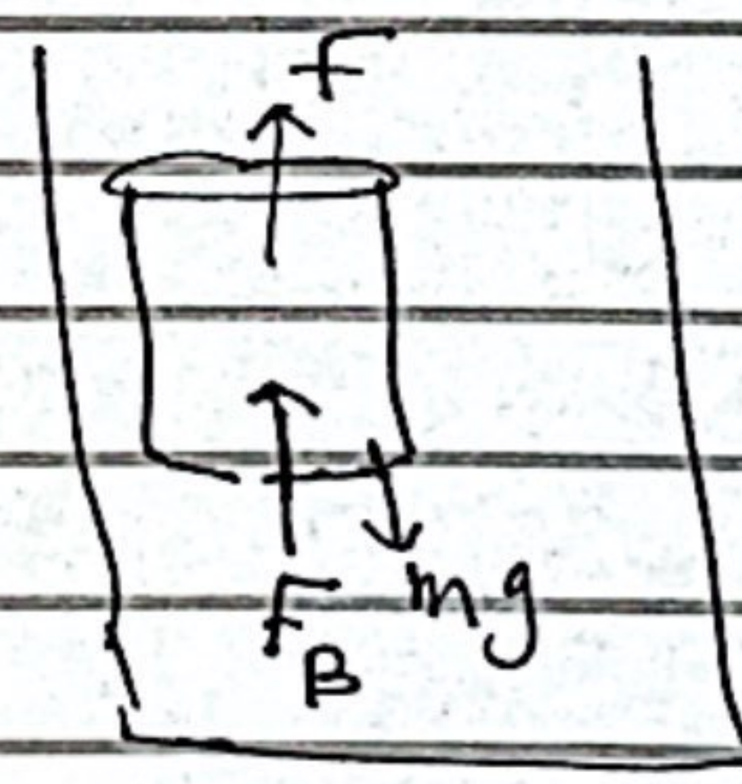
and this is equal to the weight of fluid displaced by the object.

Archimedes' Principle:

the buoyant force on an object immersed (Partially or completely) in a fluid is equal to the weight of the fluid displaced by the object.

Example 10-7

A 70-kg ancient status lies at the bottom of the sea. Its volume is $3\times10^4\ cm^3$. How much force is needed to lift it? (without acceleration)



$$\sum F = ma = 0$$

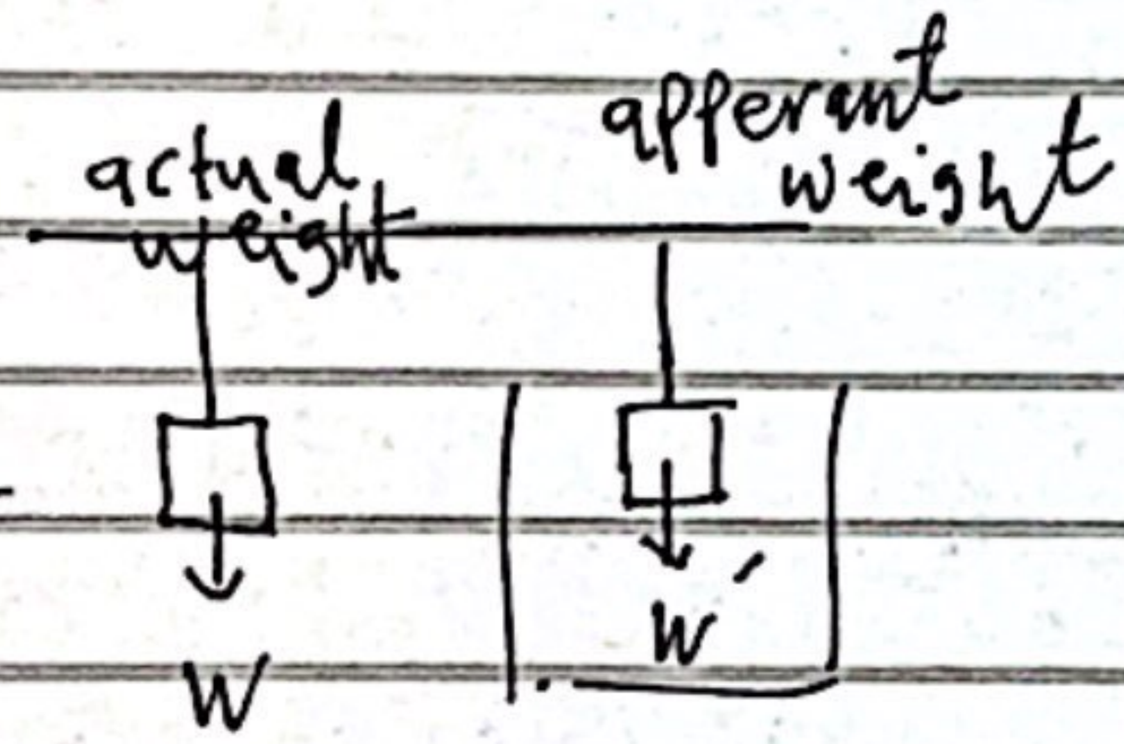
$$F + F_B - mg = 0$$

$$F = mg - F_B = mg - \rho_F g V = 70(9.8) - 1025(9.8)(3\times10^4\times10^{-6})$$
$$= 385\ N$$

Exemple 10-8

when a crown of mass 14.7 kg is submerged in water, an accurate scale reads only 13.4 kg. Is the crown made of gold? ($\rho_{gold} = 19.3 \times 10^{3}$ kg/m^3)

$F_B = W - W' = (14.7 - 13.4)(9.8)$
$= 12.74$ N



F_B = weight of the displaced water

~~the volume of displaced water~~ $= \frac{m}{\rho}$

∴ weight of the displaced water = 12.74 N

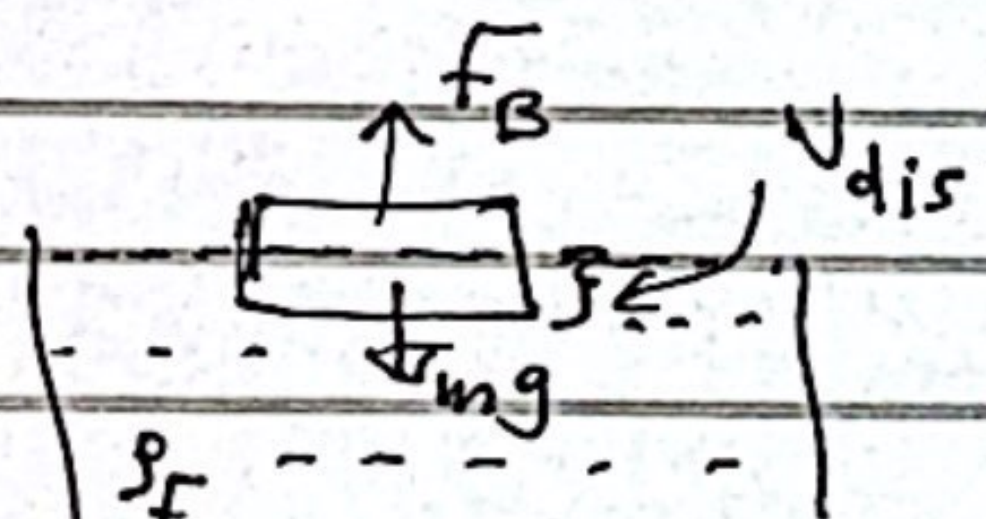
mass " " " $= \frac{12.74}{9.8} = 1.3$ kg

$\rho_{water} = \frac{m}{V} \Rightarrow V = \frac{m}{\rho_{wat}} = \frac{1.3}{1000} = 1.3 \times 10^{-3}$ m^3

$\rho_{crown} = \frac{m_{crown}}{V} = \frac{14.7}{1.3 \times 10^{-3}} = 11.3 \times 10^{3}$ kg/m^3 $\neq 19.3 \times 10^{3}$

that is the crown is not made of gold

□ Floating objects



for floating objects ($\rho_o < \rho_F$)

$F_B = m g$

$\rho_F V_{dis} g = \rho_o V_o g$

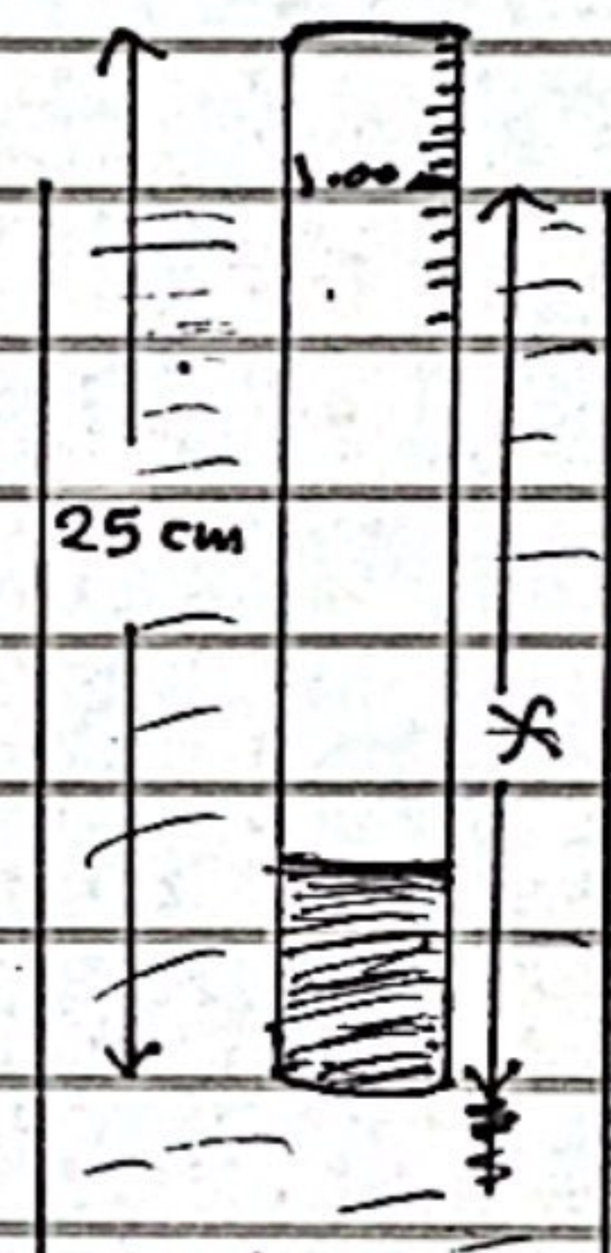
$F_B = \rho_F V_{dis} g$

$mg = \rho_o V_o g$

$\frac{V_{dis}}{V_o} = \frac{\rho_o}{\rho_F}$

Example 10-9 (hydrometer calibration)

A hydrometer is a simple instrument used to measure the specific gravity of a liquid ($\frac{\rho}{\rho_{wat}}$) by indicating how deeply the instrument sinks in the liquid. This hydrometer consists of a glass tube of long 25 cm and 2 cm^2 cross-sectional area and has a mass of 45 g. How far from the end should the 1 mark be placed?



average density of hydrometer $= \frac{m}{V} = \rho_H$

$$\rho_H = \frac{0.045}{(0.25)(2\times10^{-4})} = 900\ kg/m^3$$

$$\frac{V_{dis}}{V_o} = \frac{\rho_o}{\rho_F} = \frac{\rho_H}{\rho_F} = \frac{900}{1000} = 0.9$$

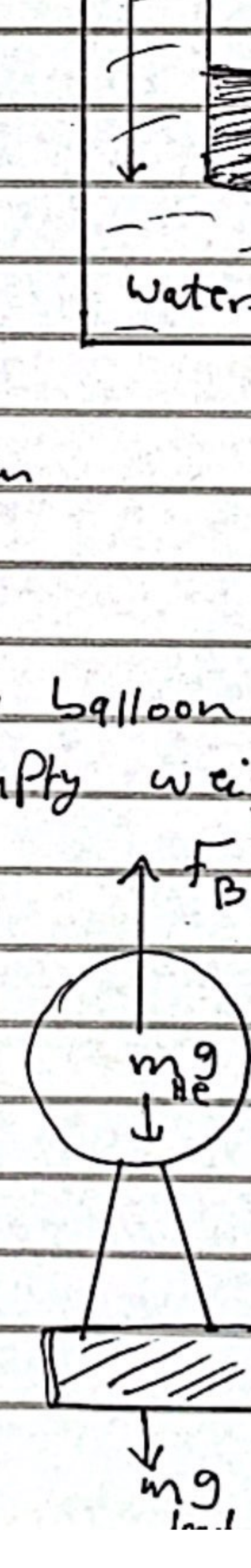
$$V_{dis} = 0.9\,V_o = 0.9\,(0.25)(2\times10^{-4})$$

$$V_{dis} = A\,X = 0.9\,(0.25)(2\times10^{-4})$$

$$\Rightarrow X = 0.9\,(0.25) = 22.5\ cm$$

Example 10-10 (helium balloon)

What volume V of helium is needed if a balloon is to lift a load of 180 kg (including its empty weight)



$\rho_{air} = 1.29\ kg/m^3$

$\rho_{He} = 0.179\ kg/m^3$

$$F_B = M_{He}\,g + m_{load}\,g$$

$$F_B = (M_{He} + m_{load})\,g$$

$$\rho_{air} V g = (M_{He} + m_{load})\,g$$

$$\rho_{air} V = \cancel{g}\,(M_{He} + 180)$$

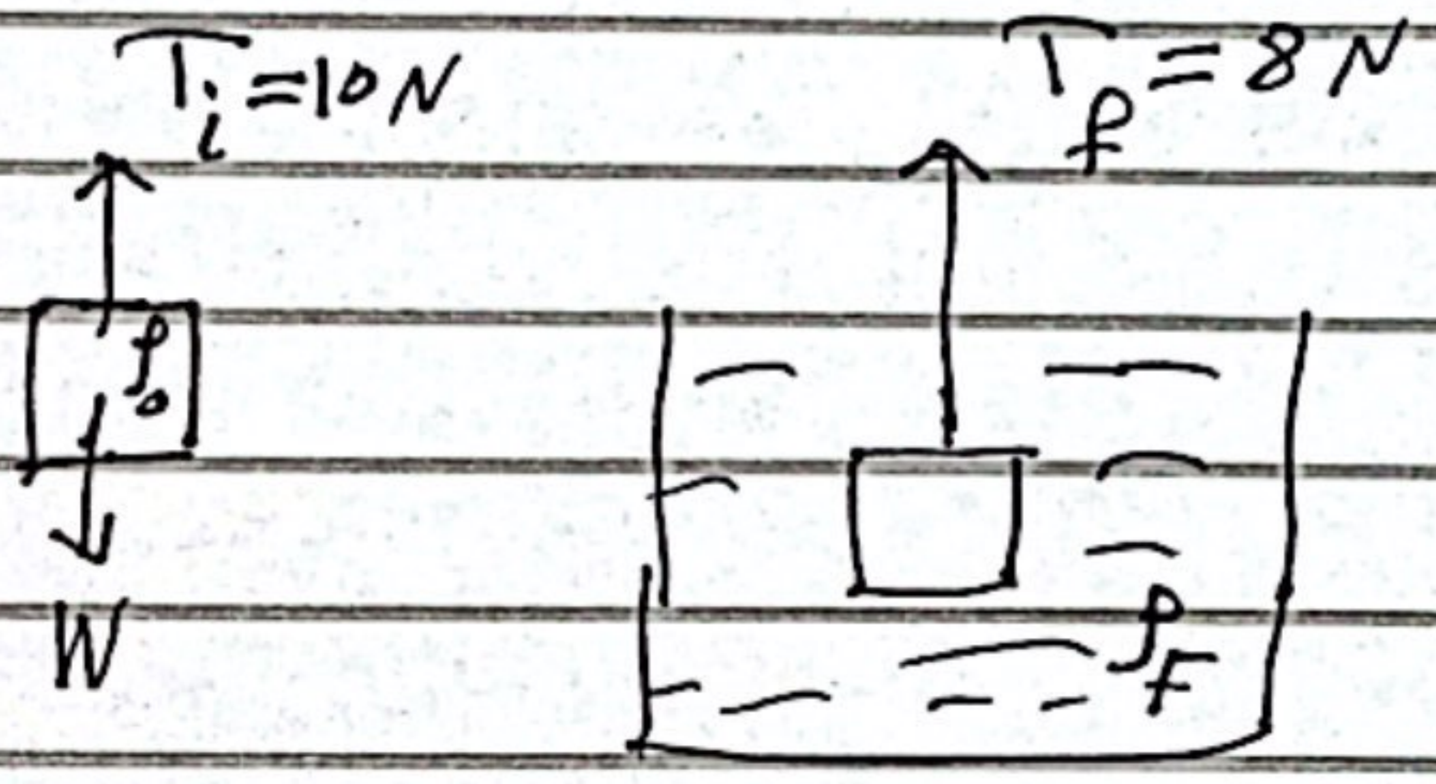
$$\rho_{air} V = (\rho_{He} V + 180)$$

$$V(\rho_{air} - \rho_{He}) = 180$$

$$V = \frac{180}{1.29 - 0.179} = 162\ m^3$$

Exercise:

A piece of metal of unknown volume V is suspended from a string $T_i = 10N$, when it submerged in water $T_f = 8N$, what is the density of the metal?



before $T_i = mg = \rho_o V g$

after $T_f + F_B = mg$

$$T_f = mg - F_B$$
$$= \rho_o V g - \rho_F V g$$
$$T_f = (\rho_o - \rho_F) V g$$

$$\frac{T_f}{T_i} = \frac{(\rho_o - \rho_F) V g}{\rho_o V g} = \frac{\rho_o - \rho_F}{\rho_o} = 1 - \frac{\rho_F}{\rho_o}$$

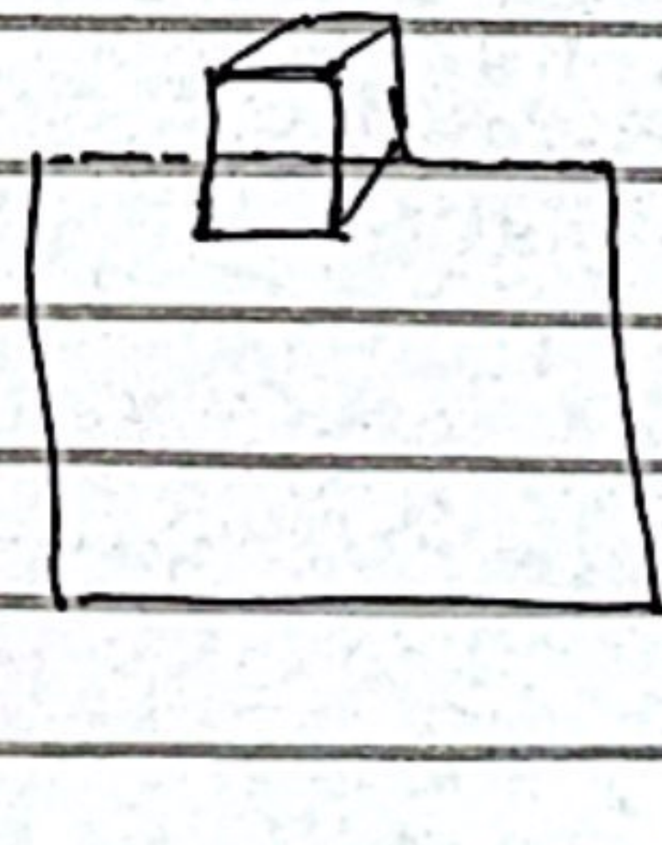
$$\frac{8}{10} = 1 - \frac{\rho_F}{\rho_o} \quad , \quad \rho_F = 10^3 \ kg/m^3$$

$$\frac{\rho_F}{\rho_o} = 1 - \frac{8}{10} = 0.2 \Rightarrow \cancel{\rho_F = 0.2\rho_o = (0.2)(10^3) =}$$

$$\rho_o = \frac{\rho_F}{0.2} = \frac{10^3}{0.2} = 5 \times 10^3 \ kg/m^3$$

Exercise

An ice cube of volume $4 \ cm^3$ is submerged halfway into water, what is the buoyant force experienced by the cube?



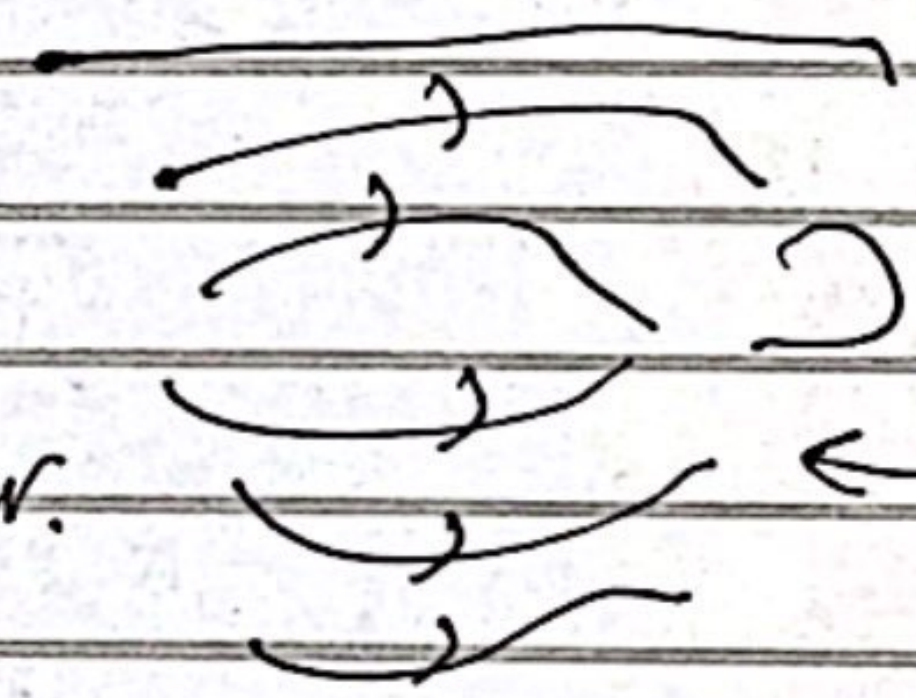
$$F_B = \rho_F V_{dis} \, g = (10^3)(2 \times 10^{-6})(9.8)$$
$$= 1.96 \times 10^{-2} N$$

10-8 Fluids in Motion, Flow Rate and the Equation of Continuity

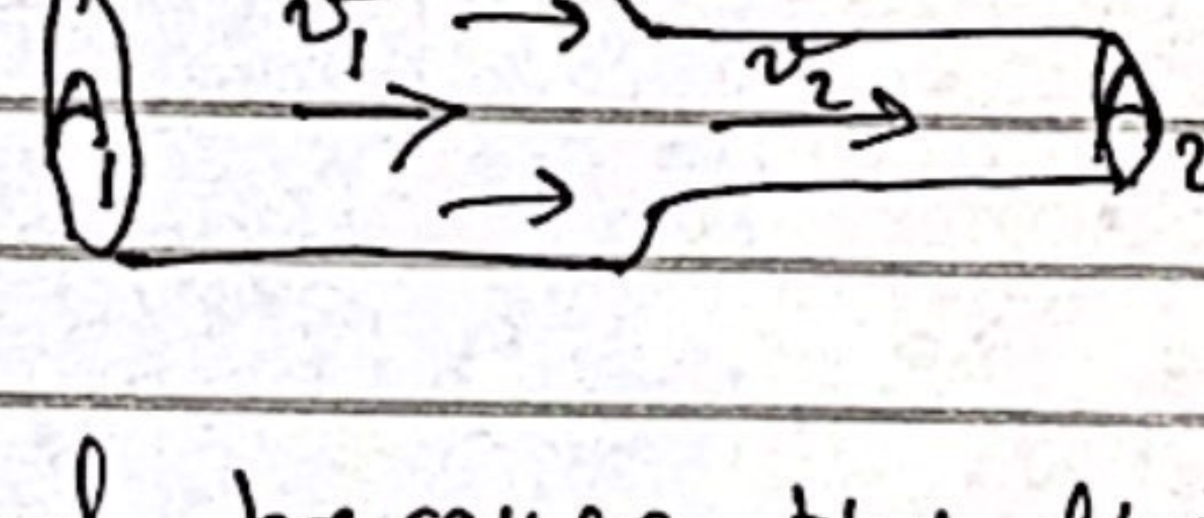
When a fluid in motion, ~~we can~~ its flow can be characterized in one of the two ways:

1) streamline or Laminor flow:
in streamline flow, each particle of the fluid follows a smooth path, and these paths do not cross one another

2) Turbulent flow
above certain speed flow becomes turbulent.
Turbulent flow has eddies, which absorb a great deal of energy and the viscosity will be much greater.



- Equation of continuity
Assume Laminor flow (streamline)
we define the mass flow rate as the mass that passes a given point per unit time



$$\text{mass flow rate} = Q = \frac{\Delta m}{\Delta t}$$

~~Volume~~ Because mass is conserved and because the flow is streamline, the flow rate into the two parts of the tube is the same

$$\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t} \Rightarrow \rho_1 \frac{\Delta V_1}{\Delta t} = \frac{\rho_2 \Delta V_2}{\Delta t}$$

$$\rho_1 \frac{A_1 \Delta x_1}{\Delta t} = \rho_2 \frac{A_2 \Delta x_2}{\Delta t}$$

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

for constant ρ

$$A_1 v_1 = A_2 v_2$$

Example 10-12 (Blood flow)

In humans, blood flows from the heart into the aorta, from which it passes into the major arteries. These branch into the small arteries, which in turn branch into myriads of tiny capillaries. The blood returns to the heart via the veins. The radius of the aorta is about 1.2 cm, and the blood passing through it has a speed of about 40 cm/s. A typical capillary has a radius of about 4×10^{-4} cm, and blood flows through it at speed of about 5×10^{-4} cm/s. Estimate the number of capillaries that are in the body.

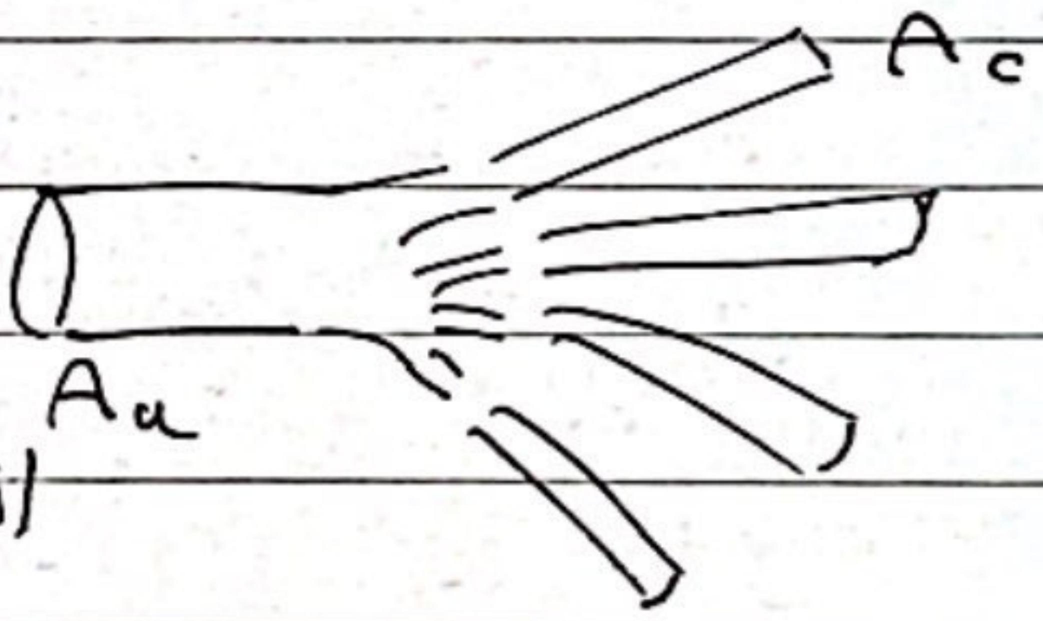
cycle of flow: heart → aorta → major arteries → small arteries

heart ← veins ← capillaries ←

$A_1 v_1 = A_2 v_2$

for capillaries for aorta

$A_1 = N A_c$ (N: number of capillaries)

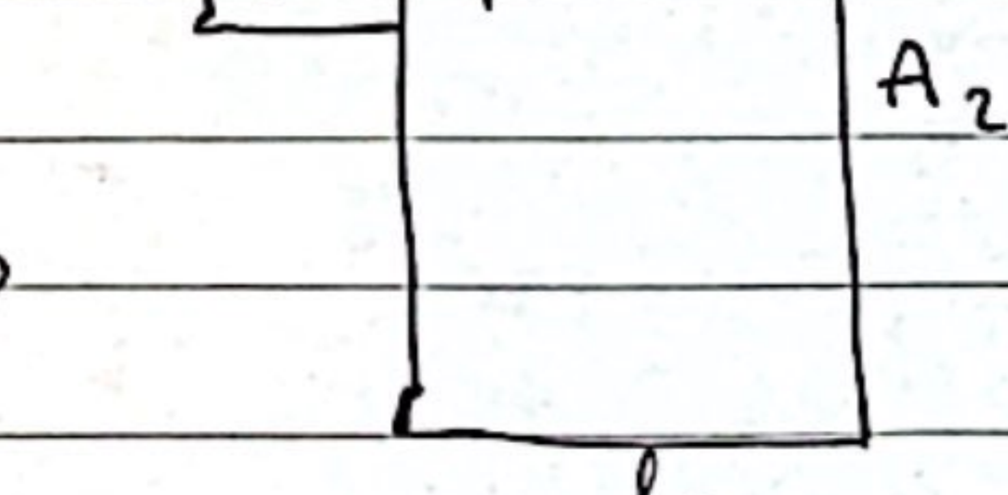


$$N A_c v_c = A_a v_a \Rightarrow N = \frac{\pi r_a^2 v_a}{\pi r_c^2 v_c} = \left(\frac{r_a}{r_c}\right)^2 \frac{v_a}{v_c} =$$

$$N = \left(\frac{1.2\times10^{-2}}{4\times10^{-4}}\right)^2 \left(\frac{0.4}{5\times10^{-4}}\right) = 7\times10^{9}$$

Example 10-13

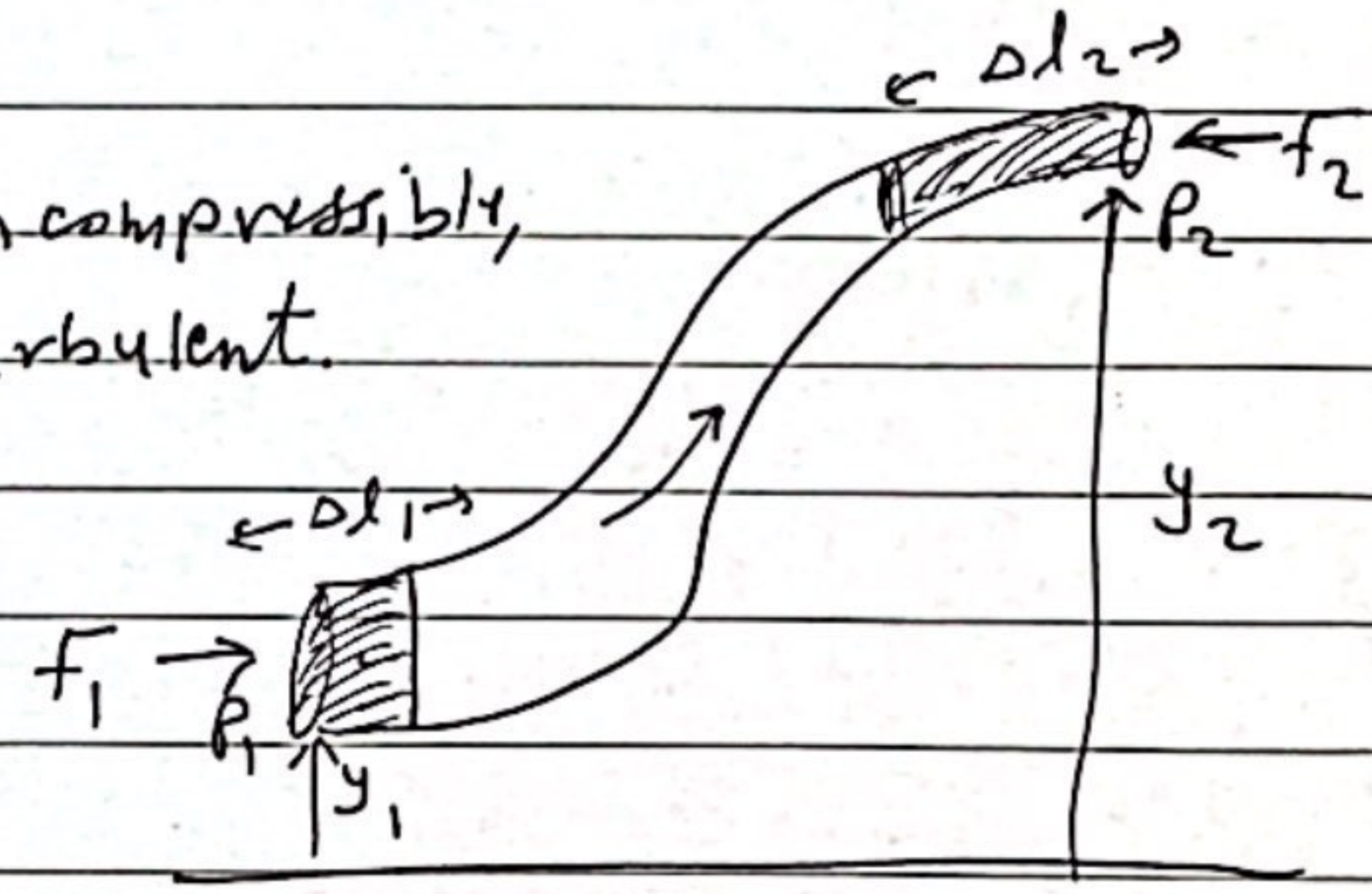
What area must a heating duct have if air moving 3 m/s along it can replenish the air every 15 minutes in a room of volume $300 m^3$?



$$A_1 v_1 = A_2 v_2 = A_2 \frac{l_2}{t} = \frac{V_2}{t}$$

$$A_1 = \frac{V_2}{v_1 t} = \frac{300}{3(15)(60)} = 0.11\ m^3$$

10-9 Bernoulli's Equation



we assume that the fluid is incompressibly, nonvisous, and flows in a nonturbulent.

consider the flow through nonuniform pipe in the time Δt.
The force on the lower end of the fluid is $P_1 A_1$.

The work done on the lower end of the fluid by the fluid behind it is

$$W_1 = F_1 \Delta X_1 = P_1 A_1 \Delta l_1 = P_1 V$$

in a similar manner, the work done on the fluid on the upper portion in the time Δt is

$$W_2 = -P_2 A_2 \Delta l_2 = -P_2 V$$

where V is the volume of the lower and upper regions which are the same.

The net work done on the fluid is

$$W_{net} = \Delta K$$

$$W_1 + W_2 + W_3 = \Delta K$$

$W_3 = -\Delta U = -mg(y_2 - y_1)$ work done by grivity

$$\Rightarrow P_1 V - P_2 V - mg(y_2 - y_1) = \frac{1}{2} m (v_2^2 - v_1^2) \quad , m = \rho V$$

$$P_1 V + mgy_1 + \frac{1}{2} m v_1^2 = P_2 V + mgy_2 + \frac{1}{2} m v_2^2$$

$$P_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2$$

Example 10-14

Water circulates throughout a house in a hot-water heating system. if the water is pumped at a speed of 0.5 m/s through a 4-cm diameter pipe in the basement under a pressure of 3 atm, what will be the flow speed and pressure in a 2.6-cm diameter pipe on the second floor 5 m above? Assume the pipes do not divide into branches

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1}{A_2} v_1 = \frac{r_1^2 \pi}{r_2^2 \pi} v_1 = \frac{(0.02)^2}{(0.013)^2} (0.5) = 1.2 \text{ m/s}$$

$$P_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2$$

$$P_2 = P_1 + \rho g (y_1 - y_2) + \frac{1}{2} \rho (v_1^2 - v_2^2)$$

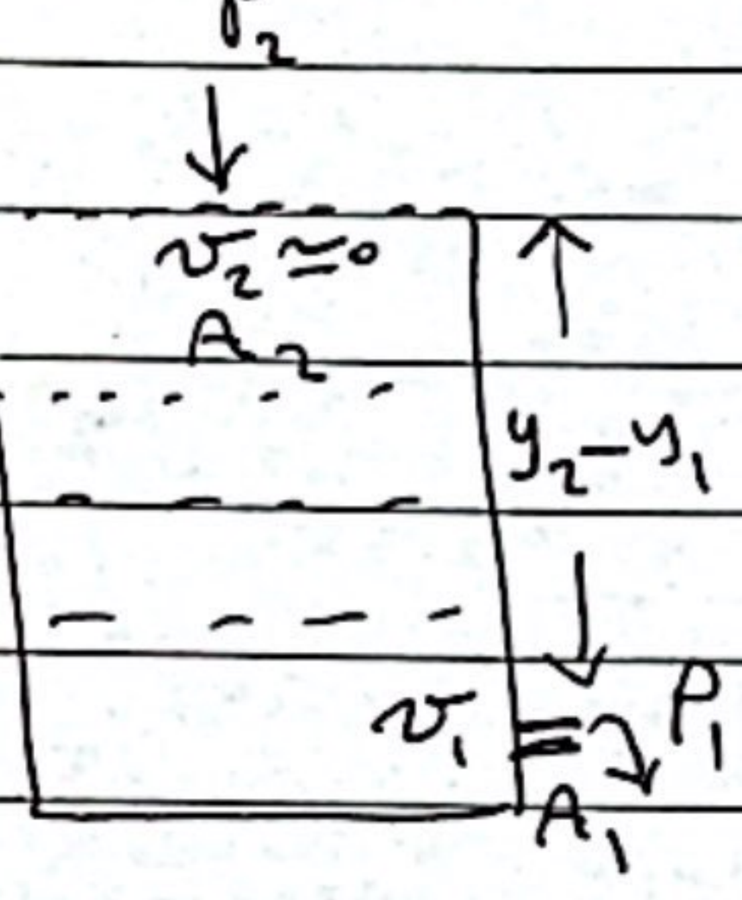
$$= (3 \times 10^5) + 10^3 (9.8)(-0.5) + \frac{1}{2}(10^3)[0.5^2 - 1.2^2]$$

$$= 2.5 \times 10^5 \text{ N/m}^2 = 2.5 \text{ atm}$$

10-10 Applications of Bernoulli's Principle: Torricelli, Airplanes, Baseballs, Blood flow

Torricelli's theorem:

Consider an open tank filled with a liquid



$P_1 = P_2$ (atmospheric pressure)

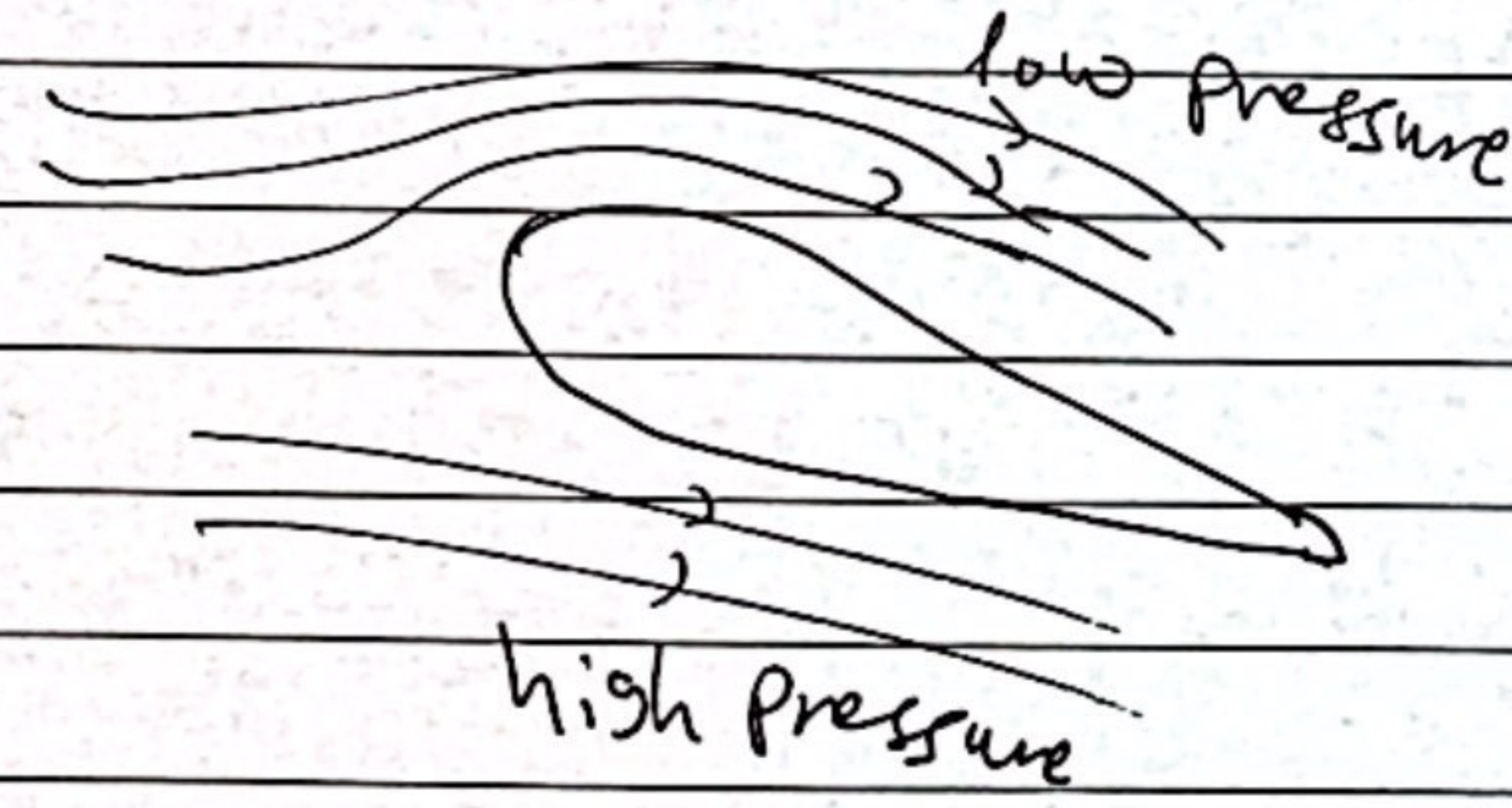
$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$v_2 \simeq 0 \quad \left(v_2 = \frac{A_1}{A_2} v_1 \simeq 0, \; A_2 \gg A_1\right)$$

$$\frac{1}{2}\rho v_1^2 + \rho g y_1 = \rho g y_2$$

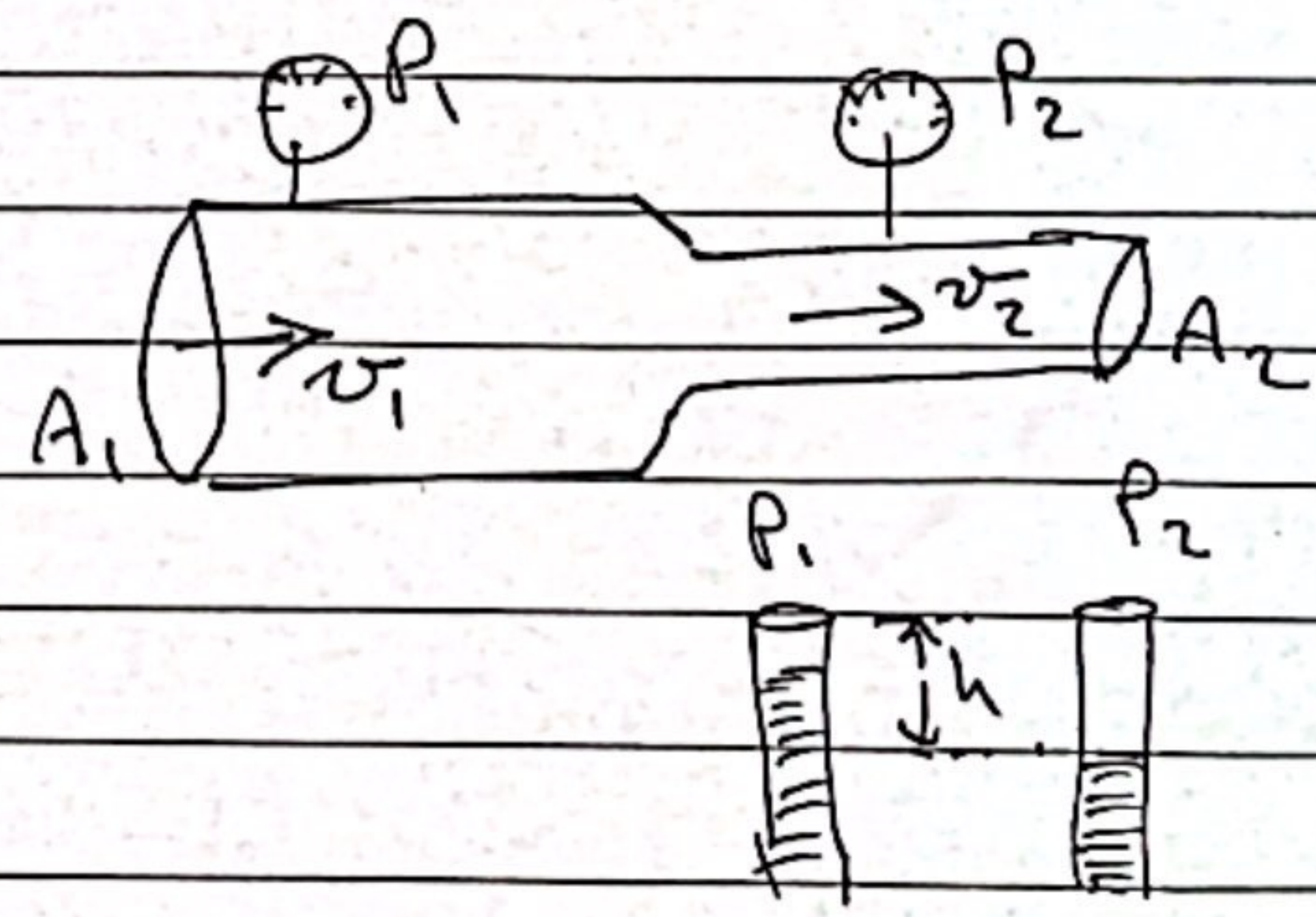
$$v_1 = \sqrt{2g(y_2 - y_1)}$$

Airplanes: lift on an airplane wing is due to the different air speeds and pressures on the two surfaces of the wing



Venturi meter:

it can be used to measure fluid flow by measuring pressure differences



$$y_1 = y_2$$

$$A_1 v_1 = A_2 v_2$$

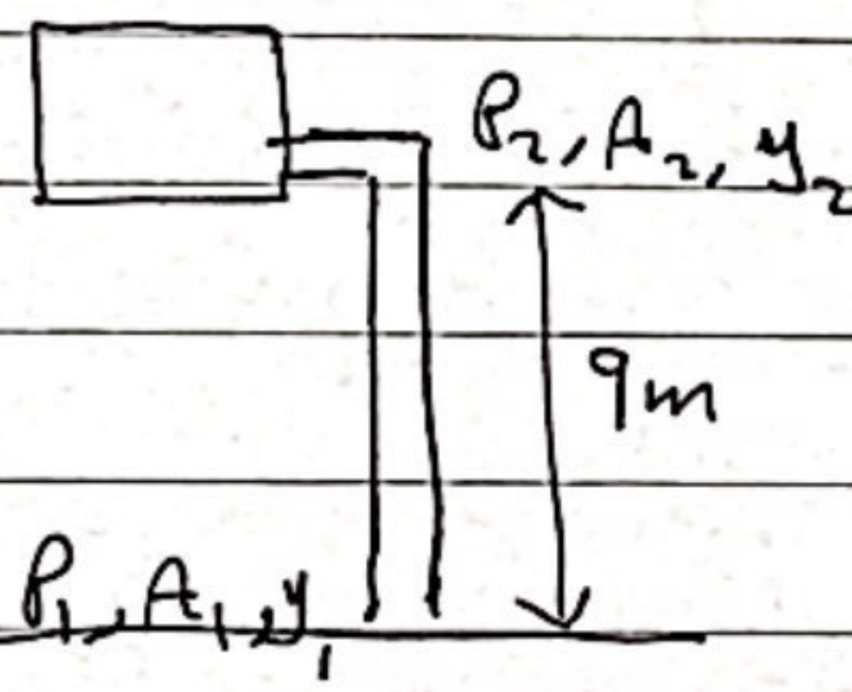
$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$v_1 = \sqrt{\frac{2\,\Delta P}{\rho\left(\left(\frac{A_1}{A_2}\right)^2 - 1\right)}}$$

$\Delta P = -\rho g h$

Exercise

In order to maintain water pressure, some houses store tap water (ماء الإسالة) in a tank in the ceiling. If a water tank is in the ceiling a two storey house (طابقين) (9 m from ground level), what is the pressure of the water at ground level if the volume flow rate is 1.5 Litres per minute? The pipes assumed to have constant cross section.



$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

since $A_1 = A_2 \Rightarrow v_1 = v_2$

$\Rightarrow$

$$P_2 - P_1 = \rho g (y_1 - y_2)$$

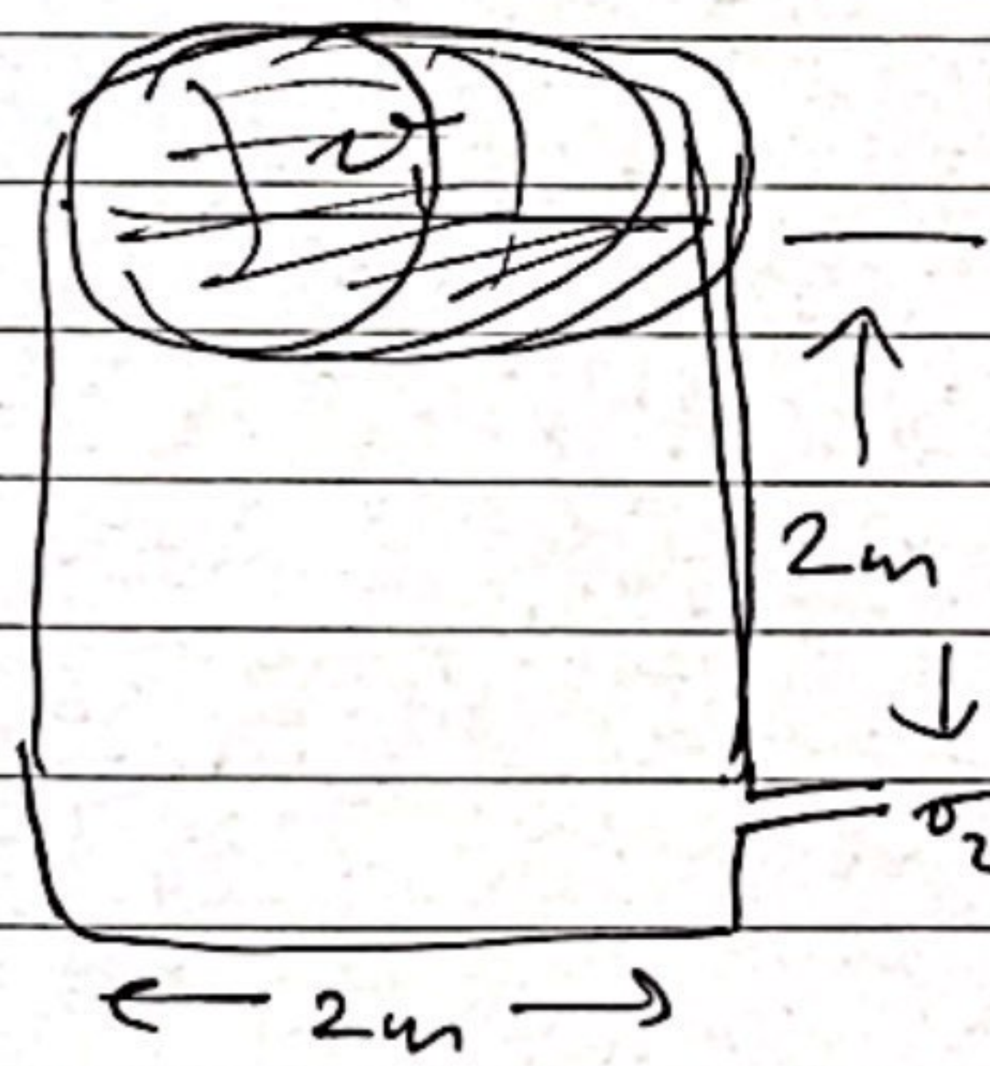
$$= 1000(9.8)(9-0) = 88.2\ kPa$$

Exercise

A circular water tank (diameter 2m) has a small circular pipe near the base with a diameter of 15 mm

a) what is the initial flow rate out of the pipe

b) when the water level has dropped to 1 m, what is the flow rate then?



a) $A_1 v_1 = A_2 v_2$

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{(2)^2}{(0.015)^2} v_1 = 17778\ v_1 \ , \quad v_2 \gg v_1$$

since $P_1 \simeq P_2$, $v_1 \simeq 0$

$$\therefore \quad v_2 = \sqrt{2 g y_2} = \sqrt{2(9.8)(2)} = 6.3\ m/s$$

b) when $y = 1$

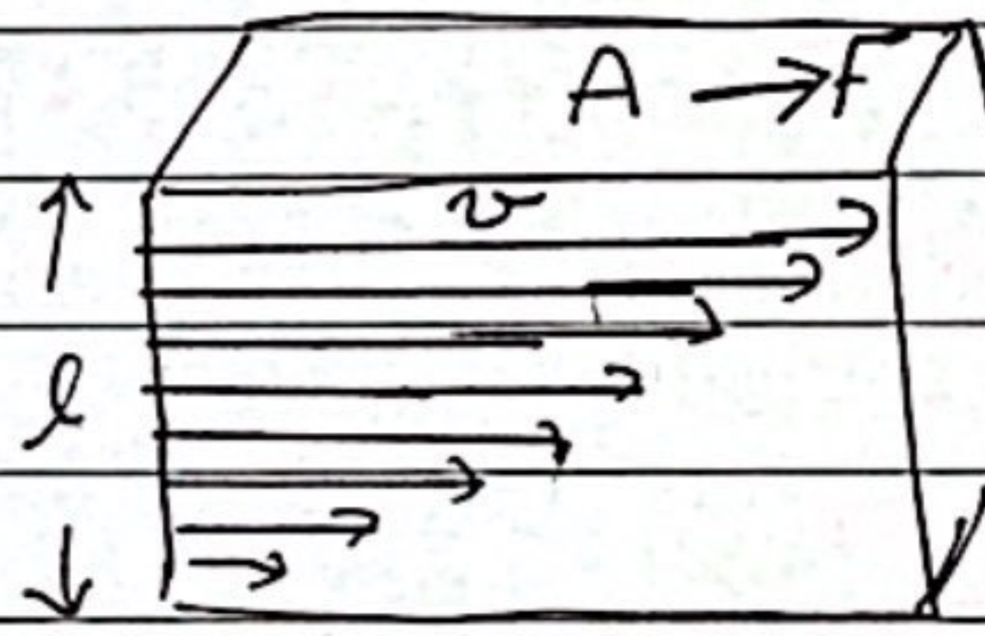
$$v_2 = \sqrt{2gy} = \sqrt{2(9.8)(1)} = 4.4\ m/s$$

10-11 Viscosity

Real fluids have internal friction forces called viscosity. and are not incompressible.

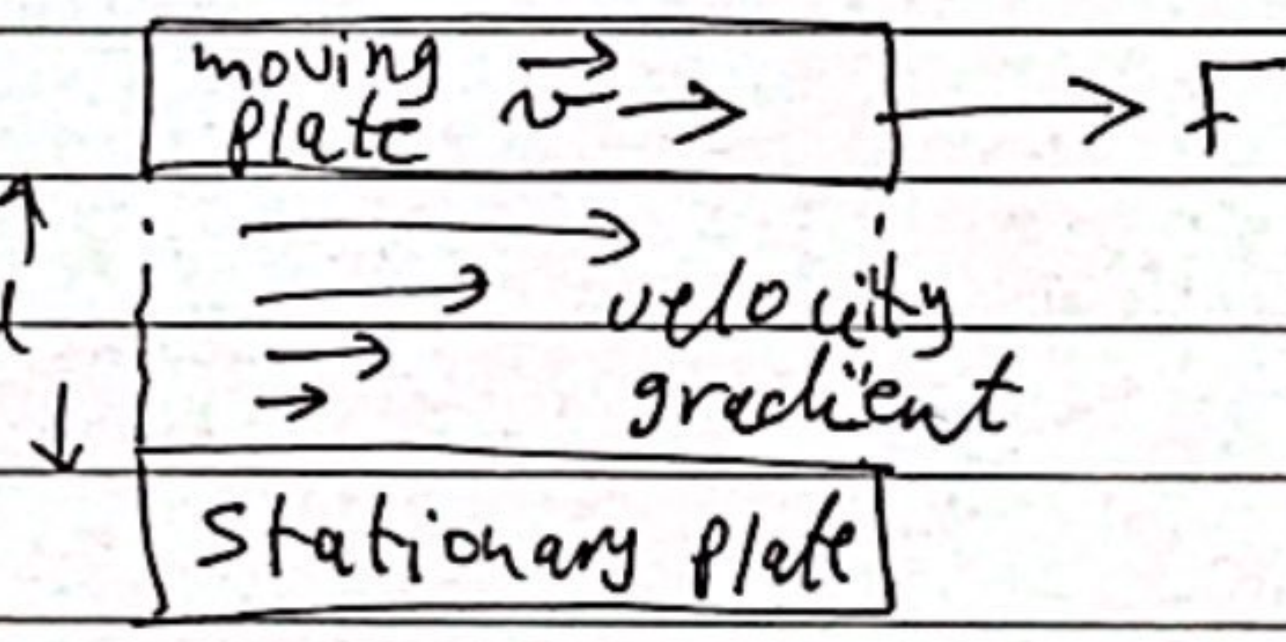
So far we have only studied the properties of ideal fluids. Additional forces come into play when real fluids are in motion.

Viscosity is the internal friction in a fluid which tends to prevent it from flowing when subjected to an applied force.



consider a layer of liquid between two solid surfaces in which the lower surface is fixed and the upper surface moves to the right with a velocity v. The force F has the proportionals

$F \sim A$, $F \sim \frac{v}{l}$



$$F = \eta \frac{Av}{l} \quad \text{or} \quad \eta = \frac{Fl}{Av}$$

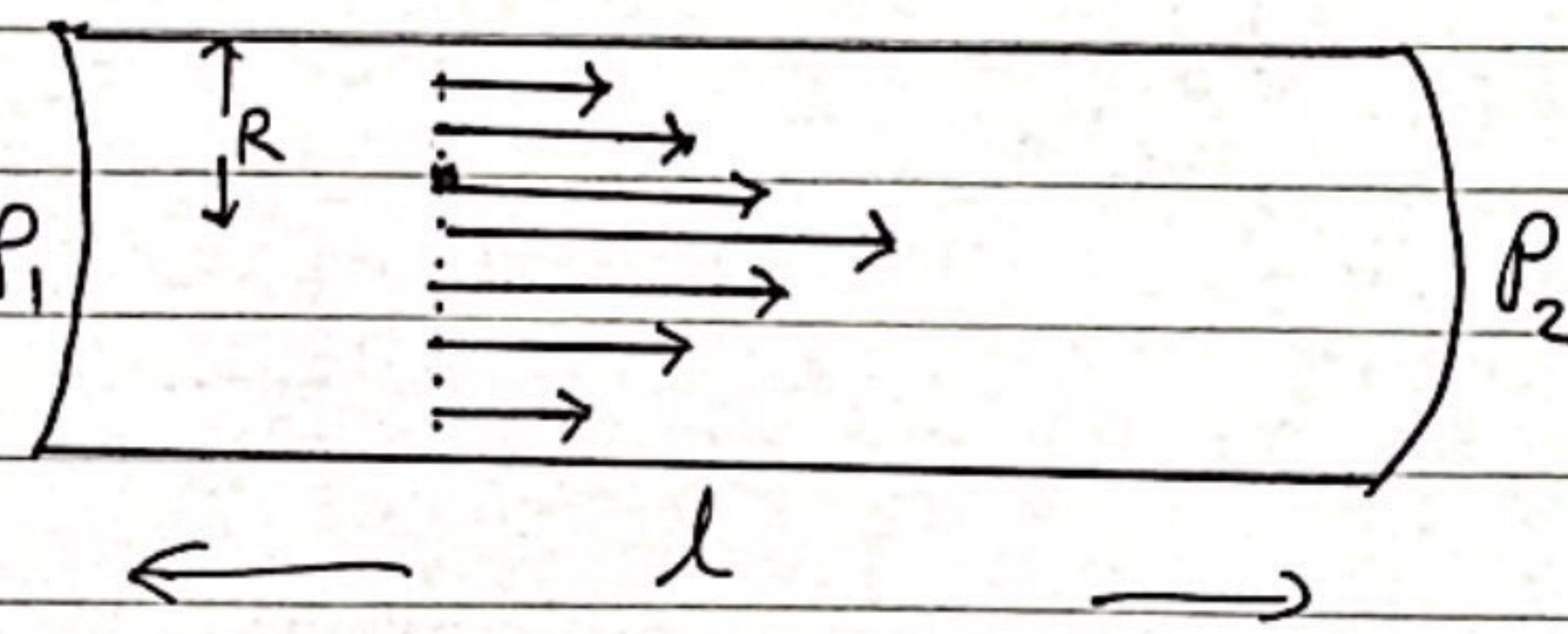
η: is the coefficient of viscosity

the SI unit of η is $N \cdot s/m^2 = P_a \cdot s$

in cgs system the unit is $dyne \cdot s/cm^2 \equiv P$ (poise)

10-12 Flow in Tubes: Poiseuille's Equation, Blood Flow

Consider a horizontal fluid flow through a tube, if there is no viscosity, fluid will flow through a level tube without an applied force.



Due to viscosity, a pressure difference is required for flow.

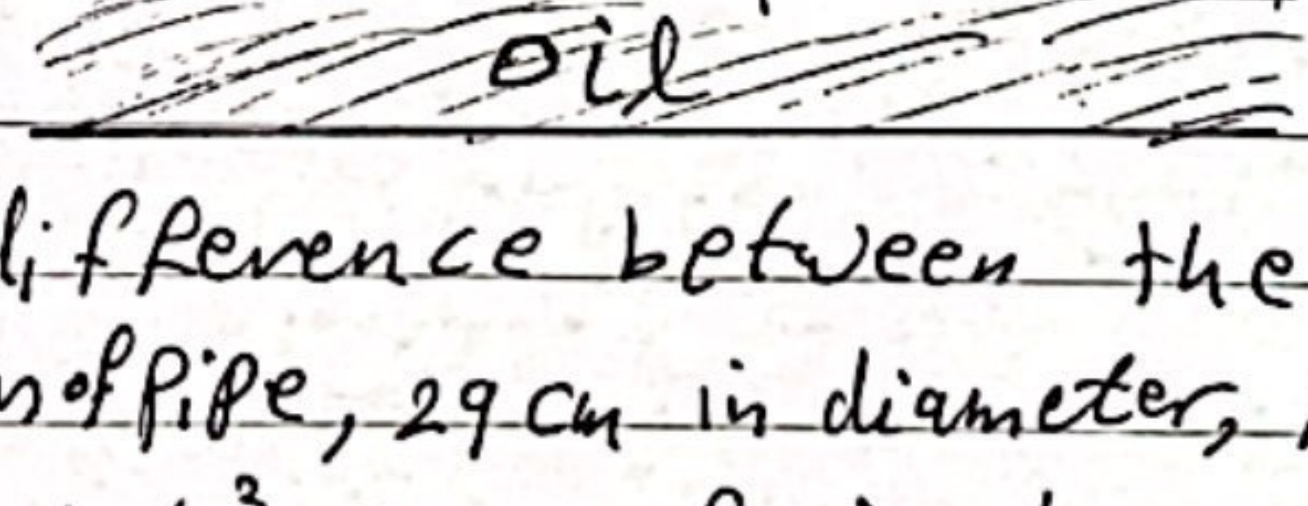
The flow rate is proportional to the pressure difference, inversely proportional to the length of the tube, and proportional to the fourth power of the radius of the tube.

$$Q = \frac{\pi R^4 (P_1 - P_2)}{8 \eta l} \quad \text{(Poiseuille's equation)}$$

Exercise

A metal plate of area $0.02\,m^2$ moves at constant speed of $0.1\,m/s$ ~~100 m/s~~ parallel to another much larger plate. The $0.1\,mm$ space between the plates is filled with an oil of viscosity $0.15\,P_a \cdot s$. What is the force driving the moving plate?

$$F = \eta A \frac{v}{l} = (0.15)(0.02)\frac{0.1}{10^{-4}} = 3N$$



Prob. 62

Exercise: What must be the pressure difference between the two ends of a 1.6 Km section of pipe, 29 cm in diameter, if it is to transport oil ($\rho = 950\ Kg/m^3$, $\eta = 0.2\ P_a \cdot s$) at a rate of $650\ cm^3/s$

$$Q = \pi R^4 \Delta P / 8 \eta l$$

$$\Delta P = \frac{8 \eta l Q}{\pi R^4} = \frac{8(0.2)(1600)(650 \times 10^{-6})}{\pi (14.5 \times 10^{-2})^4} =$$

example 9.18 serway

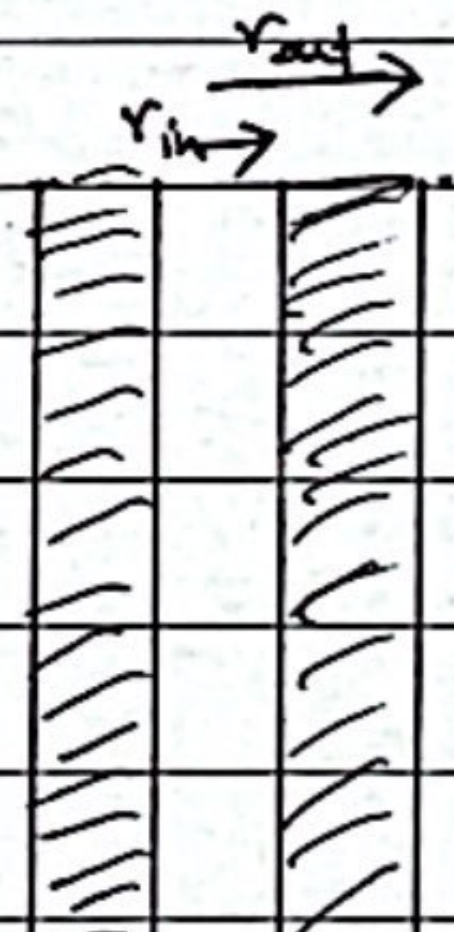
Exercise: A patient receives a blood transfusion through a needle of radius 0.2 mm and length 2 cm. The density of blood is 1050 kg/m³. The bottle supplying the blood is 0.5 m above the patient's arm. What is the rate of flow through the needle?

$$\Delta P = P_1 - P_2 = \rho g h = (1050)(9.8)(0.5) = 5.15 \times 10^{3}\ Pa$$

$$Q = \frac{\pi R^4 (P_1 - P_2)}{8 \eta l} = \frac{\pi (2\times10^{-4})^4 (5.15\times10^{3})}{8 (2.7\times10^{-3})(2\times10^{-2})} = 6\times10^{-8}\ m^3/s$$

Exercise (Prob. 58)

A viscometer consists of two concentric cylinders, 10.2 cm and 10.6 cm in diameter. A liquid fills the space between them to a depth of 12 cm. The outer cylinder is fixed, and a torque of 0.024 m.N keeps the inner cylinder turning at a steady rotational speed of 57 rev/min. What is the viscosity of the liquid?



$$F = \eta A \frac{v}{l}$$

$$\eta = \frac{F l}{A v} = \frac{\left(\frac{\tau}{r_{in}}\right)(r_{out} - r_{in})}{(2\pi r_{avg} h)(\omega r_{in})}$$

$$\eta = \frac{\left(\frac{0.024}{0.051}\right)(0.2\times10^{-2})}{(2\pi)(0.052)(0.120)\left(57 \frac{rev}{min} \frac{2\pi r}{rev} \frac{1\ min}{60\ s}\right)(0.05)}$$

$$\eta = 7.9\times10^{-2}\ Pa \cdot s$$