بسم الله الرحمن الرحيم

Correlation
and
linear regression

Pearson’s Correlation Coefficient (r)

- $$r = \frac{n \sum XY - (\sum X)(\sum Y)}{\sqrt{[n\sum X^2 - (\sum X)^2]\,[n \sum Y^2 - (\sum Y)^2]}}$$

Correlation is defined
as the degree or strength of relationship between two characteristics in a population

- **The aim is**
- to **investigate the linear association between two**
- **continuous quantitative variables**.

Correlation therefore measures the closeness of the association.

For the correlation to be obtained we need the followings;

a. **One population**

b. **two characteristics**

c. **both should be continuous type (quantitative data)**

d. **both should be changing (variables) (not constant)**

e. **There must be some sort of relationship between two**

- ✓ **in order to obtain the strength of this relationship**

- ❑ **After that we need to determine**
- ❖ **which of the two** variables **is X** and
- ❖ which one **is Y according to the following;**

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X Independent:	Y Dependent:
The change in X is independent on the change in Y	**The change in Y is dependent on the change in X**
Less changing in a short period of time (more constant)	**More changing in a short period of time (more changing)**
As the cause	**As the effect**

- **After that we need to draw a scatter diagram in** order to **ascertain** the **presence of correlation** and

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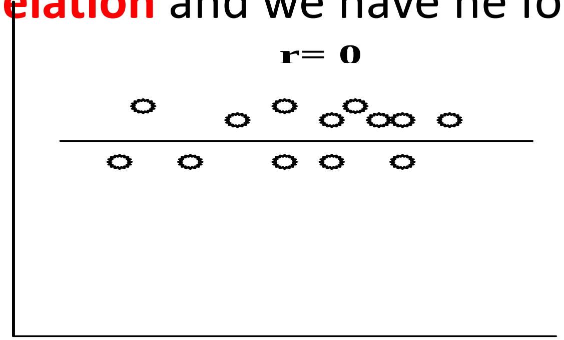
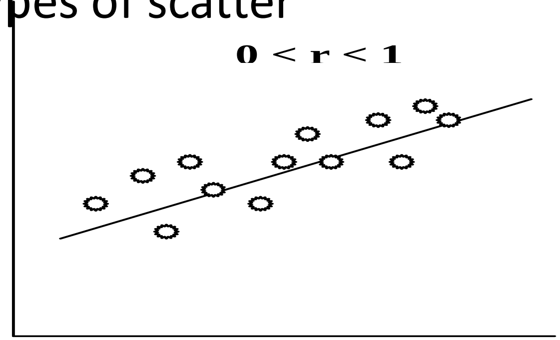
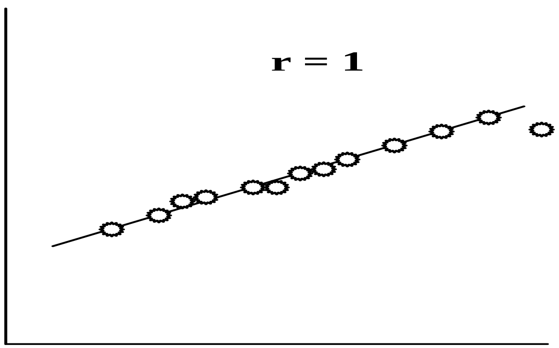


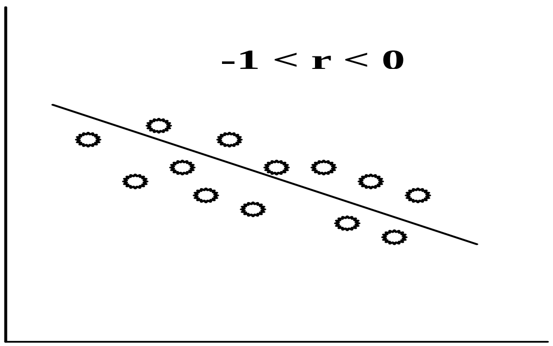
a) No correlation



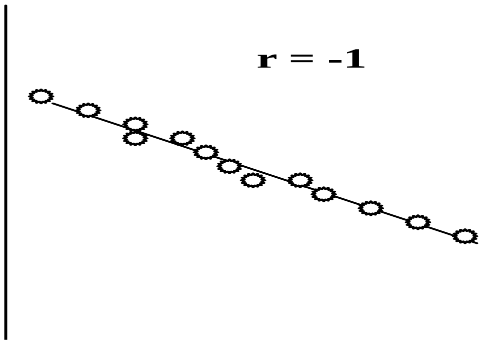
b) Direct positive correlation



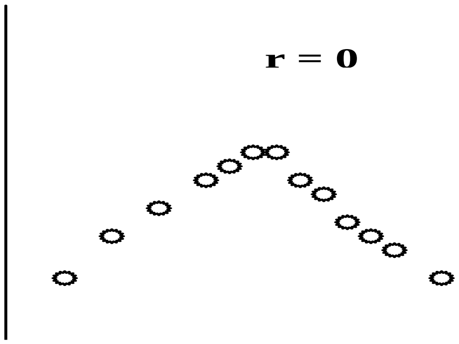
c) Perfect direct positive correlation



d) Inverse negative correlation



e) Perfect inverse negative correlation



f) Strong but non-linear relationship exists

Pearson's Correlation Coefficient (r)

- $$r = \frac{n \sum XY - (\sum X)(\sum Y)}{\sqrt{[n\sum X^2 - (\sum X)^2]\,[n \sum Y^2 - (\sum Y)^2]}}$$

r only **measures the linear relationship so we have to draw a scatter diagram** first **to identify non-linear relationship**

Linear regression;

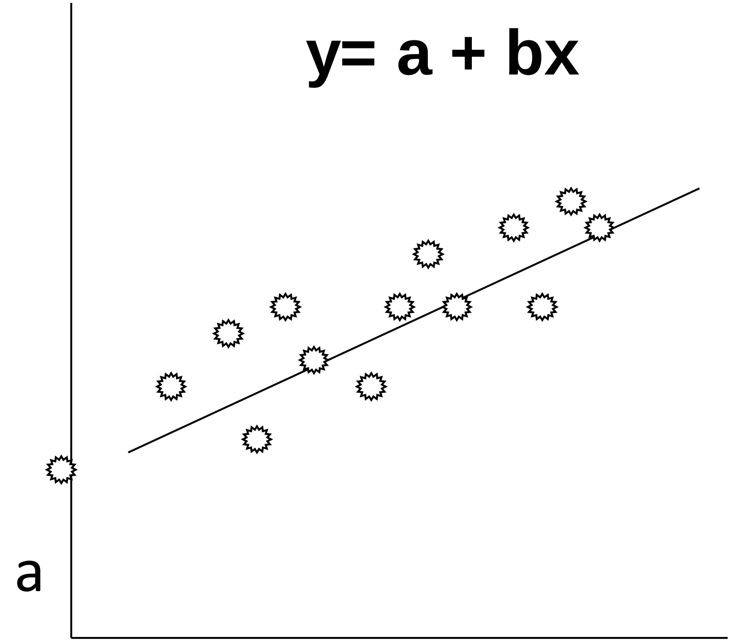
Gives the equation of straight line that best describes it and enables the prediction of one variable from the other.

a= constant, **called y-intercept**, **it is the place where the regression line intercept with y axis**

b=regression coefficient

x= any value of X variable

y= any value of Y variable



Interpretation of r:

- r **is always a number between -1 and +1**
- r is **positive if x and y** tend to **be high or low together, and the larger its value, the closer the association**
- r is **negative if high value of y tend to go with low values of x and vice versa**
- r **only measures the linear relationship so we have to draw a scatter diagram first to identify non-linear relationship.**

Pearson's Correlation Coefficient (r)

The Pearson correlation coefficient (r) is the most common way of measuring a linear correlation

-
- **The r^2** (The coefficient of determination), i.e. when value of **$r=0.58$, then $r^2=0.34$**,

this means that 34% of the variation in the values of y may be accounted for by **knowing values of x or** vice versa

Pearson’s Correlation Coefficient (r)

- **The value of (r) indicates the strength of the relationship**
- **<0.2 : very weak**
- **0.2- <0.4 : weak**
- **0.4- <0.7 : moderate**
- **0.7- <0.9 : strong**
- **≥0.9 : very strong**

The Pearson correlation coefficient (r) is the most common way of measuring a linear correlation e.g; The body weight (Kg) and plasma volume (Liter) of 8 healthy men are presented in this table;

No	**Body weight (Kg)**		**Plasma volume (Liter)**		
	X	**X^2**	**Y**	**Y^2**	**X x Y**
1	**58**	**3364**	**2.75**	**7.56**	**159.50**
2	**70**	**4900**	**2.86**	**8.18**	**200.20**
3	**74**	**5476**	**3.37**	**11.36**	**249.38**
4	**63.5**	**4032.25**	**2.76**	**7.62**	**175.26**
5	**62**	**3844**	**2.62**	**6.86**	**162.44**
6	**70.5**	**4970.25**	**3.49**	**12.18**	**246.05**
7	**71**	**5041**	**3.05**	**9.30**	**216.55**
8	**66**	**4356**	**3.12**	**9.73**	**205.92**
	$\sum x=535$	$\sum x^2=35983.5$	$\sum y=24.02$	$\sum y^2=72.798$	$\sum x.y=1615.292$

Pearson's Correlation Coefficient (r)

- $$r = \frac{n \sum XY - (\sum X)(\sum Y)}{\sqrt{[n\sum X^2 - (\sum X)^2]\,[n \sum Y^2 - (\sum Y)^2]}}$$

$$r = \frac{\sum (X - \bar{X})(Y - \bar{Y})}{\sqrt{[\sum (X - \bar{X})^2 \, . \, \sum (Y - \bar{Y})^2}}$$

$$r = \frac{SP\,xy}{\sqrt{(SSx \; . \; SSy)}} = \frac{SP\,xy}{\sqrt{(SQx \; . \; SQy)}}$$

SP = Sum of products of X and Y

SS=SQ= Sum of squares of X or of Y

$SPxy = \sum x.y - (\sum x).(\sum y) / n \Rightarrow 1615.292 - (535x24.02)/ 8 \Rightarrow 8.9545$

$SQx = \sum x^2 - (\sum x)^2 / n \quad ==> 35983.5 - (535)^2 / 8 \quad ==> 0.675$

$SQy = \sum y^2 - (\sum y)^2 / n \quad ==> 72.798 - (24.02)^2 / 8 \quad ==> 205.375$

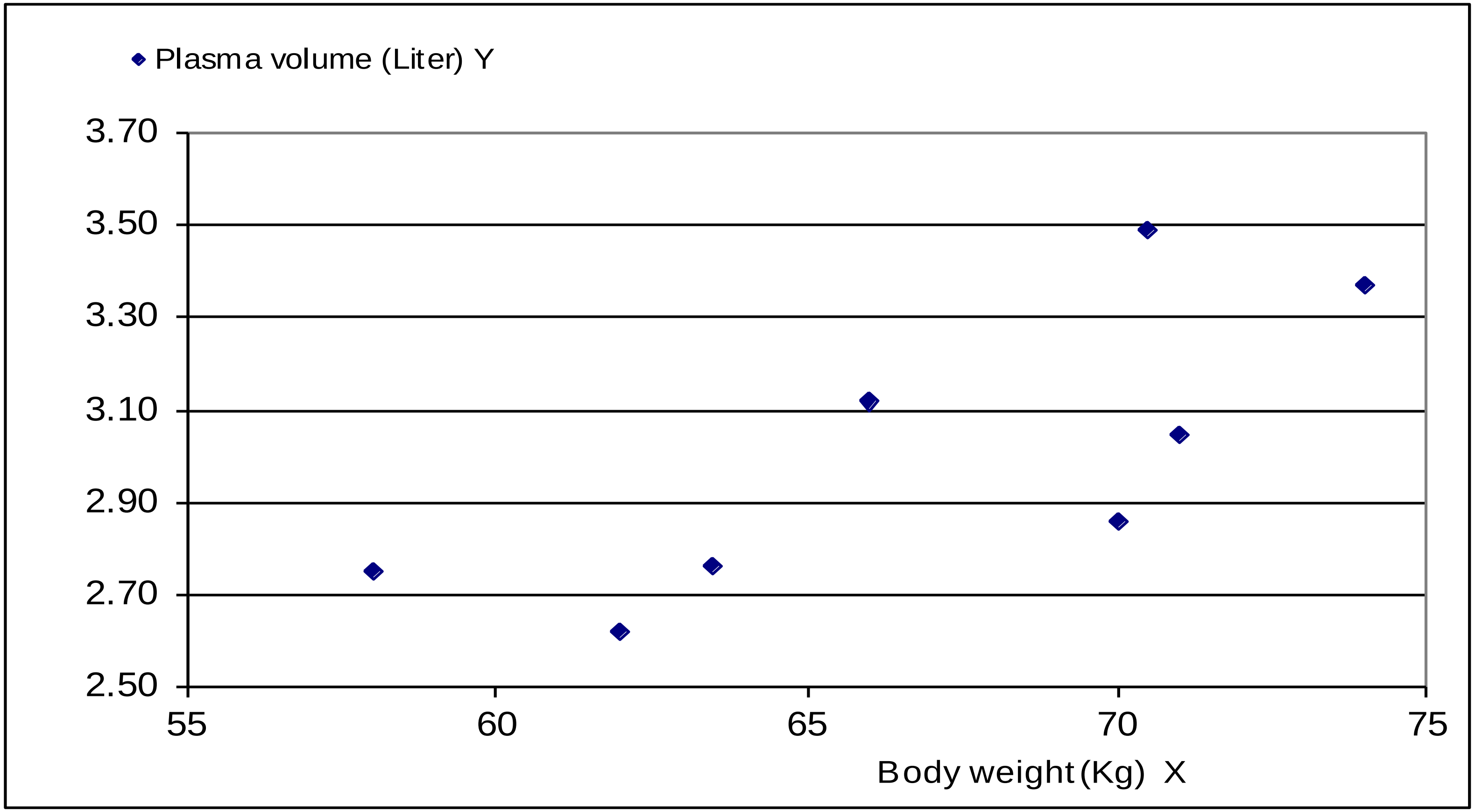
SP = Sum of products of X and Y

SS=SQ= Sum of squares of X or of Y

$$r = \frac{\sum x.y - (\sum x).(\sum y) / n}{\sqrt{[\sum x^2 - (\sum x)^2/ n] [\sum y^2 - (\sum y)^2/ n]}}$$

$r = 8.9545 / \sqrt{(0.675 \times 205.375)} \implies + 0.759$

There is strong direct relationship between weight (Kg) and plasma volume (Liter)



Scatter diagram of plasma volume and body weight showing linear regression line

Regression:

Regression:

Regression: it is the best fit for the relationship between two characteristics in a population.

Regression Line: The best line that fits the relationship between two characteristics in a population. Usually determined by the equation of straight line of the first degree which is→ y=a+bx

Y → any value on y axis, the dependent variable

a→constant, the intercept, the value of y when x equals to zero, it is the distance between the X axis and the point at which the regression line or its extension cuts the Y axis.

.b=regression coefficient= SPxy/SSX → SPxy/SQx

X → any value on X axis, the independant variable

We can get the equation according to the following

~~¥~~=a + b~~X~~

~~¥~~= ∑y/n = 24.02/8 → 3.0025

~~X~~=∑x/n= 535/8 → 66.875

b=SPxy/SQx = 8.9545/205.375→0.0436

~~¥~~=a + b~~X~~

3.0025=a + 0.0436 x 66.875

3.0025 = a + 2.916

a = 3.0025-2.916 → 0.0865

Y = 0.0865 + 0.0436 X

Regression coefficient “b” means that; for each one unit change in x axis there is about (b) unit change in y axis

For each one Kg increase in weight, there is about 0.0867 Liter increase in plasma volume

$Y = a + bX$

$3.0025 = a + 0.0436 \times 66.875$

$3.0025 = a + 2.916$

$a = 3.0025 - 2.916$ ⍰ 0.0865

$$Y = 0.0865 + 0.0436\ X$$

Thank you for attention

year 1ST
medical students