Chapter 13

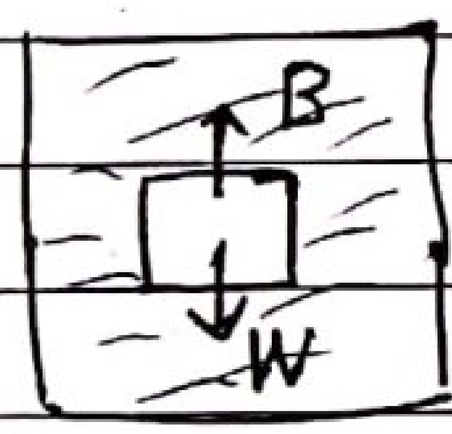
The Mechanics of Nonviscous Fluids

Archimede's principle:

An object floating or submerged in a fluid experiences an upward or buoyant force (قوة الطفو) due to the fluid.

The Buoyant force B is given by

$$B = \rho_o V g$$



where

ρ_o is the density of fluid

g is the acceleration of gravity

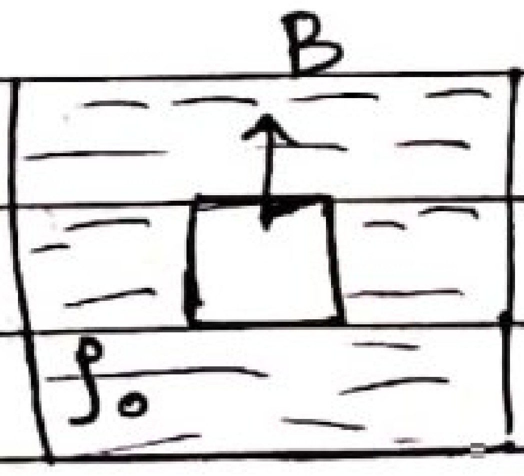
V is the volume of the object

Archimede's principle states that

the submerged object or floating object will lose from its real weight an amount equals the weight of the displaced fluid.

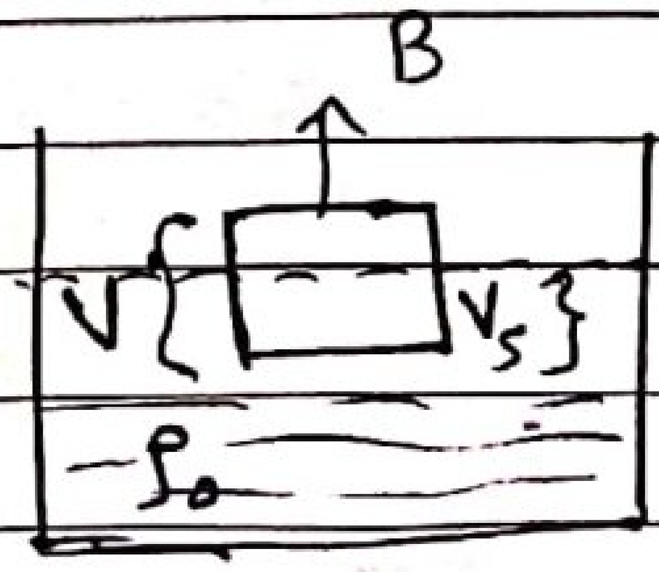
① For submerged objects

$$B = \rho_o V g$$



② For floating objects

$$B = \rho_o V_s g$$



where V_s is the portion of V that is submerged

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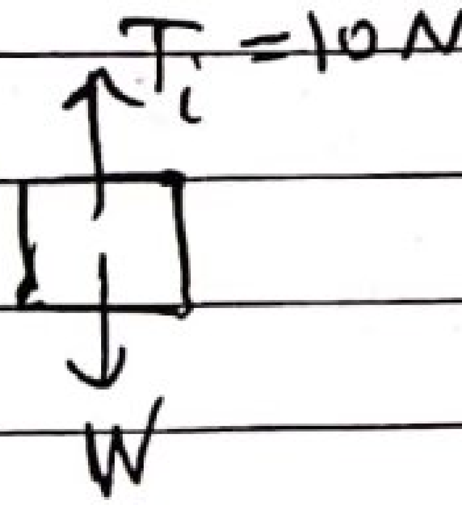
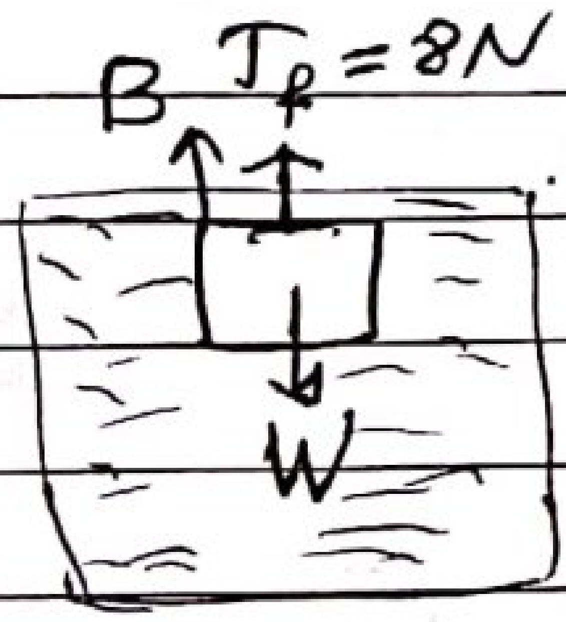
Example:

A piece of metal of unknown volume V is suspended from a string $T_i = 10\ N$, when it submerged in water $T_f = 8\ N$, what is the density of the metal

solution





before submerged: $T_i = W = \rho V g$

after submerged: $B + T_f = W$

$$\Rightarrow T_f = W - B$$
$$= \rho V g - \rho_o V g$$
$$= (\rho - \rho_o) V g$$

$$\frac{T_f}{T_i} = \frac{(\rho - \rho_o) V g}{\rho V g} = \frac{\rho - \rho_o}{\rho} = 1 - \frac{\rho_o}{\rho}$$

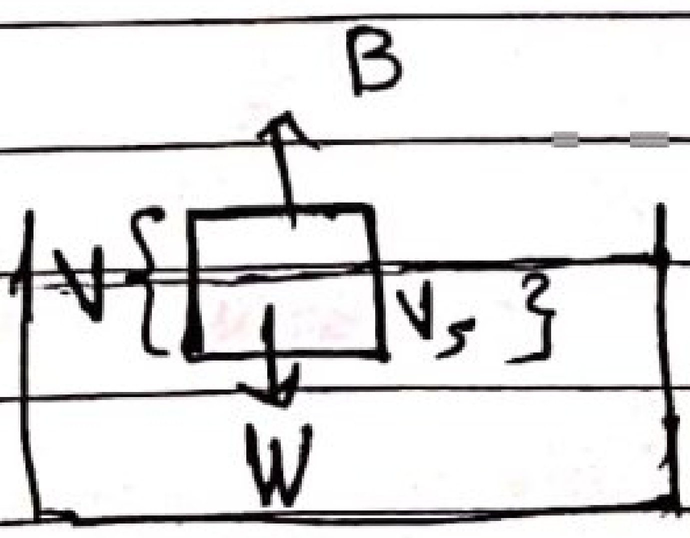
$$\frac{8}{10} = 1 - \frac{\rho_o}{\rho} \Rightarrow \rho = \rho_o \frac{10}{10-8}$$

but $\rho_o = 10^3\ kg/m^3$

$$\Rightarrow \rho = 10^3 \frac{10}{2} = 5000\ kg/m^3 = 5 \times 10^3\ kg/m^3$$

Example:

The density of ice is $920\ kg/m^3$ while that of water is $1000\ kg/m^3$, what fraction of an iceberg is submerged



$B = W$

$$\rho_o V_s g = \rho V g \Rightarrow \frac{V_s}{V} = \frac{\rho}{\rho_o} = \frac{920}{1000} = 0.92$$

Fraction of submerged $= \frac{V_s}{V} = 0.92$

Example:

A cube of volume 4 cm^3 is submerged halfway into water, what is the buoyant force experienced by the cube?

$$B = \rho_o V_s g = (10^3)(2 \times 10^{-6})(9.8) = 19.6 \times 10^{-3}\ N$$

Example:

An ice cube floats in a glass of water, what fraction of ~~water~~ the ice cube lies above the water level?

$$\text{fraction above} = 1 - \frac{V_s}{V} = 1 - \frac{\rho}{\rho_o} = 1 - \frac{920}{1000} = 0.08$$

The Equation of continuity

if we define the flow rate Q as the volume of fluid flowing past a point in a channel per unit time and has units of m^3/s as

$$Q = \frac{\Delta V}{\Delta t} \quad (1)$$

then the equation of continuity states that, if an incompressible fluid enter one end of a channel at a rate Q_1, it must leave the other end at a rate Q_2, which is the same. Mathematically

$$Q_1 = Q_2 \quad (2)$$

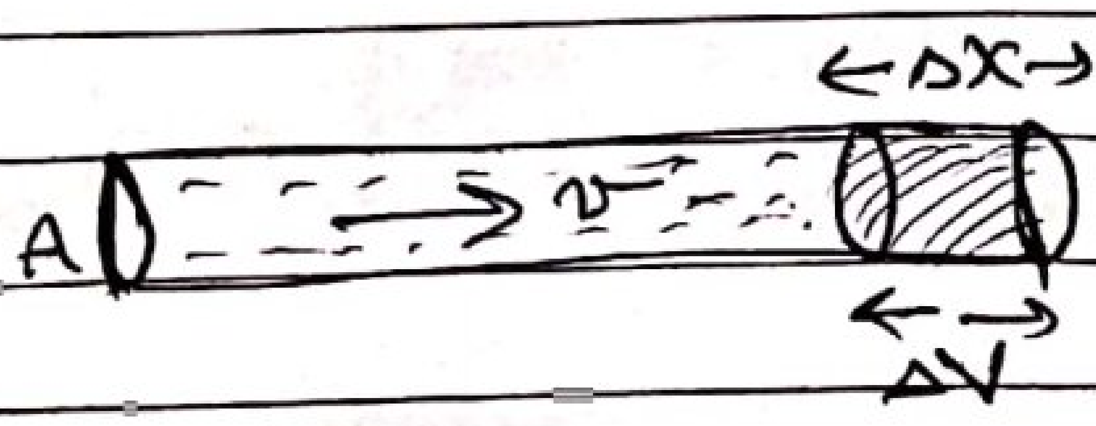
consider a section of a tube with a constant cross-sectional area A. In a time Δt, the fluid moves a distance $\Delta x = v\Delta t$ where v is the velocity of fluid. But the volume of fluid leaving the tube is $\cancel{\Delta V = A\Delta x = Av\Delta t}$ —(3)

~~but from (1)~~ $\Delta V = A\,\Delta x = Av\,\Delta t \quad (3)$

but from (1) $\Delta V = Q\,\Delta t \quad (4)$

then from (3) and (4)

$$\boxed{Q = Av} \quad (5)$$



From eq. (2)

$$\boxed{A_1 v_1 = A_2 v_2} \quad (6)$$

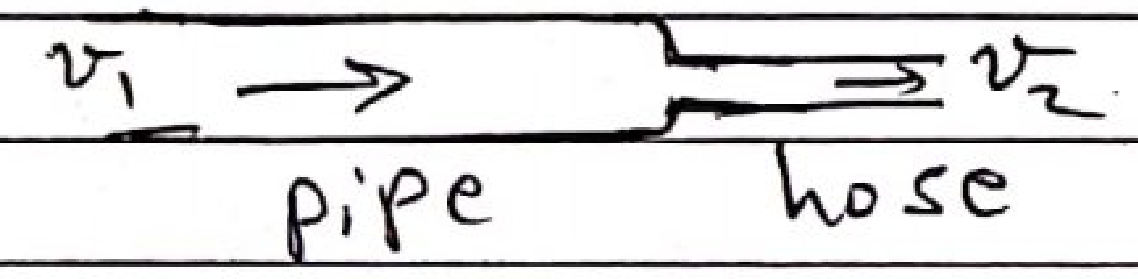
Example:

A water pipe leading up to a hose has a radius of 1 cm. Water leaves the hose at $Q = 3$ lit/min.

a) Find the velocity of water in the pipe

b) the hose has a radius of 0.5 cm, what is the velocity of the water in the hose?

solution



a) $Q_1 = \frac{\Delta V}{\Delta t} = \frac{3\times10^{-3}}{60}\ m^3/s$

$Q_1 = 5\times10^{-5}\ m^3/s$

but $Q_1 = A v_1 \Rightarrow v_1 = \frac{Q_1}{A} = \frac{5\times10^{-5}}{\pi r^2}$

$v_1 = \frac{5\times10^{-5}}{\pi(0.01)^2} = 0.159\ m/s$

b) $A_1 v_1 = A_2 v_2$

$v_2 = \frac{A_1}{A_2} v_1 = \left(\frac{r_1}{r_2}\right)^2 v_1 = \left(\frac{1}{0.5}\right)^2 (0.159)$

$v_2 = 0.636\ m/s$

Example:

A water hose 2 cm in diameter is used to fill a 20 liter bucket, if it takes 1 min to fill the bucket (دلو), what is the speed at which water leave the hose?

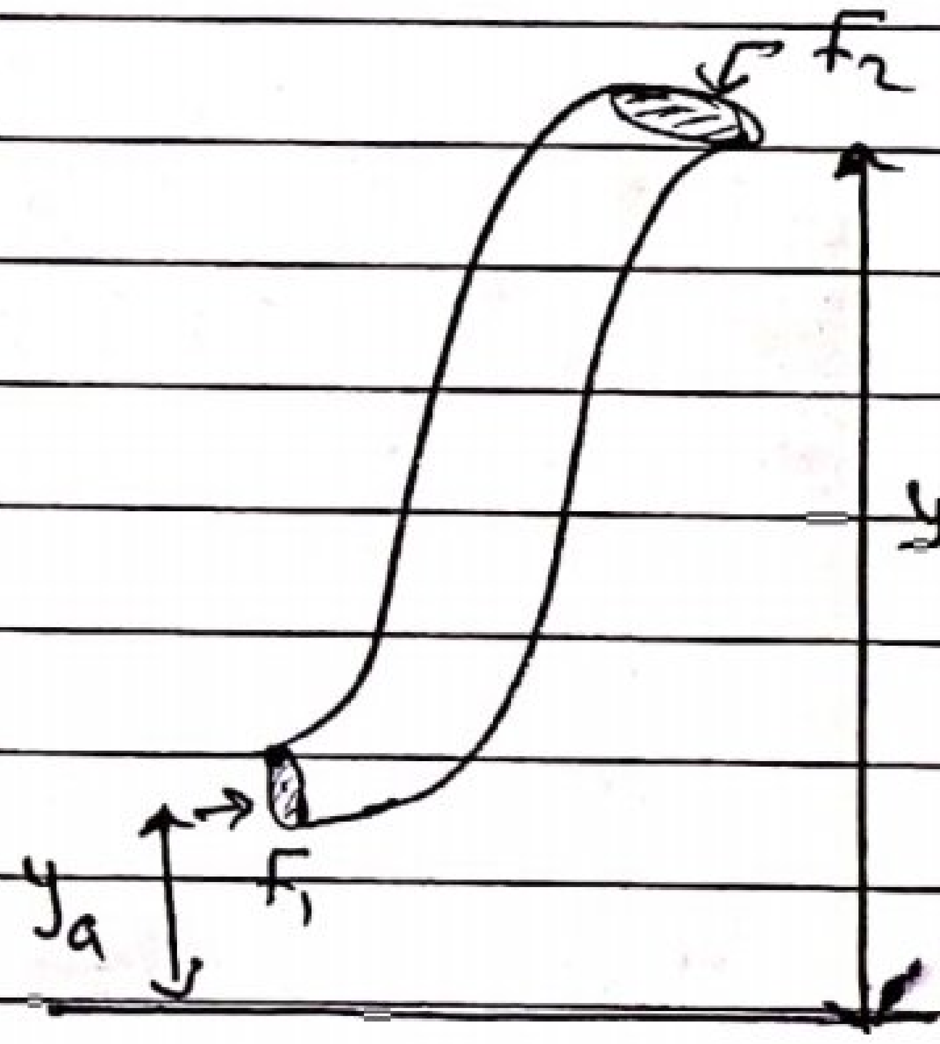
$Q = Av \Rightarrow v = \frac{Q}{A} = \frac{2\times10^{3}/60}{(1)^2\pi} = 106$ cm/s
$= 1.06$ m/s

Bernoulli's Equation

It states that " the work done on a fluid as it flows from one place to another is equal to the change in its mechanical energy "

It needs the following conditions to be satisfied

1) the fluid is nonviscous
2) the fluid is noncompressible (ρ = constant)
3) the flow is streamline not turbulent (eddies)
4) the velocity of the fluid at any point does not change during the motion (steady-state)



$$W = \Delta E = \Delta U + \Delta K$$

$U_i = mgy_a$, $U_f = mgy_b$

$K_i = \frac{1}{2} m v_a^2$, $K_f = \frac{1}{2} m v_b^2$

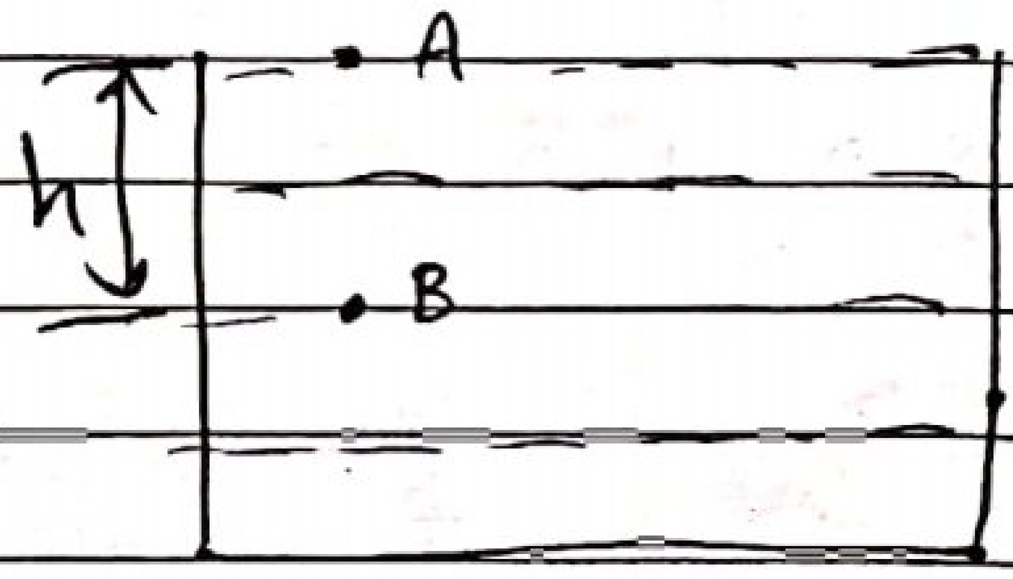
$$W = \Sigma F \Delta x = (F_1 - F_2)\Delta x$$
$$= (P_a - P_b) A \Delta x$$

$$\Rightarrow \boxed{P_a + \rho g y_a + \frac{1}{2}\rho v_a^2 = P_b + \rho g y_b + \frac{1}{2}\rho v_b^2}$$

- Fluid at rest

$v = 0$

$$P_A + \rho g y_A = P_B + \rho g y_B$$



$$P_A = P_B + \rho g (y_B - y_A)$$

or $\boxed{P_B = P_A + \rho g h}$

Example:
What is the pressure on a swimmer 5 m below the surface of a lake?

$$P_B = P_A + \rho g h \,, \quad P_A = P_{atm} = 1.013 \times 10^5 \; Pa$$

$$= (1.013 \times 10^5) + (1000)(9.8)(5) = 1.5 \times 10^5 \; Pa$$

Example:
The pressure 1 m above the floor is measured to be the normal atmospheric pressure $1.013 \times 10^5 \; Pa$. How much greater is the pressure at the floor

$$P_{Floor} = P_{atm} + \rho g h$$

$$= (1.013 \times 10^5) + (1.29)(9.8)(1)$$

$$\approx 1.013 \times 10^5 \; Pa \quad (\text{negligible})$$

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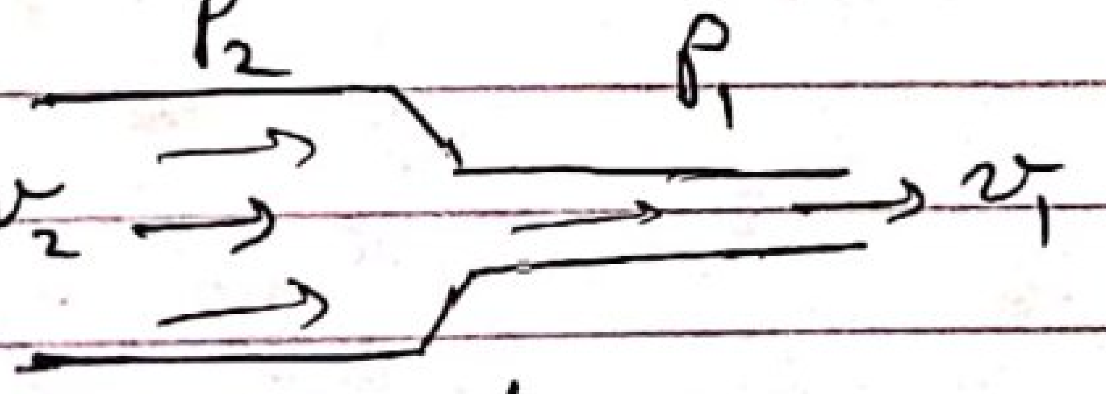
Example: (Venturi tube)



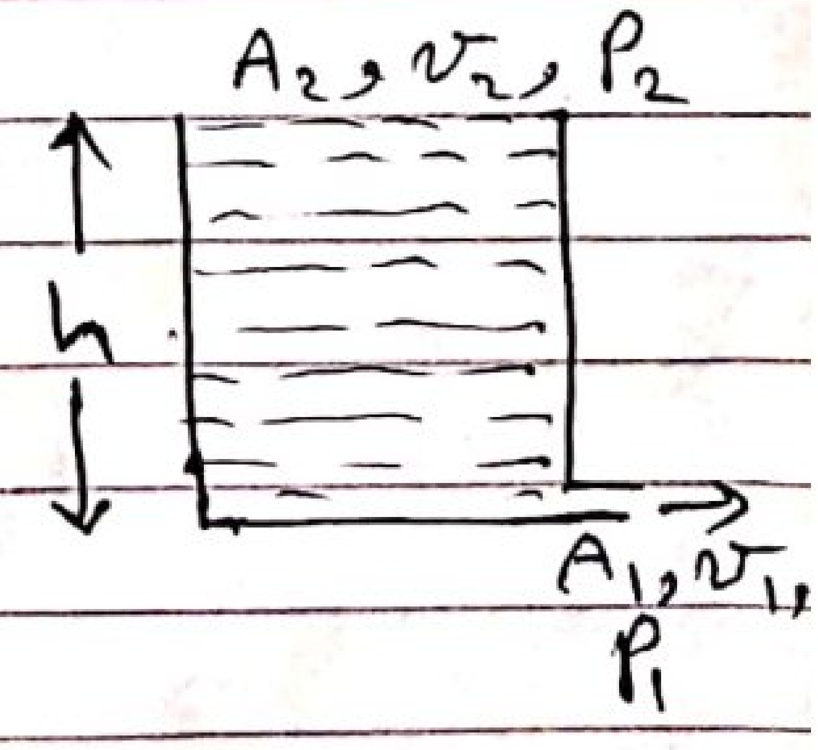
Venturi tube is a horizontal pipe used to measure flow velocities in an incompressible fluid

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 \quad \text{—(1)}$$
$$A_1 v_1 = A_2 v_2 \quad \text{—(2)}$$

$$(1) \Rightarrow P_1 + \frac{1}{2}\rho \left(\frac{v_2 A_2}{A_1}\right)^2 = P_2 + \frac{1}{2}\rho v_2^2$$
$$\Rightarrow v_2 = A_1 \sqrt{\frac{2(P_1 - P_2)}{\rho (A_1^2 - A_2^2)}}$$

Example:

Water flow from a hole at a bottom of an open tank of height h. Find the speed of water out of the hole



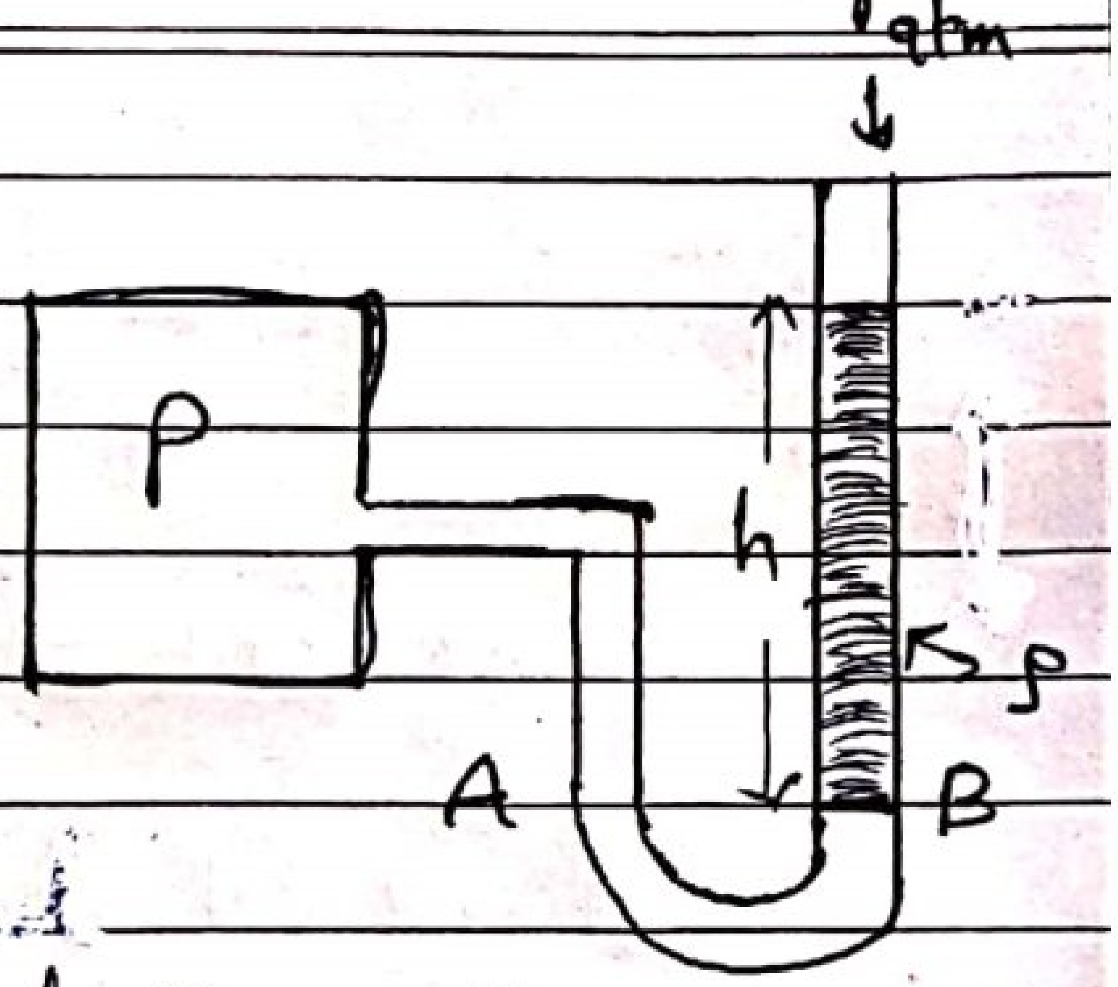
$P_1 \simeq P_2$ (atmospheric pressure)

$$\rho g y_1 + \frac{1}{2}\rho v_1^2 = \rho g y_2 + \frac{1}{2}\rho v_2^2 \quad \text{—(1)}$$
$$A_1 v_1 = A_2 v_2 \quad \text{—(2)}$$

$$v_2 = \frac{A_1}{A_2} v_1 \simeq 0$$

$$(1) \Rightarrow \rho g y_1 + \frac{1}{2}\rho v_1^2 = \rho g y_2$$
$$\rho g (y_2 - y_1) = \frac{1}{2}\rho v_1^2$$
$$v_1^2 \simeq 2\rho g h$$
$$v_1 \simeq \sqrt{2gh}$$

The Manometer



The open-tube manometer is a U-shaped tube used to measuring gas pressure.
It contains mercury or water or oil for low pressures.
One end of the tube is open to the atmosphere ~~pressure~~ and the other end is contact with the gas

Now
$$P_A = P_B$$
$$P = P_{atm} + \rho g h$$

or gauge pressure
$$P - P_{atm} = \rho g h$$

Example: What is the gauge pressure in a U-shaped tube if the level of oil ($\rho = 830\ kg/m^3$) in the open side is raised a distance of 5 cm

$$P = P_{atm} + \rho g h$$

gauge pressure: $P - P_{atm} = \rho g h$
$$= (830)(9.8)(0.05)$$
$$= 406\ Pa$$