Chapter 4

Dynamics: Newton's Laws of Motion

4.1 □ Force: Any kind of a push or a pull on an object.

4.2 □ Newton's first law of motion
Every object continues in its state of rest or of a uniform velocity in a straight line, as long as no net force acts on it. (law of inertia).

4.3 □ Mass: mass is a measure of inertia of an object. in SI units, the unit of mass is kilogram (kg)

□ Weight: it is the force exerted by the gravity on an object ($W = mg$).

4.4 □ Newton's second law of motion
The acceleration of an object is directly proportional to the net force acting on it, and is inversely proportional to the objects mass.

$$\sum \vec{F} = m\vec{a}, \quad \sum F_x = ma_x, \quad \sum F_y = ma_y, \quad \sum F_z = ma_z$$

SI: the unit of force is Newton (N), where
$1\ N = 1\ kg \cdot m/s^2$

cgs: the unit of force is dyne ($= g \cdot cm/s^2$)
British: " " " " " pound (Lb),

$1\ Lb = slug \cdot ft/s^2$

conversion factors: $1\ dyne = 10^{-5}\ N$
$1\ Lb = 4.45\ N$
$1\ slug = 14.6\ kg$

Example 4.2

Estimate the net force needed to accelerate

a) a 1000-kg car at $\frac{1}{2}g$

b) a 200-gram apple at the same rate.

Solution

a) $F = ma = 1000\left(\frac{1}{2}(9.81)\right) \simeq 5000\ N$

b) $F = ma = 0.2\left(\frac{1}{2}(9.81)\right) \simeq 1\ N$

Example 4.3

What average net force is required to bring a 1500-kg car to rest from a speed of 100 Km/h within a distance of 55 m ?

Solution

we have $v_0 = 100\ Km/h = 27.8\ m/s$, $v_f = 0$, $\Delta x = 55\ m$

$$v_f^2 = v_0^2 + 2a\Delta x$$

$$0 = (27.8)^2 + 2a(55) \Rightarrow a = -7\ m/s^2$$

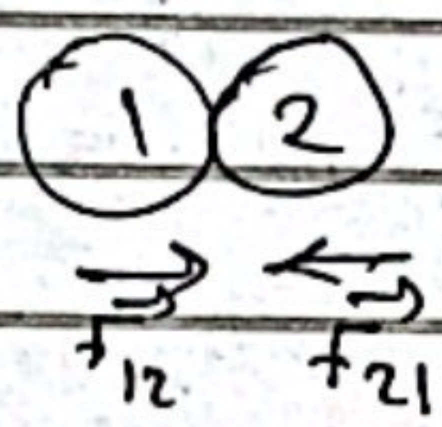
$$\Sigma F = ma = 1500(-7) = -1.1 \times 10^4\ N$$

the negative sign means: that the force must be exerted in the direction ~~of~~ opposite to the initial velocity.

4.5 Newton's Third Law of Motion

whenever one object exerts a force on a second object, the second object exerts an equal force in the opposite direction on the first.
(Action and reaction forces are equal and opposite)

$$\vec{F}_{12} = -\vec{F}_{21}$$



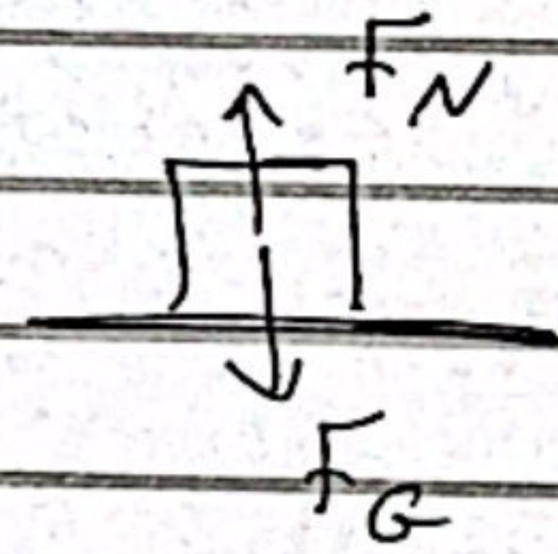
4.6 Weight - the force of gravity and the normal force

the gravitational force on an object is

$$\vec{F}_G = m\vec{g}$$

when a contact force acts perpendicular to the common surface of contact it is referred to as the normal force $\vec{F}_N$

At equilibrium : $F_N = F_G$

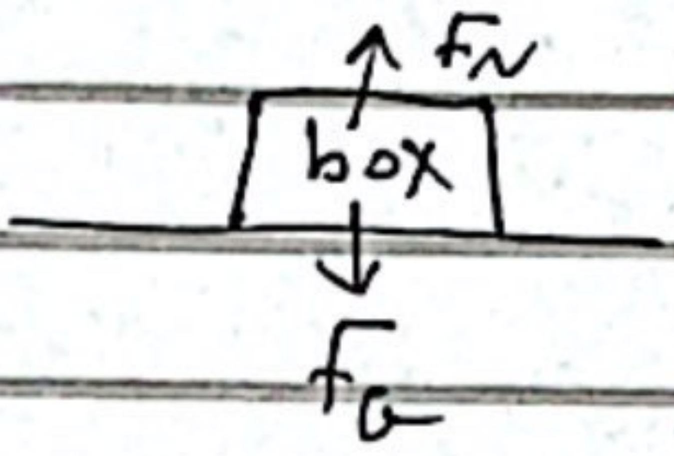


Example 4-6
A friend has given you a special gift, a box of mass 10 kg with a mystery surprise inside. The box is resting on the smooth (frictionless) horizontal surface of a table.
a) Determine the weight of the box and the normal force exerted on it by the table.
b) Now your friend pushes down on the box with a force of 40 N, what now is the normal force exerted by the table?
c) if your friend pulls upward on the box with a force of 40 N, again determine the normal force exerted on the box by the table

solution

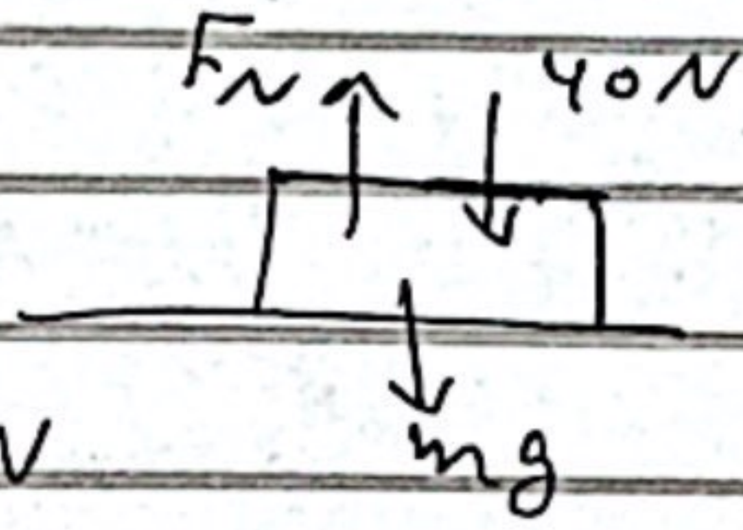
a) $\sum F_y = ma_y = 0$, $F_G = mg = 10(9.8) = 98\ N$

$F_N - mg = 0$



$F_N = mg = 98\ N$

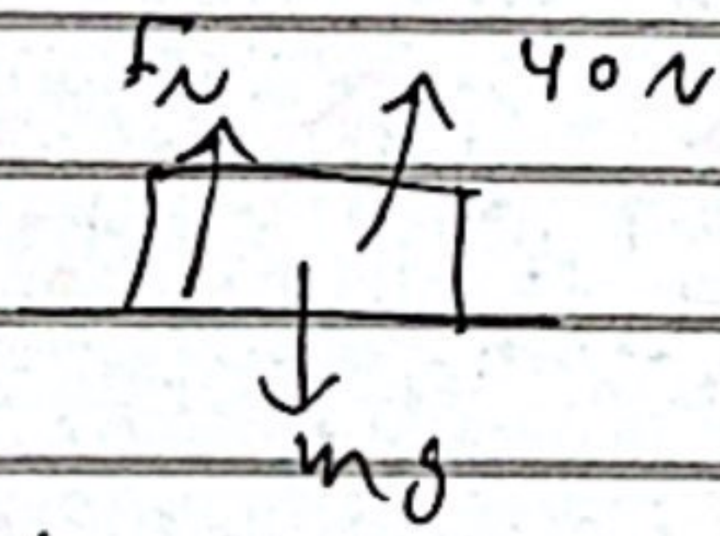
b) $\sum F_y = 0$



$F_N - mg - 40 = 0$

$F_N = mg + 40 = 98 + 40 = 138\ N$

c) $\sum F_y = ma_y = 0$

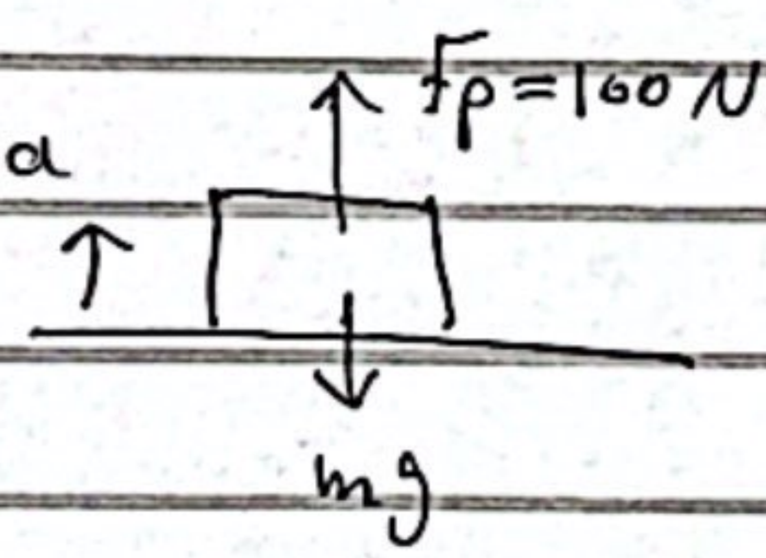


$F_N - mg + 40 = 0$

$F_N = mg - 40 = 98 - 40 = 58\ N$

Example 4-7

What happens when a person pulls upward on the box in example 4-6C with a force equal to, or greater than, the box's weight? for example, let $F_P = 100N$ rather than the 40N.



$\sum F_y = F_N - mg + F_P$

$= F_N - 98 + 100$

if $\sum F_y = 0 \Rightarrow F_N = -2\ N$

but the least F_N can be is zero

setting $F_N = 0$, then

$\sum F_y = F_P - mg = 100 - 98 = 2\ N$

$a_y = \frac{\sum F_y}{m} = \frac{2}{10} = 0.2\ m/s^2$

Example 4-8 (Apparent weight loss)

A 65-kg woman descends (نزل) in an elevator that briefly accelerates at 0.2 g downward. she stands on a scale that reads in kg

a) During this acceleration, what is her weight and what does the scale read?

b) what does the scale read when the elevator descends at a constant speed of 2 m/s?

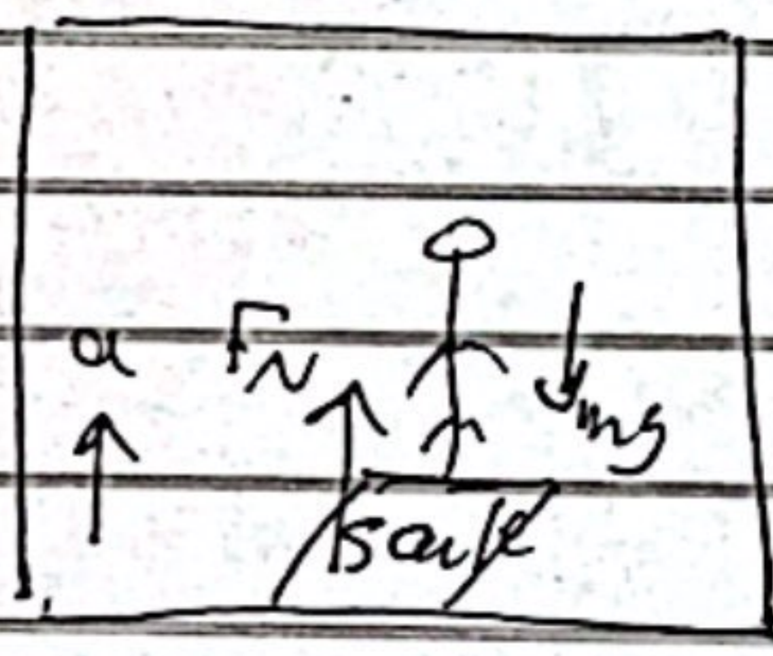
solution

$$\Sigma F = ma$$

$$mg - F_N = m(0.2g)$$

$$F_N = mg - 0.2\,mg$$

$$= 0.8\,mg \quad (\text{upward}) = 520\,N$$



her weight is $mg = 65(9.8) = 640\,N$

but the scale, needing to exert a force of only $0.8\,mg$ and $m = 52$ kg

b) $\Sigma F = ma = 0$

$$F_N - mg = 0$$

$$F_N = mg = 65(9.8) = 640\ N$$

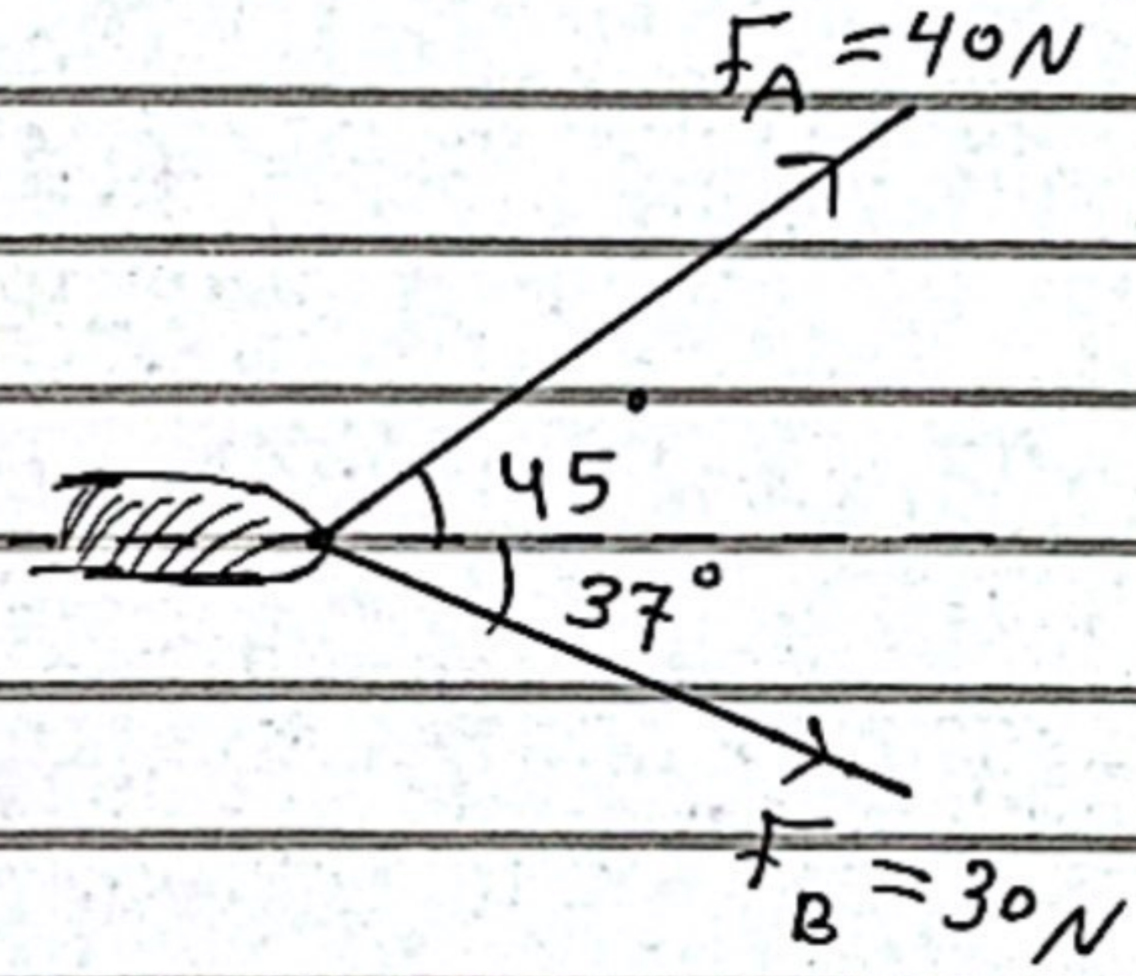
* the scale in (a) gives a reading of 52 kg (as an apparent mass), but her mass doesn't change as a result of the acceleration: it stays at 65 kg.

4-7 Solving Problems with Newton's Laws: Free-Body Diagrams

Example 4-9 (Adding Force vectors)

Calculate the sum of the two forces exerted on the boat by workers A and B

Solution



$F_{Ax} = F_A \cos 45 = 40(0.707) = 28.3N$

$F_{Ay} = F_A \sin 45 = 40(0.707) = 28.3N$

$F_{Bx} = F_B \cos 37 = 30(0.8) = 24N$

$F_{By} = -F_B \sin = -30(0.6) = -18.1N$

with $\vec{F}_R = \vec{F}_A + \vec{F}_B$

$F_{Rx} = F_{Ax} + F_{Bx} = 28.3 + 24 = 52.3N$

$F_{Ry} = F_{Ay} + F_{By} = 28.3 - 18.1 = 10.2N$

the magnitude of the resultant force is

$$F_R = \sqrt{F_{RX}^2 + F_{RY}^2} = \sqrt{(52.3)^2 + (10.2)^2} = 53.3 \ N$$

with a direction (with respect to the x-axis)

$$\theta = \tan^{-1} \frac{F_{Ry}}{F_{RX}} = \tan^{-1} \frac{10.2}{52.3} = 11°$$

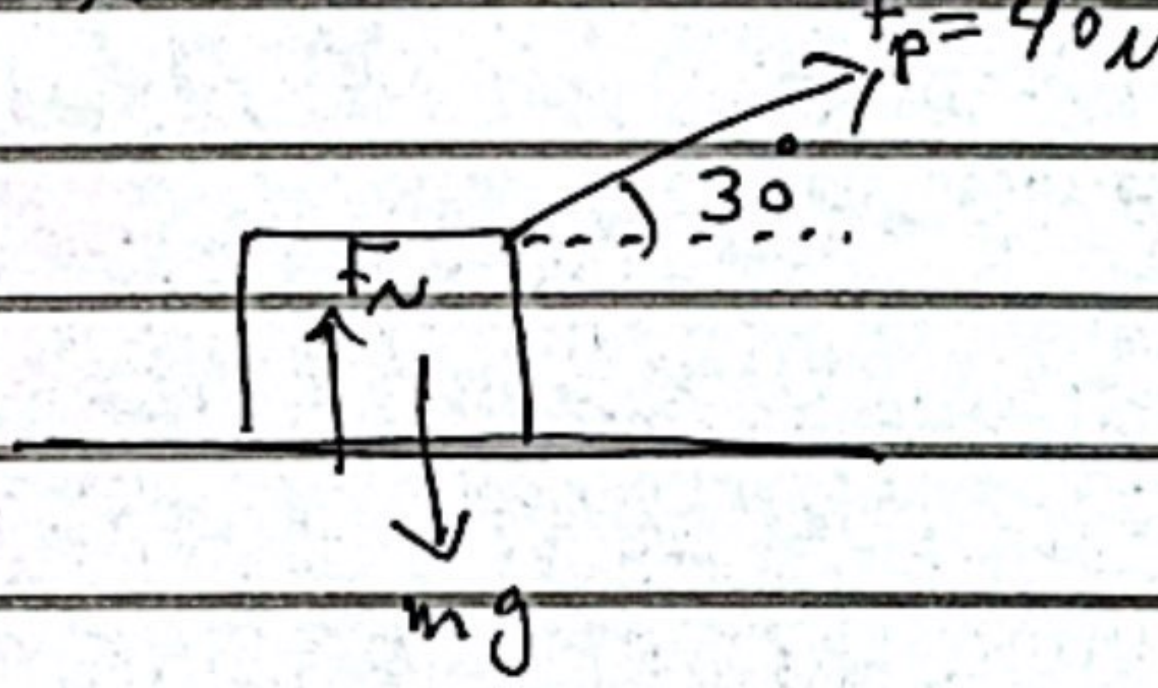
thus, the net force on the boat has magnitude 53.3N and acts at an ~~angle~~ 11° angle to the x-axis

Example 4-11

A box of mass 10 kg is pulled by an attached cord along the smooth surface of a table. The magnitude of the force is $F_P = 40N$ and with an angle at $30°$.

a) calculate the acceleration of the box

b) " the magnitude of the upward force F_N exerted by the table on a box



solution

a) $F_{Px} = 40 \cos 30 = 34.6 \ N$

$F_{Py} = 40 \sin 30 = 20 \ N$

$\Sigma F_x = ma_x$

$F_{Px} = ma_x \Rightarrow a_x = \frac{F_{Px}}{m} = \frac{34.6}{10} = 3.46 \ m/s^2$

b) $\Sigma F_y = ma_y = 0$

$F_N + F_{Py} - mg = 0$

$F_N = mg - F_{Py}$

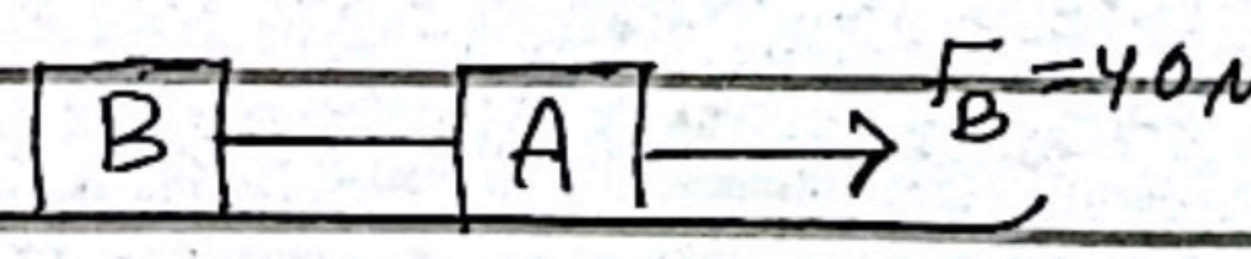
$= 10(9.8) - 20 = 78 \ N$

Example 4-12

Two boxes, A and B, are connected by a lightweight cord and are resting on a smooth (frictionless) table. The boxes have masses of 12 kg and 10 kg. A horizontal force F_P of 40 N is applied to the 10 kg box. Find

a) the acceleration of each box

b) the tension in the cord connecting the boxes



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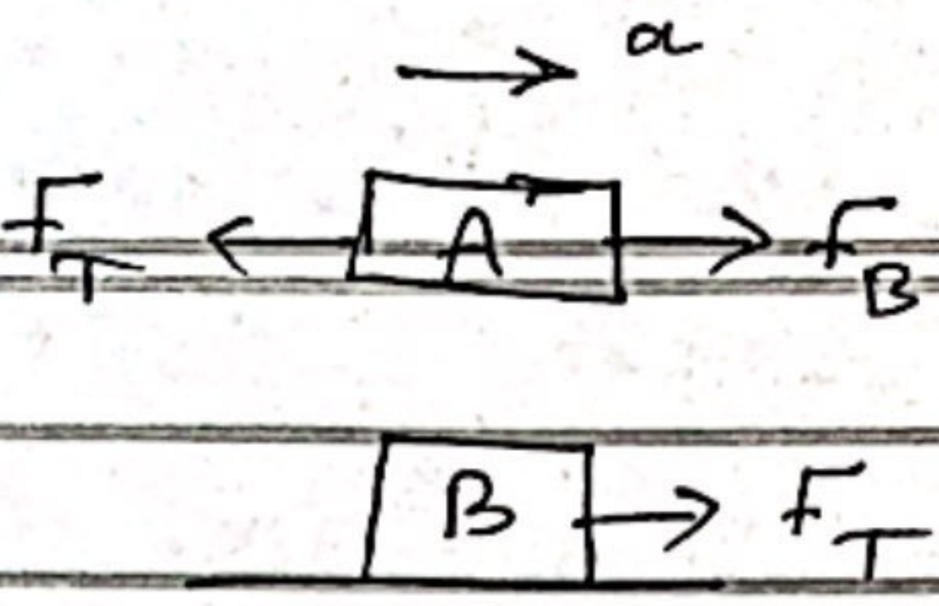
box A: $F_B - F_T = m_A a$ —(1)

box B: $F_T = m_B a$ —(2)



(1) + (2) $\Rightarrow$ $F_B = (m_A + m_B)\, a$

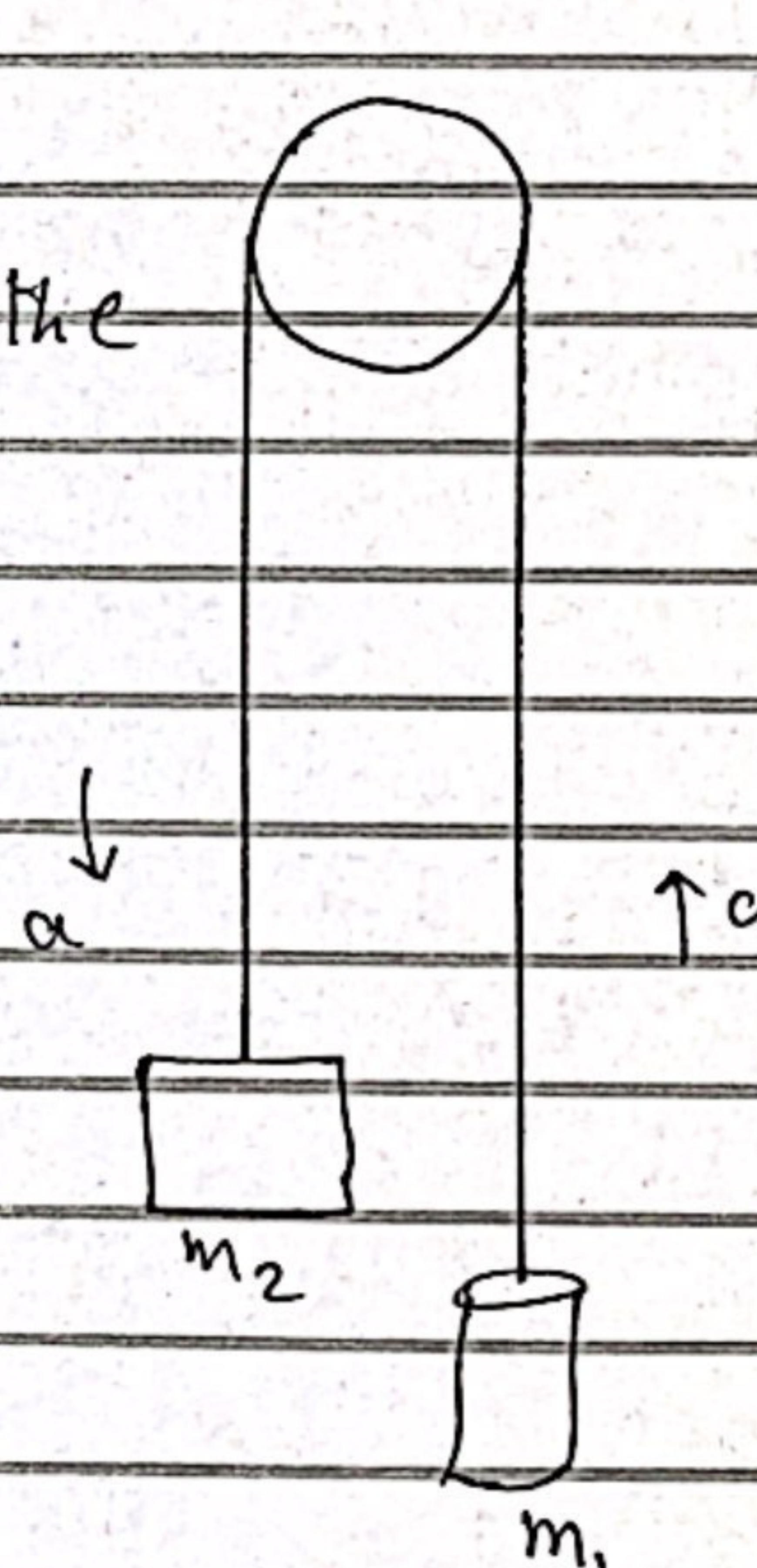
$$a = \frac{F_B}{m_A + m_B} = \frac{40}{10+12} = 1.82 \ m/s^2$$

from eq.(2): $F_T = m_B a = 12(1.82) = 21.8 \ N$

Example 4-12 (Atwood machine)

a) calculate the acceleration of the elevator

b) find the tension in the cable



m_1: $F_T - m_1 g = m_1 a$ —(1)

m_2: $m_2 g - F_T = m_2 a$ —(2)

(1) + (2) $\Rightarrow$ $(m_2 - m_1) g = (m_1 + m_2) a$

$$a = \frac{(m_2 - m_1)\, g}{m_1 + m_2}$$

$m_1 = 1000$ kg

$m_2 = 1100$ kg

$$a = \frac{(1100 - 1000)\, g}{1100 + 1000} = 0.68 \ m/s^2$$

(1) $\Rightarrow$ $F_T = m_1 g + m_1 a = m_1 (g + a)$

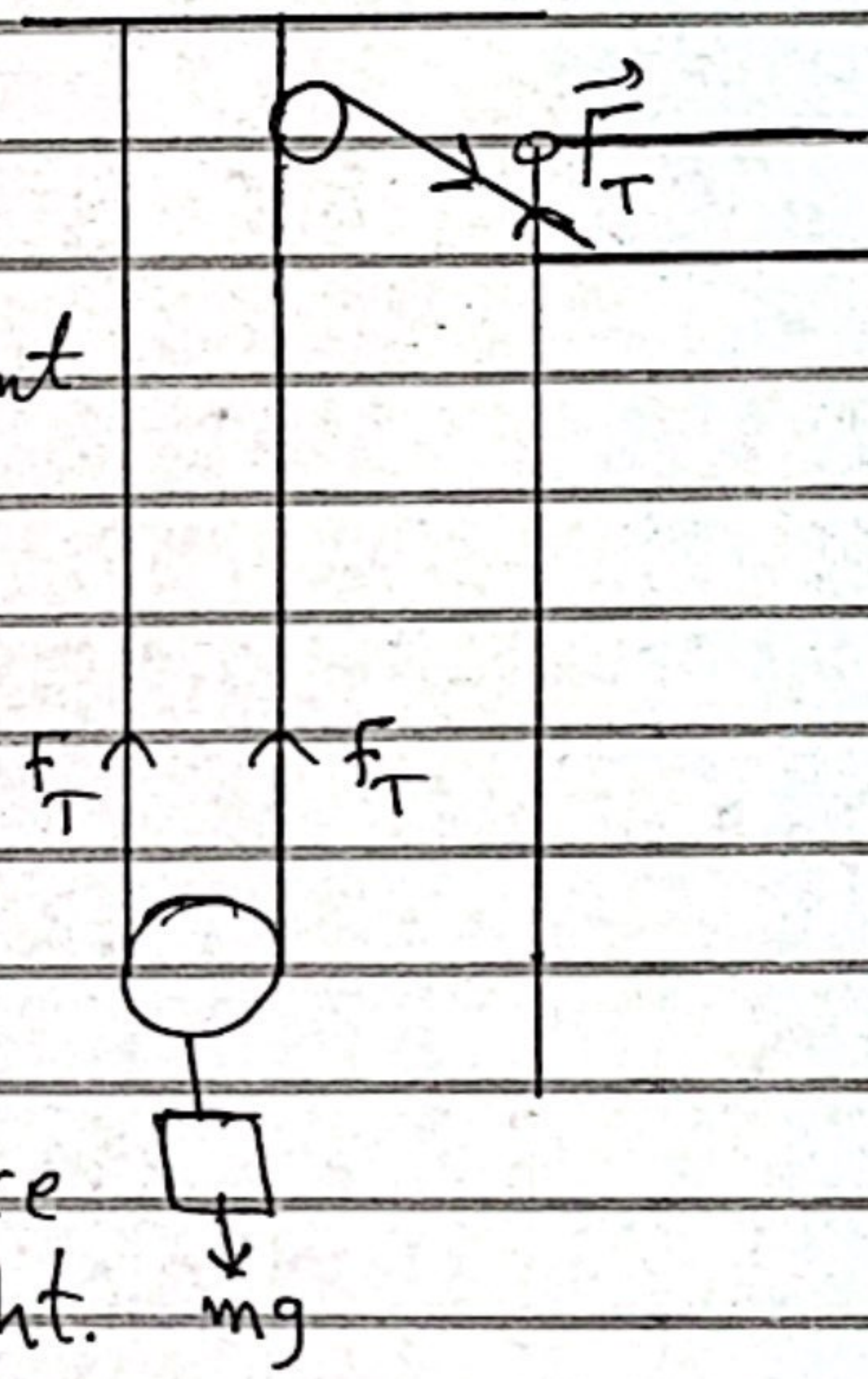
$= 1000(9.8 + 0.68) = 10\,500 \ N$

Example 4-14

A mover is trying to lift a piano up to a second apartment. He is using a rope looped over two pulleys as shown. What force must he exert on the rope to slowly lift the piano's 1600-N weight?

$\sum F = ma$

$2F_T - mg = ma$



to move the piano with constant speed (set $a=0$), thus,

$2F_T - mg = 0$

$F_T = \frac{mg}{2} = \frac{1600}{2} = 800\ N$

the piano mover can exert a force equal to half the piano's weight.

Example 4-15

A small mass m hangs from a thin string and can swing like a pendulum. What angle θ does the string make

a) when the car accelerates at a constant $a = 1.2\ m/s^2$

b) when the car moves at constant velocity, $v = 90\ km/h$

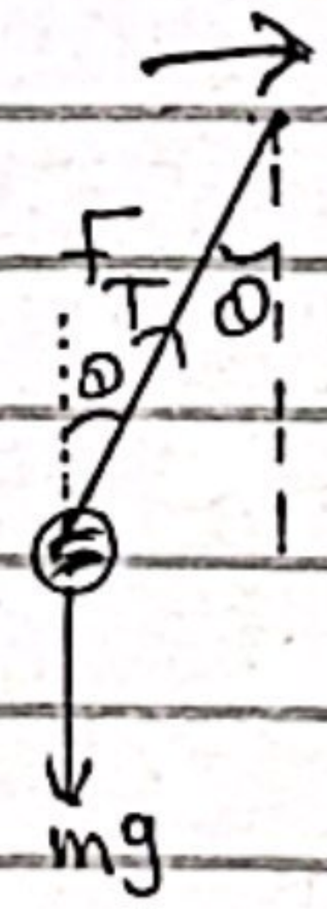
solution

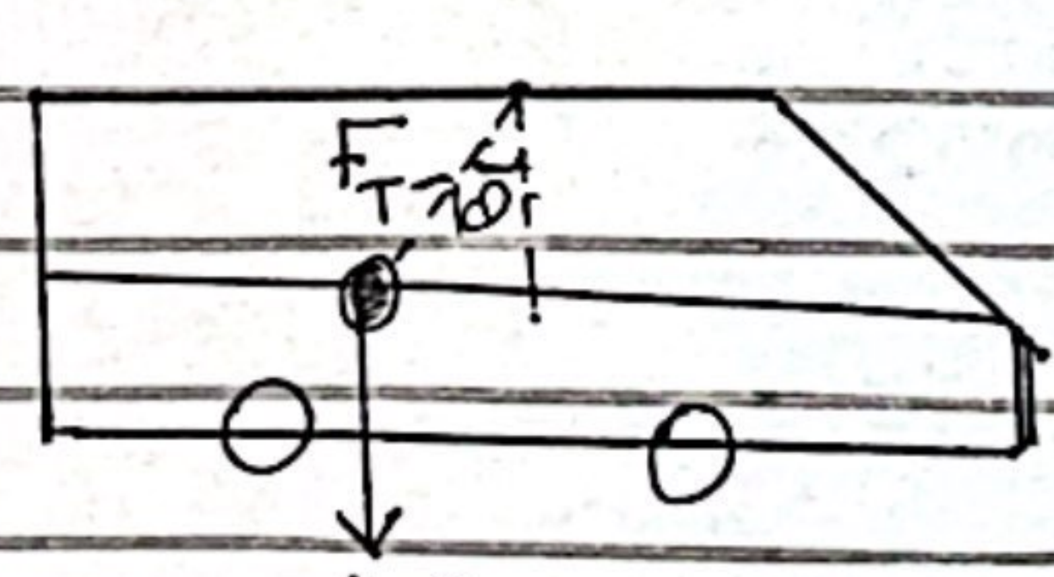
a) $\sum F_x = ma$

$F_T \sin\theta = ma$ —— ①

$\sum F_y = 0$

$F_T \cos\theta - mg = 0$ —— ②





From (1) and (2)

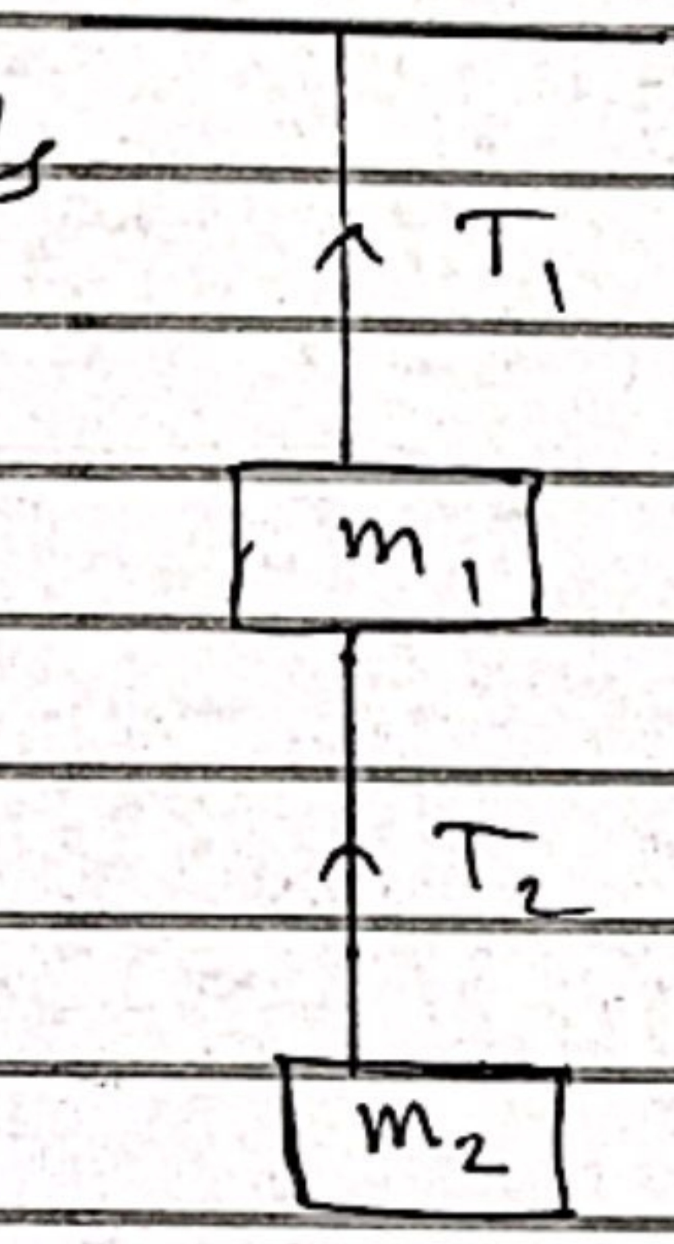
$\tan\theta = \frac{a}{g} = \frac{1.2}{9.8}$

$\theta = 7^\circ$

b) for constant velocity $a = 0$, that is
$\tan\theta = 0 \Rightarrow \theta = 0^\circ$

Exercise (1)

Two masses hang by two cords as shown. Find the tensions T_1 and T_2



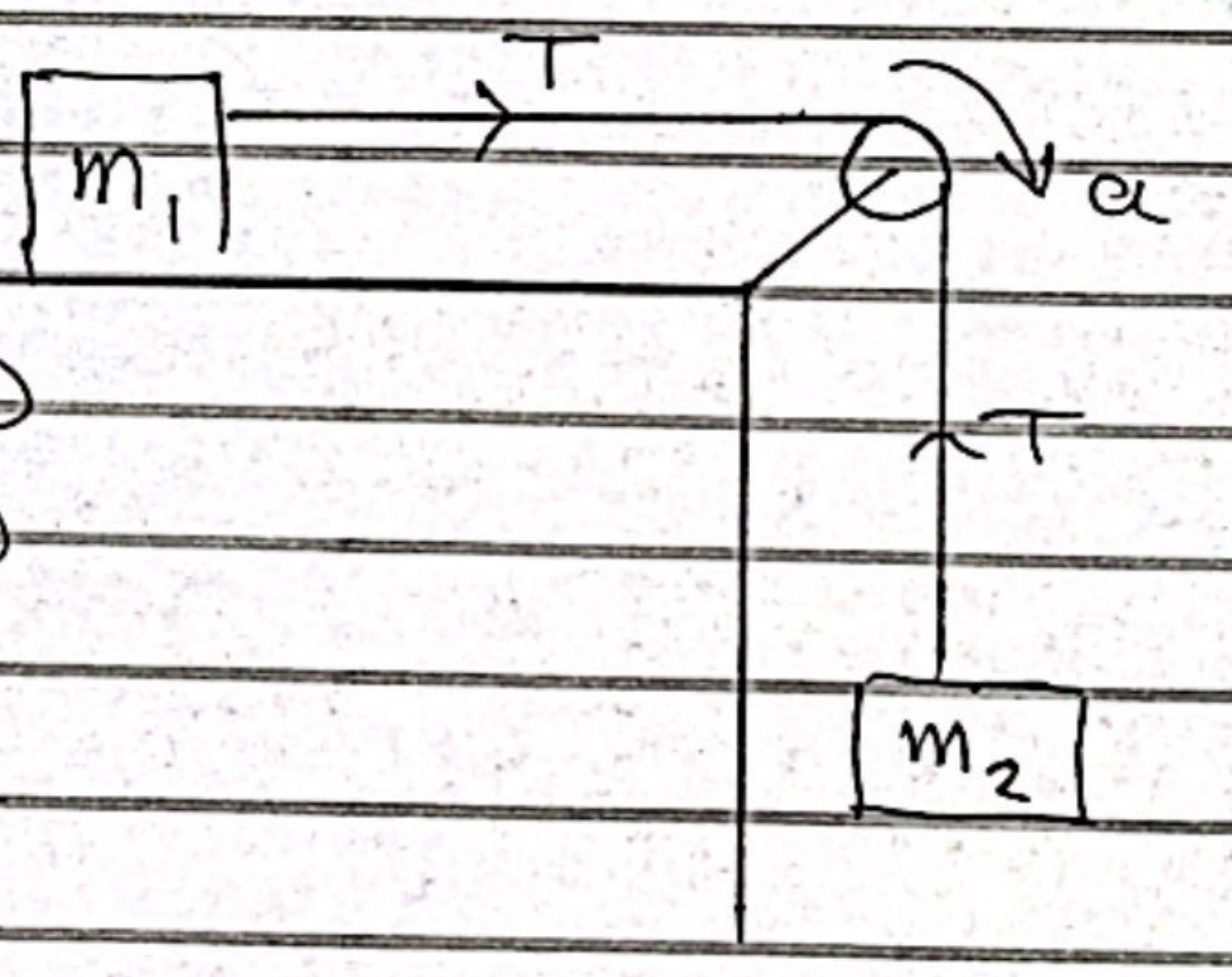
for m_1: $T_1 - m_1 g - T_2 = 0$ —— ①
for m_2: $T_2 - m_2 g = 0$ —— ②

$\Rightarrow T_2 = m_2 g = 5(9.8) = 49\ N$
$T_1 = T_2 + m_1 g = 49 + 49 = 98\ N$

$m_1 = m_2 = 5\ kg$

Exercise (2)

Find the acceleration and tension



m_2: $T - m_2 g = -m_2 a$ —— ①
m_1: $T = m_1 a$ —— ②

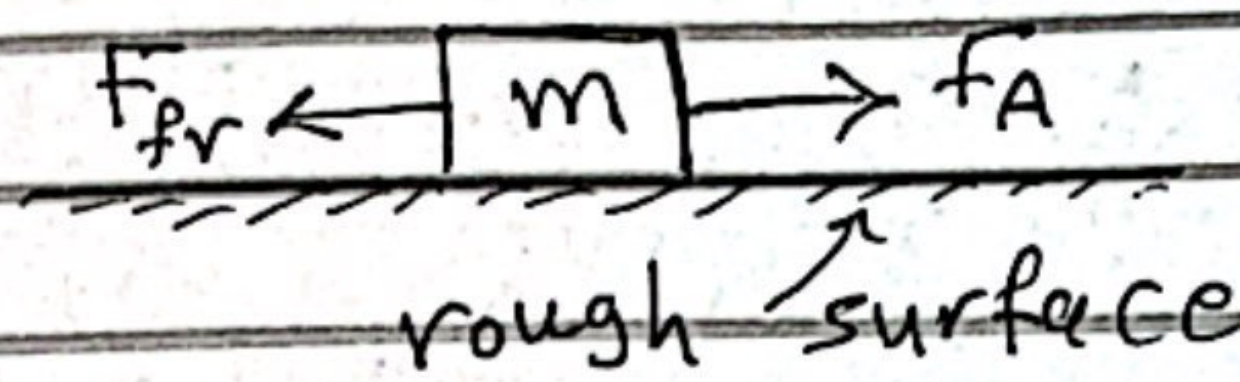
② − ① $\Rightarrow m_2 g = (m_1 + m_2) a$

$$a = \frac{m_2}{m_1 + m_2} g$$

$$T = m_1 a = m_1 \frac{m_2 g}{m_1 + m_2} = \frac{m_1 m_2}{m_1 + m_2} g$$

4-8 Problems Involving Friction, Inclines

consider a mass m at rest on a rough surface. A Force F_A is trying to move the mass but it remains at rest

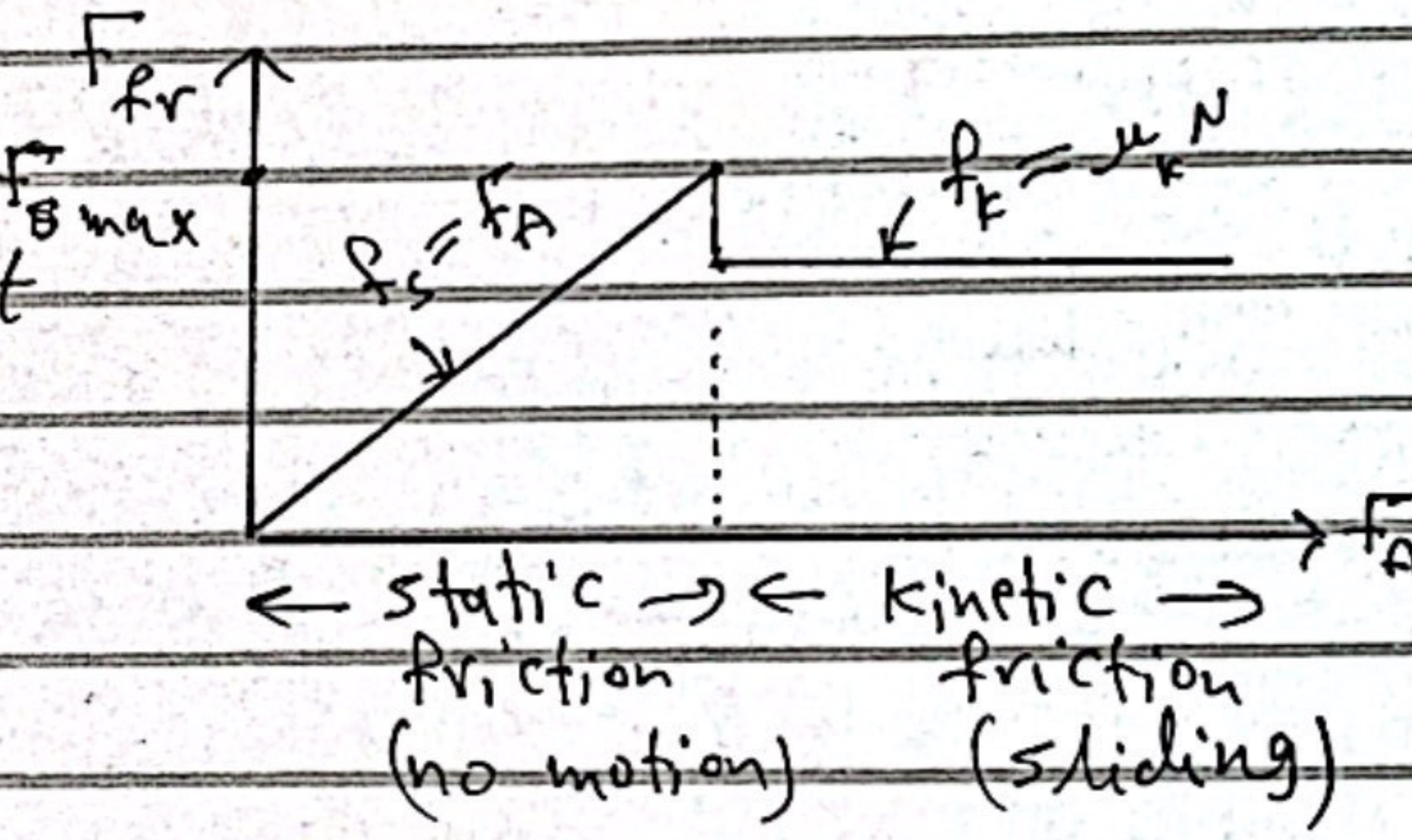


with newton's 2nd law

$$\Sigma F_x = 0 \Rightarrow F_A - F_{fr} = 0 \Rightarrow F_A = F_{fr}$$

As long as the mass is stationary $\Rightarrow F_A = F_{fr}$

The force of friction that acts while the object is at rest is called static friction (f_s)



$f_s \leq \mu_s N$

$f_{s_{max}} = \mu_s N = \mu_s mg$

where μ_s is the coefficient of static friction.

As soon as the object starts moving (~~with constant velocity~~), we have a kinetic friction (f_k)

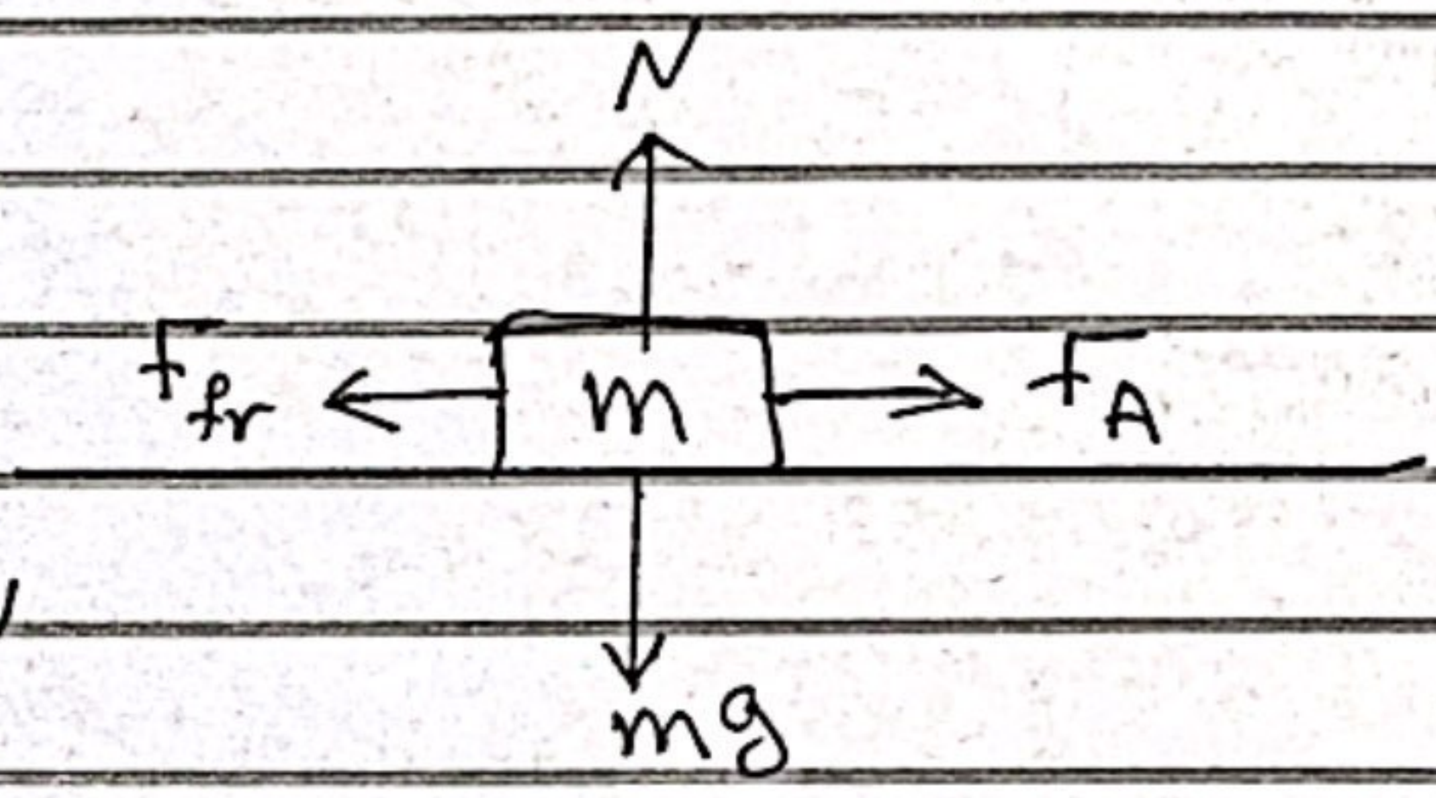
$f_k = \mu_k N$, $\mu_k \equiv$ is the coefficient of kinetic friction

note: $\mu_s > \mu_k$

Example 4-16

A box of mass 10 kg rests on a horizontal floor. The coefficient of static friction is $\mu_s = 0.4$ and the 〃 〃 kinetic 〃 is $\mu_k = 0.3$. Determine the force of friction acting on the box if a horizontal applied force F_A is exerted on it of magnitude

a) 0   b) 10 N   c) 20 N   d) 38 N   e) 40 N



$\Sigma F_y = m a_y = 0$

$N - mg = 0$

$N = mg = 10(9.8) = 98\ N$

since $f_{s_{max}} = \mu_s N = 0.4(98) = 39\ N$, so that for $F_A < 39\ N$, the object is not moving and hence $f_{fr} = f_s = F_A$

a) $F_A = 0 \Rightarrow f_s = 0$

b) $F_A = 10N \Rightarrow f_s = 10N$

c) $F_A = 20N \Rightarrow f_s = 20N$

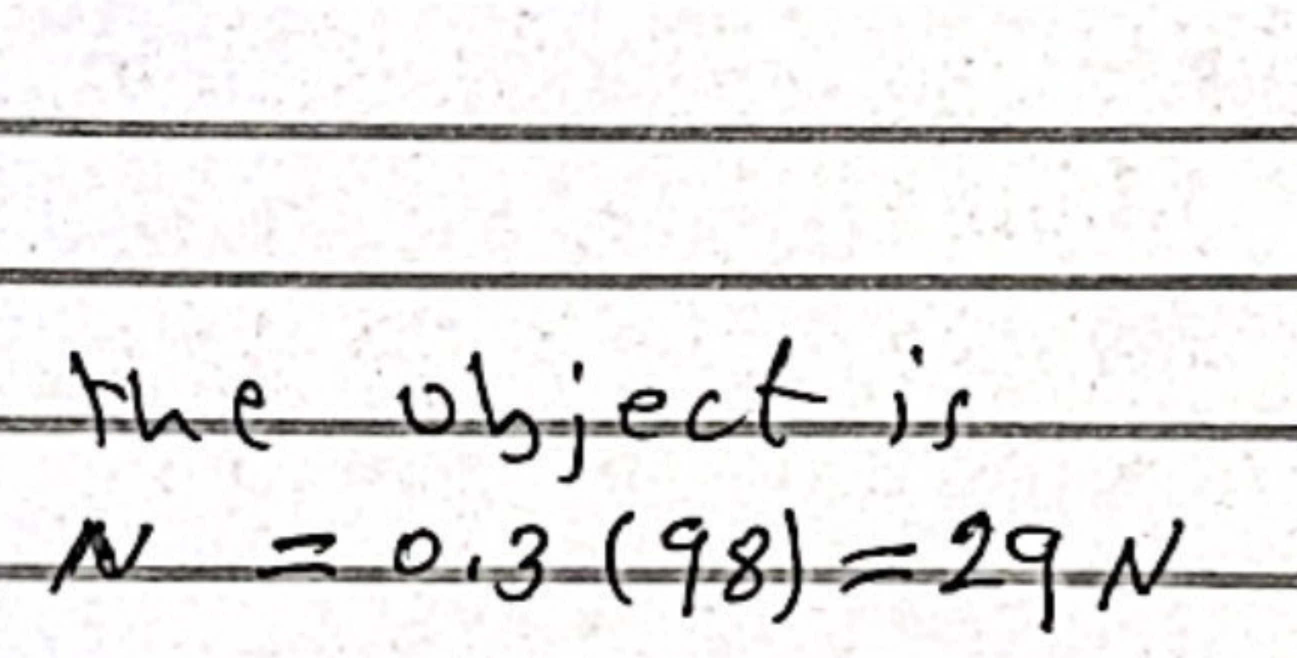
d) $F_A = 38N \Rightarrow f_s = 38N$

e) $F_A = 40N > 39\ N$, thus the object is in moving, and $f_k = \mu_k N = 0.3(98) = 29N$

$\Sigma F_x = m a_x \Rightarrow F_A - f_k = m a_x$

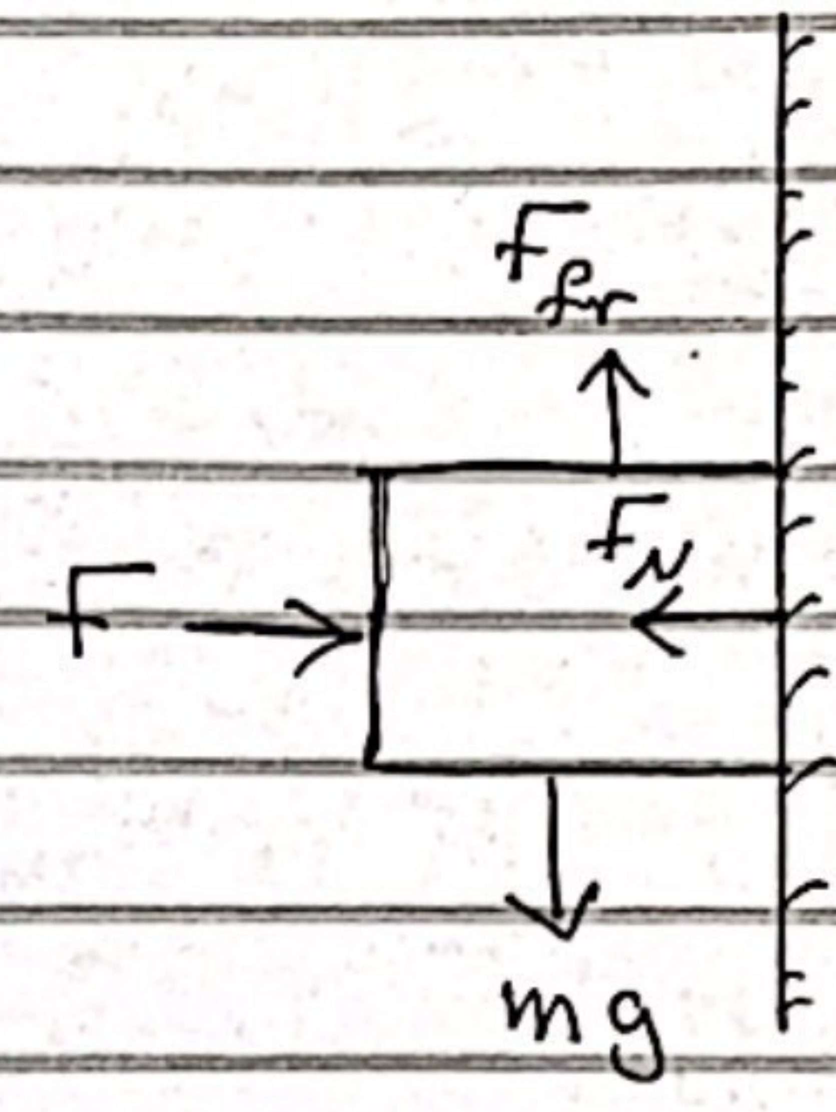
$40 - 29 = 10\, a_x$

$a_x = \frac{11}{10} = 1.1\ m/s^2$



Example 4-17

You can hold a box against a rough wall and prevent it from slipping down by pressing hard horizontally. If $\mu_s = 0.4$ and $mg = 20\,N$, what minimum force F will keep the box from falling



To keep the box from slipping

$$\sum F_y = 0 \Rightarrow f_s - mg = 0 \Rightarrow f_s = mg = 20\,N$$

$$\sum F_x = 0 \Rightarrow F - F_N = 0 \Rightarrow F = F_N$$

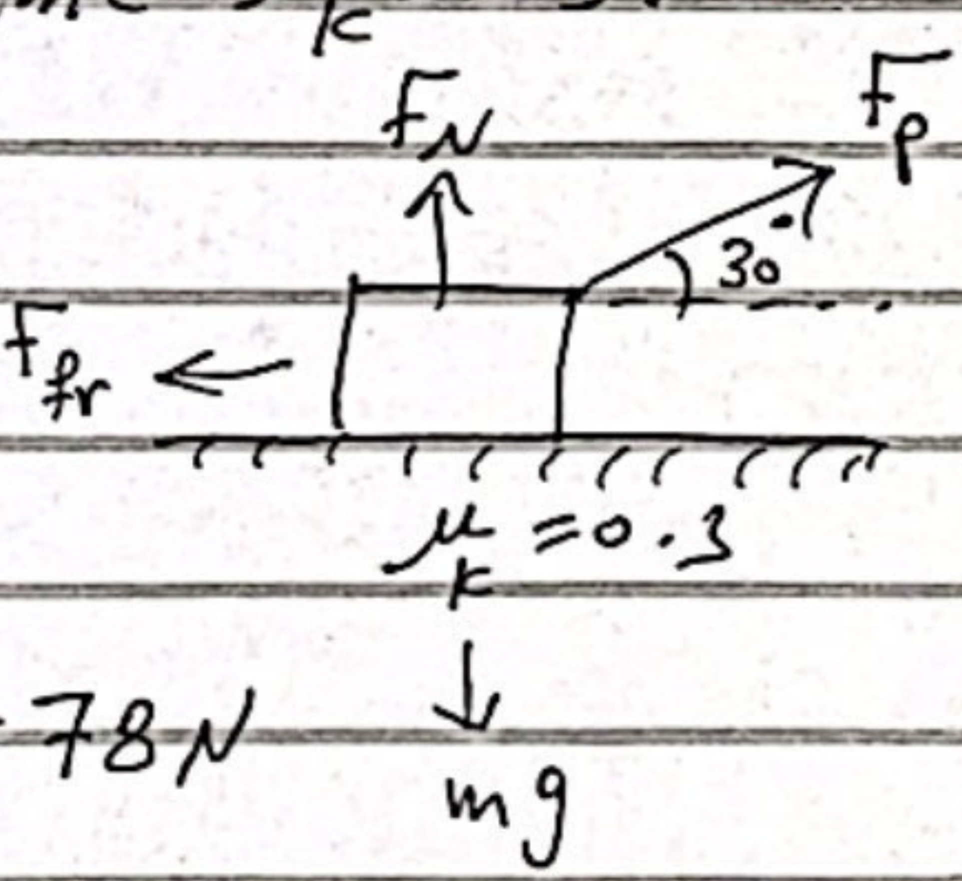
but $f_s \le \mu_s F_N$

$$f_s \le \mu_s F \Rightarrow F \geq \frac{f_s}{\mu_s}$$

$$F_{min} = \frac{f_s}{\mu_s} = \frac{20}{0.4} = 50\,N$$

Example 4-18 (Pulling against friction)

A 10-kg box is pulled a long a horizontal surface by a force $F_p = 40\,N$ applied at 30°, assume $\mu_k = 0.3$. calculate the acceleration.



$$\sum F_y = m a_y = 0$$

$$F_N - mg + F_p \sin 30 = 0$$

$$F_N = mg - F_p \sin 30 = 10(9.8) - 40(0.5) = 78\,N$$

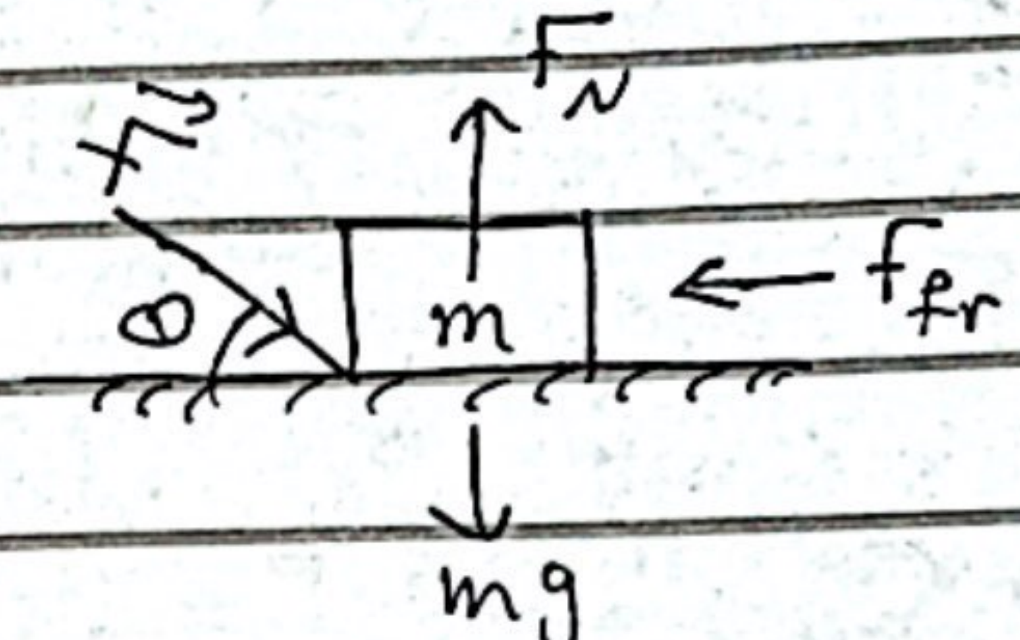
$$\sum F_x = m a_x$$

$$F_p \cos 30 - F_{fr} = m a_x$$

$$F_p \cos 30 - \mu_k F_N = m a_x$$

$$40 \cos 30 - 0.3(78) = 10\, a_x \Rightarrow a_x = 1.1\,m/s^2$$

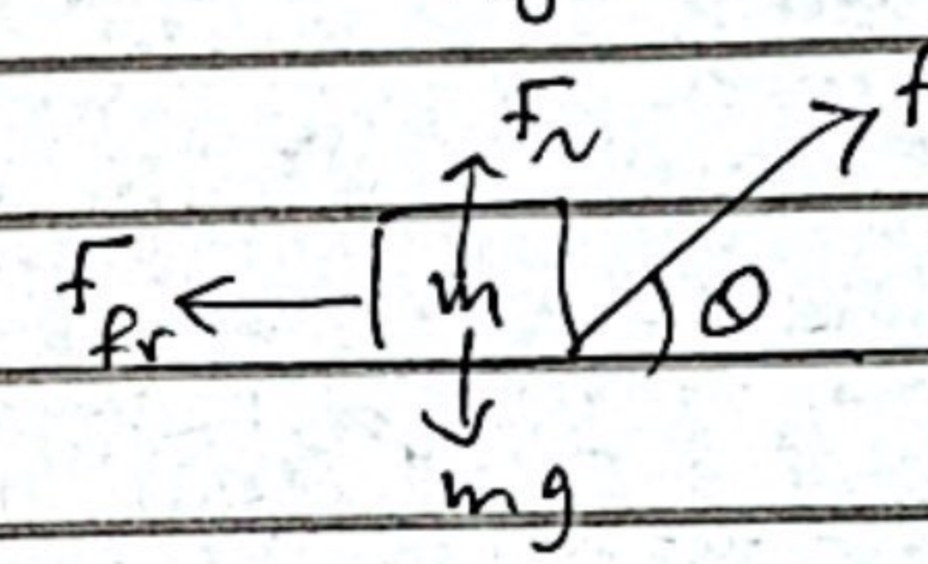
Example 4-19 (To push or to pull a sled?)
Your little sister wants a ride on her sled. If you are on flat ground, will you exert less force if you push her or pull her?



since $f_{fr} = \mu_k F_N$, we have

a) Push case $\sum F_y = 0$

$$F_N - mg - F\sin\theta = 0$$



$$F_N = mg + F\sin\theta$$

b) Pull case $\sum F_y = 0$

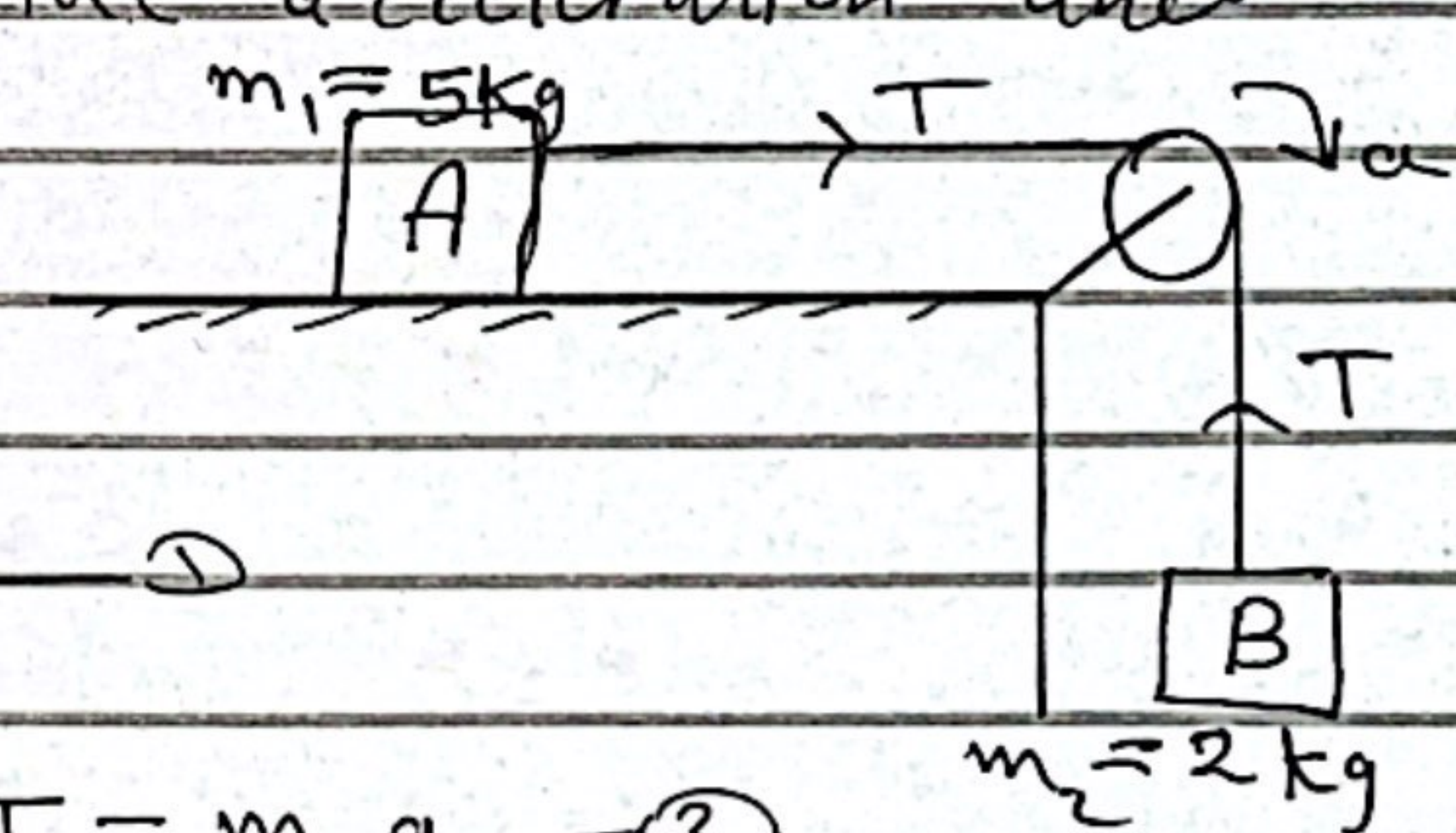
$$F_N + F\sin\theta - mg = 0$$

$$F_N = mg - F\sin\theta$$

$$\therefore (F_N)_{pull} < (F_N)_{push} \Rightarrow (f_{fr})_{pull} < (f_{fr})_{push}$$

So you exert less force if you pull her.

Example 4-20
Two boxes are connected by a cord running over a pulley. The coefficient of kinetic friction between box A and the table is 0.2. if we ignore the mass of the cord and the pulley (that is the same tension and acceleration), what is the acceleration and the tension of the cord



box A: $\sum F_x = m_1 a$

$$T - f_{fr} = m_1 a \quad \text{—} ①$$

$$\sum F_y = 0 \Rightarrow F_N = m_1 g$$

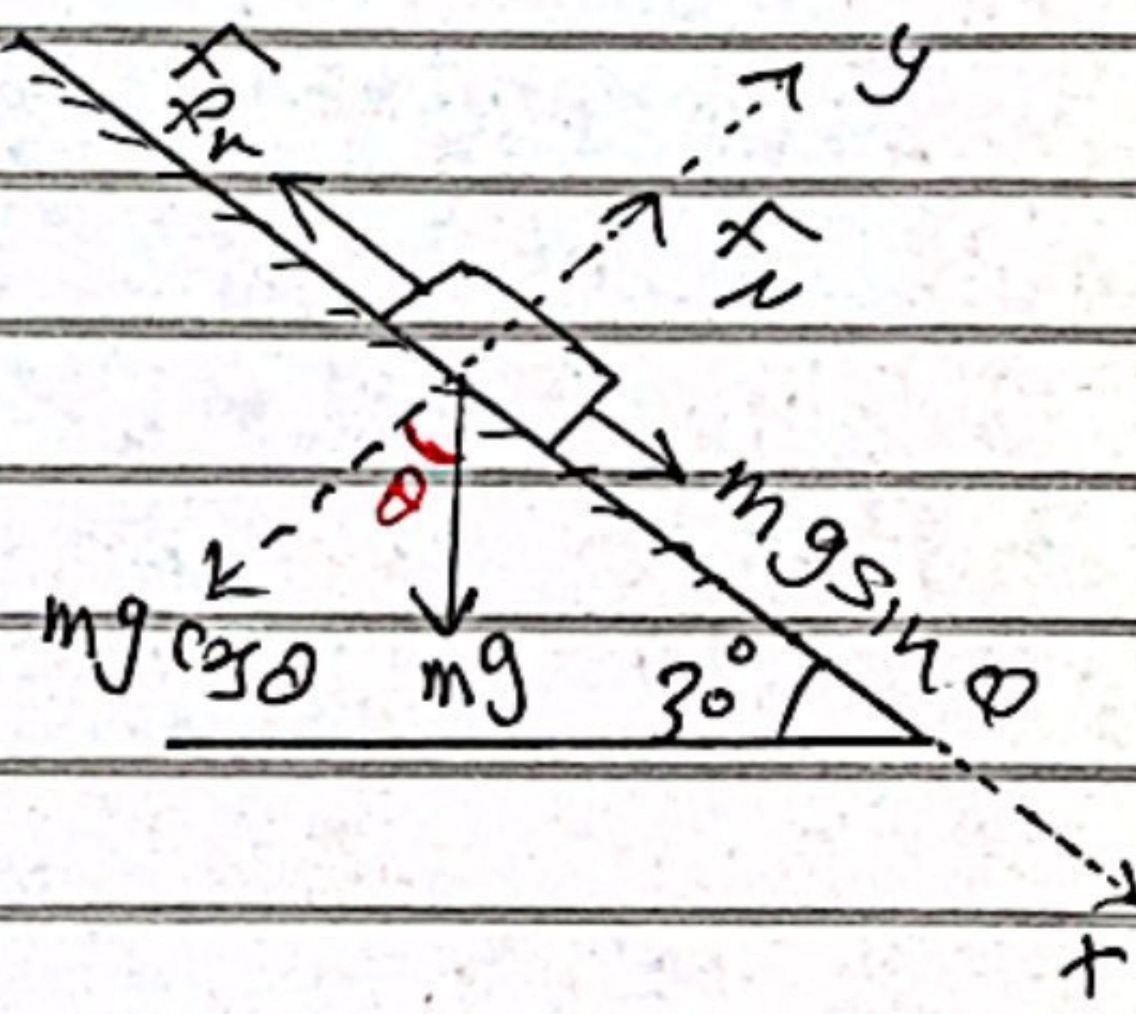
box B: $\sum F_y = m_2 a \Rightarrow m_2 g - T = m_2 a$ — ②

$$① + ② \Rightarrow a = \frac{m_2 g - f_{fr}}{m_1 + m_2} = \frac{m_2 g - \mu m_1 g}{m_1 + m_2} = 1.4 \text{ m/s}^2$$

(1) $\Rightarrow T = 17$ N

Example 4-21 (The skier)

The skier has begun descending the $30°$ slope. If the coefficient of kinetic friction is 0.1, what is her acceleration?



$$\sum F_x = ma_x$$

$$mg\sin\theta - F_{fr} = ma_x \quad \text{——} \ (1)$$

$$\sum F_y = ma_y = 0$$

$$F_N - mg\cos\theta = 0 \quad \text{——} \ (2)$$

$$(1) \Rightarrow \quad mg\sin\theta - \mu_k F_N = ma_x$$

$$mg\sin\theta - \mu_k\, mg\cos\theta = ma_x$$

$$a_x = g\sin\theta - \mu_k\, g\cos\theta$$

$$= 9.8(0.5) - 0.1(9.8)(0.866) = 4 \ m/s^2$$