

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ



السلام عليكم ورحمة الله وبركاته

LXVI

t-Tests

Part 4

Paired t-Test

Paired t-Test

In many medical studies, individuals are **followed over time** to see if there is a **change in the value** of some continuous variable.

❖ **Typically**, this occurs in a “**before** and **after**” experiment such as one testing to see

if there was a decrease in average blood pressure after treatment or

to see if there was a **reduction in weight** after the use of a special diet.

❖ In this type of **comparison**, an individual **patient serves** as his or her **own control**.

☐ The appropriate statistical test for this type of data is the **paired t-test**.

The paired t-test is more robust

Cont. ...Paired t-Test

The paired t-test is more robust than pooled Student's t-test

- ❖ **because** it considers the **variation** from only **one group** of people, whereas pooled **Student's t-test** considers **variation** from **two groups**.

Variation that is detected in the **paired t-test** is presumably

- **(probably) attributable** to the **intervention** or to **changes over time** in the same person



CALCULATION OF THE VALUE OF T in Paired t-Test

To calculate a paired t-test, a **new variable** must be created. This **variable, called d**, is the **difference between the values** before and after the intervention for each individual studied.

- ❖ The paired t-test is a test of the null hypothesis that, on the average, the difference is equal to **0**,
- ❖ which is what would be expected if there were no change over time.
- ❖ Using the symbol **d** to indicate the mean observed **difference between** the **before** and **after** values,
- ❖ the formula for the paired t-test is as follows:



$$t = \frac{m_1 - m_2}{S.E \text{ of difference}}$$

$$t = \frac{\text{mean of the difference}}{S.E \text{ of difference}}$$

$$t = \frac{\sum d}{N} = \frac{\bar{d}}{S.E \text{ of}}$$

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The formulas for **pooled Student's t -test** and the **paired t -test** are similar the **ratio of a difference to the variation** around that difference (the standard error).

- ❖ **In pooled Student's t -test**, each of the two distributions to be **compared contributes to the variation** of the difference, and the two variances must be added.
- ❖ **In the paired t -test**, there is only one frequency distribution, that of the **before-after difference** in each person.
- ❖ **In the paired t -test**, because only one mean is calculated (**d**), only **1** degree of freedom is lost; the formula for the degrees of freedom is **$N - 1$** .

III- Difference between two means of dependent sample

We have one sample (group) **under two different condition**, or two different events or two different occasions

- **Therefore each individual in the sample having two observations or two values .**
- ❖ **Each value related to certain occasion or event .**
- ❖ **Each individual having two values .**
- ❖ **Each individual having paired of value or observation**



We would like to know if there is a **significance difference in the variable** (B. sugar, Hb level) **between two occasions or conditions** .

❖ Whether **changing of the condition** **have an effect on the variable value** that we are interest about .

❖ We are measuring and testing the significance difference between **two means** of **two** different **occasions** .

❑ **We use what we call it Paired t test** .

❖ We have **one sample (N)** under **two** different conditions

❖ **Measuring** the difference between **two values** of each individuals

$$t = \frac{m_1 - m_2}{S.E \text{ of difference}}$$

$$t = \frac{\text{mean of the difference}}{S.E \text{ of difference}}$$

$$t = \frac{\sum d}{N} = \frac{\bar{d}}{S.E \text{ of } d}$$

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Example

Five Geriatric patients put under Zinc Sulphate, their body weight was measured before treatment, then 10 WS after treatment by 200 mg Zinc Sulphate T d s .

At α level of 0.05, could we conclude that Zinc Sulphate affect body Wt. of geriatric patients

B. Wt lb

Pt	Before	After	d	d^2
1	140	143	+3	9
2	138	136	-2	4
3	142	138	-4	16
4	130	125	-5	25
5	152	150	-2	4

$$\sum d = -10 \quad \sum d^2 = 58$$

$$\bar{d} = -2$$

Data

-Quantitative data .Representing mean B. Wt (lb) of five geriatric patients before treatment and after treatment .

Assumption

We assume that **dependent sample** has been chosen randomly from normal distribution population under two different conditions .

Formulation of hypothesis

Ho

There is **no significance** difference in the mean body Wt. of the Geriatric patients before treatment and after treatment , .

There is **no effect of zinc** sulphate treatment on B. Wt. of geriatric patients **between two conditions**



□ The **difference in mean B. Wt.** (before and after) is due to **Sampling Error, Sampling Variability and Chance Factor.**

HA

There is a **significance difference in mean B. Wt.** between **two conditions** .

This difference is **due** to the **effect of Zinc Sulphate** on the **B. Wt.**

Level of significance

$$\alpha = 0.05 \quad \frac{\alpha}{2} = \frac{0.05}{2} = 0.025$$

$$d.F = N - 1 = 5 - 1 = 4$$

$$\text{tabulat } t_{0.025}^4 = 2.77$$

Apply Proper test

t test

paired t test

$$t = \frac{\bar{d}}{S.E \text{ of } \bar{d}}$$

$$= \frac{\sum d}{N} = \frac{-10}{5} = -2$$

$$S.E \text{ of } d = \frac{S.D \text{ of } Ib}{\sqrt{N}}$$

$$S.D = \sqrt{\frac{\sum d^2 - \frac{(\sum d)^2}{N}}{N - 1}} = \sqrt{\frac{58 - \frac{(-10)^2}{5}}{5 - 1}} = \sqrt{\frac{58 - 20}{4}} = \sqrt{\frac{38}{4}}$$

$$S.D = \mp 3.08$$

$$t = \frac{-2}{\frac{3.08}{\sqrt{5}}} = \frac{-2}{1.38} = -1.45$$

Decision

Calculated t fall in area of acceptance



accept H_0

- ❖ There is **no significance** difference in the mean B. Wt. **between two condition (before and after treatment)** .
Calculated $t <$ tabulated t .
Calculated t falls **in front of critical region** .
- ❖ Area or % of **influencing factor** decreased (less than 95%)
- ❖ **While the chance** factor increased (more than 5%) .
 $P > 0.05$.

Conclusion

There is **no significance effect** of Zinc Sulphate on the B. Wt. of the Geriatric patients .

Example

In order to determine the effect of certain oral contraceptive on weight gain, 9 healthy females were weighed prior to the start of medication and again at the end of the three month period .

SUBJECT	INITIAL WT.(POUND)	3 MONTH WT.(POUND)
1	120	123
2	141	143
3	130	140
4	150	145
5	135	140
6	140	143
7	120	118
8	140	141
9	130	132

Is there a sufficient evidence to conclude that female experience a weight gain following three months of oral contraception

.Alpha=0.05

Example

Serum digoxin level in 9 healthy males following intravenous injection of the drug was measured after 4 hours and again after 8 hours . the following data was obtained

Person	<u>After 4 hours</u>	<u>After 8 hours</u>
1	1	1
2	1.3	1.3
3	0.9	0.7
4	1	1
5	1	0.9
6	0.9	0.8
7	1.3	1
8	1.1	1
9	1	1

A-Is there a statistical significant difference in the serum digoxin level concentration at the end of 4 hours and at the end of 8 hours,
Alpha = 0.05 .

B--- Present the above data properly

t-test table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
Z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%

