DONE BY :
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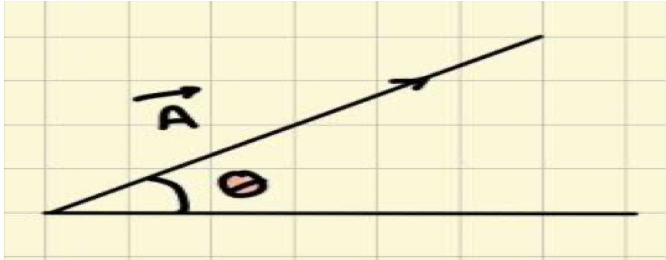
Motion in two dimensions

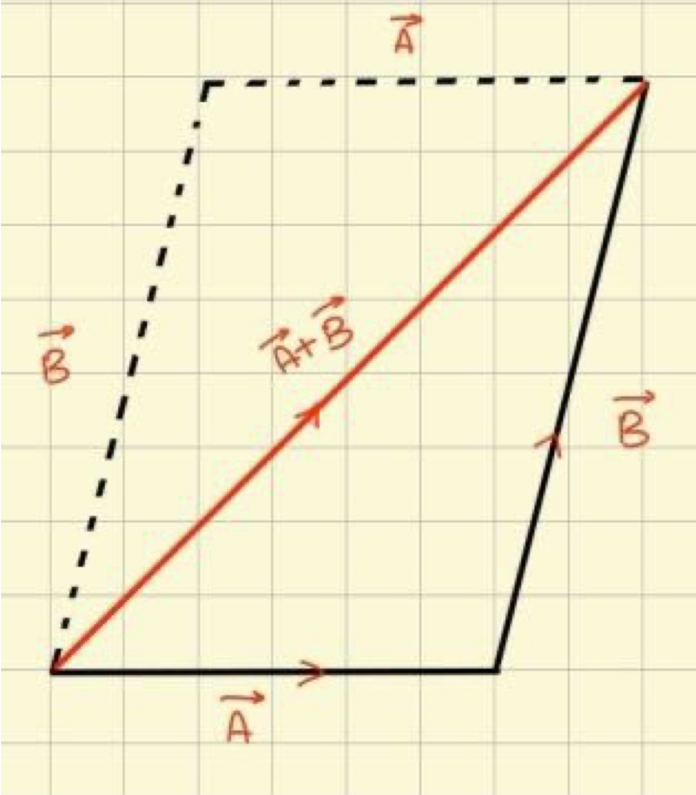
Introduction to vectors

- **Vector** : is defined as a physical quantity which has both magnitude and direction (forces , velocity , acceleration ….)
- **Scalar** : is defined as a physical quantity which has magnitude only (mass , time , energy …..)
- Vectors are denoted by $\vec{A}$ or A (highlighted)
- Magnitude of vectors $|\vec{A}|$ or A
- ***A vector*** is pictured in a diagram by an arrow with a length is proportional to the magnitude and an angle for the direction

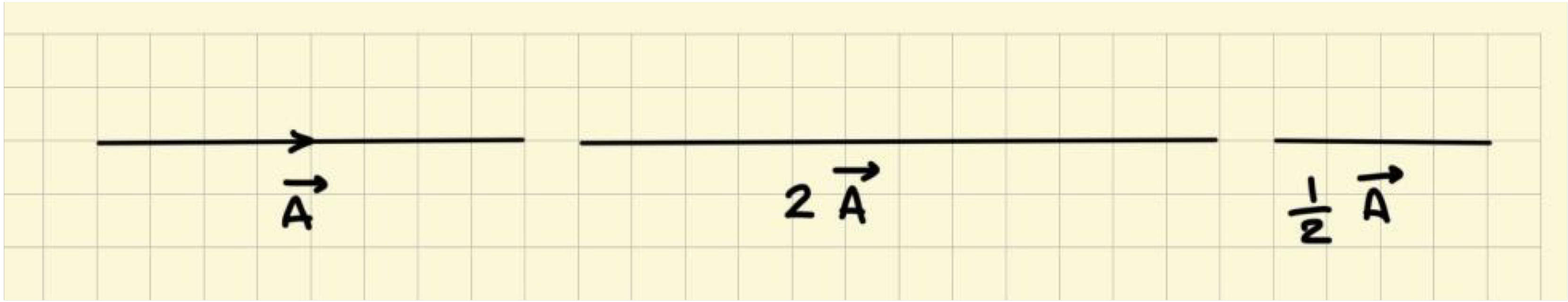


Addition of vectors (graphically)

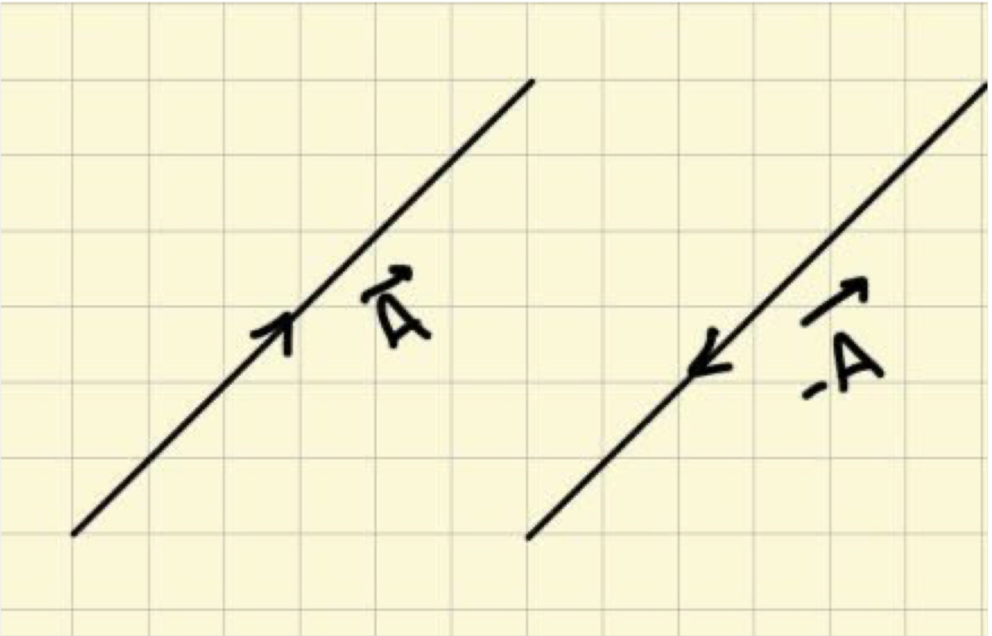
- Note that $\vec{A} + \vec{B} = \vec{B} + \vec{A}$



Multiplication of a vector by a scalar

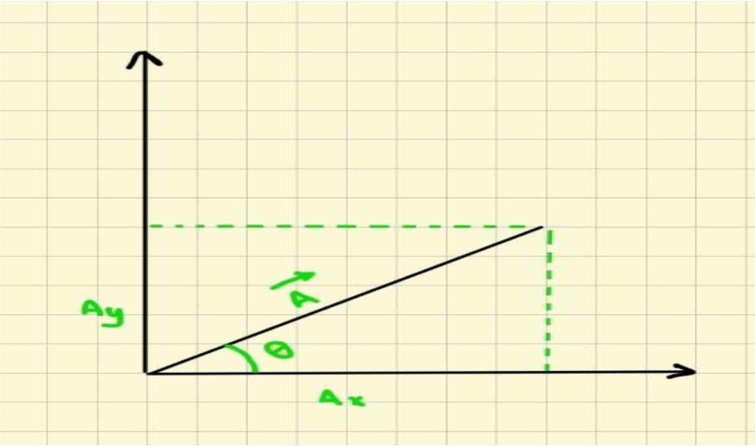


Negative of a vector



Components of a vector

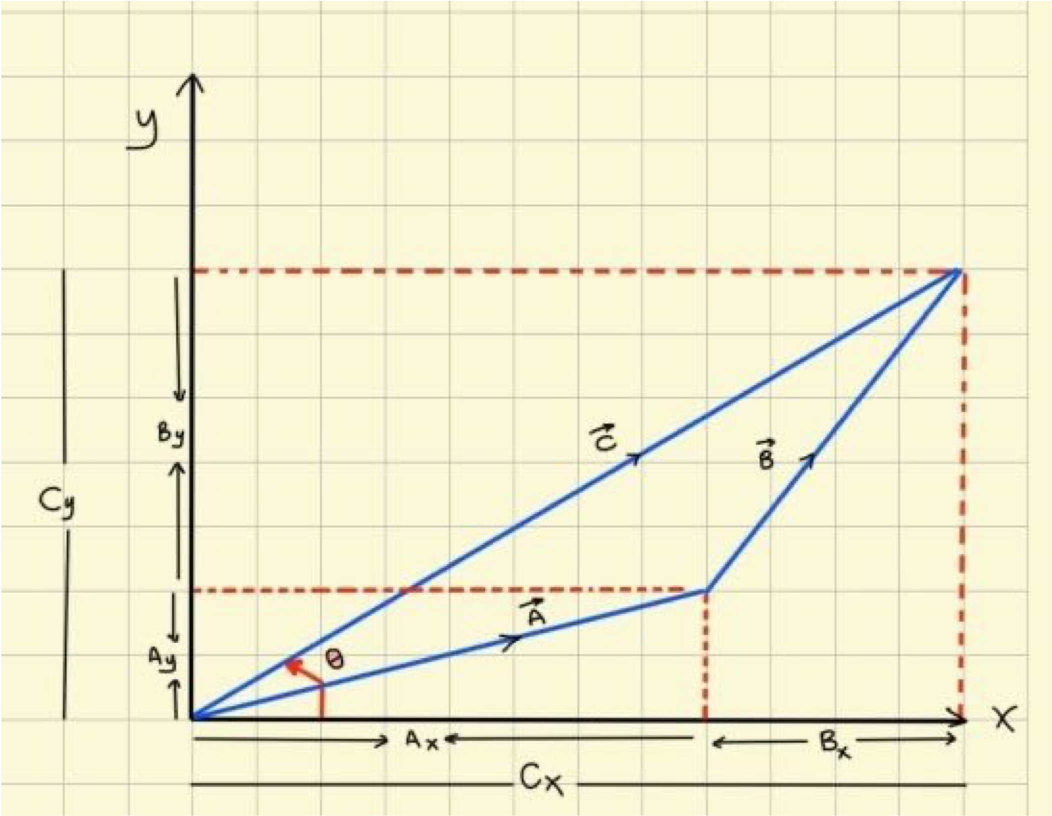
- $A_x = A * \cos\theta$ (x – components)
- $A_y = A * \sin\theta$ (y – components)
- Where θ is the angle measured from the positive x – axis counter clockwise



Uni vectors ($\hat{x}$, $\hat{y}$)

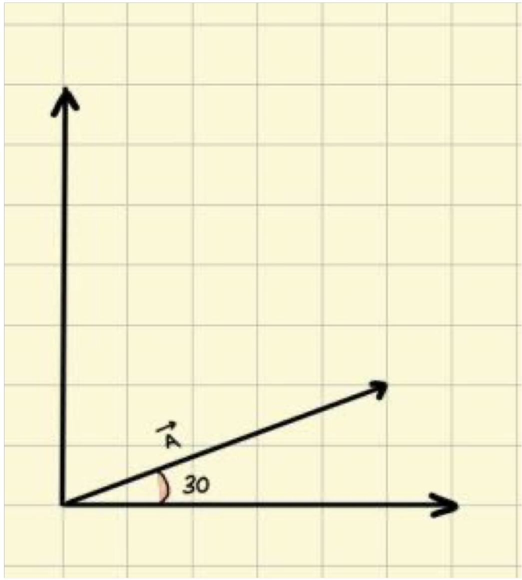
- The vector $\vec{A}$ can be written in terms of unit vectors as :
- $\vec{A} = (a_x * \hat{x}) + (a_y * \hat{y})$
- Where $\hat{X}$ is a unit vector in the x – direction
- Where $\hat{Y}$ is a unit vector in the y – direction

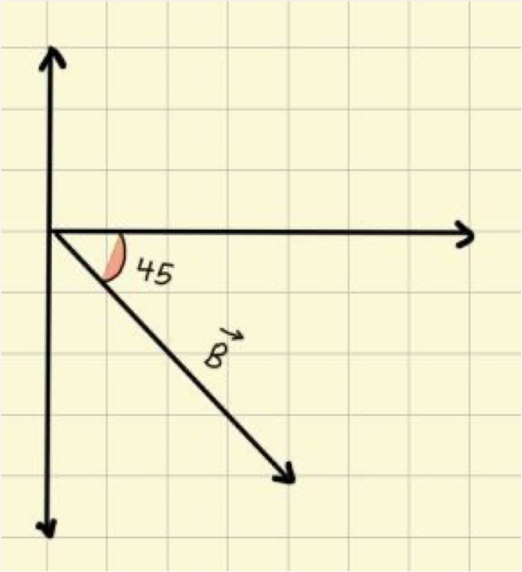
- **The sum of two vectors may be obtained as :**
- $\vec{C} = \vec{A} + \vec{B} = (A_X * \hat{X} + A_Y * \hat{Y}) + (B_X * \hat{X} + B_Y * \hat{Y}) = (C_X * \hat{X}) + (C_Y * \hat{Y})$
- **With :**
- $C_X = A_X + B_X$
- $C_Y = A_Y + B_Y$
- ____________
- $C = |\vec{C}| = \sqrt{C_X^2 + C_Y^2}$
- $\theta = \tan^{-1} c_y/c_x$



• Example :

- 1) find the components of the vectors $\vec{A}$ and $\vec{B}$ if $|\vec{A}| = 2$, $|\vec{B}| = 3$
- 2) find the sum resultant of $\vec{A}$ and $\vec{B}$

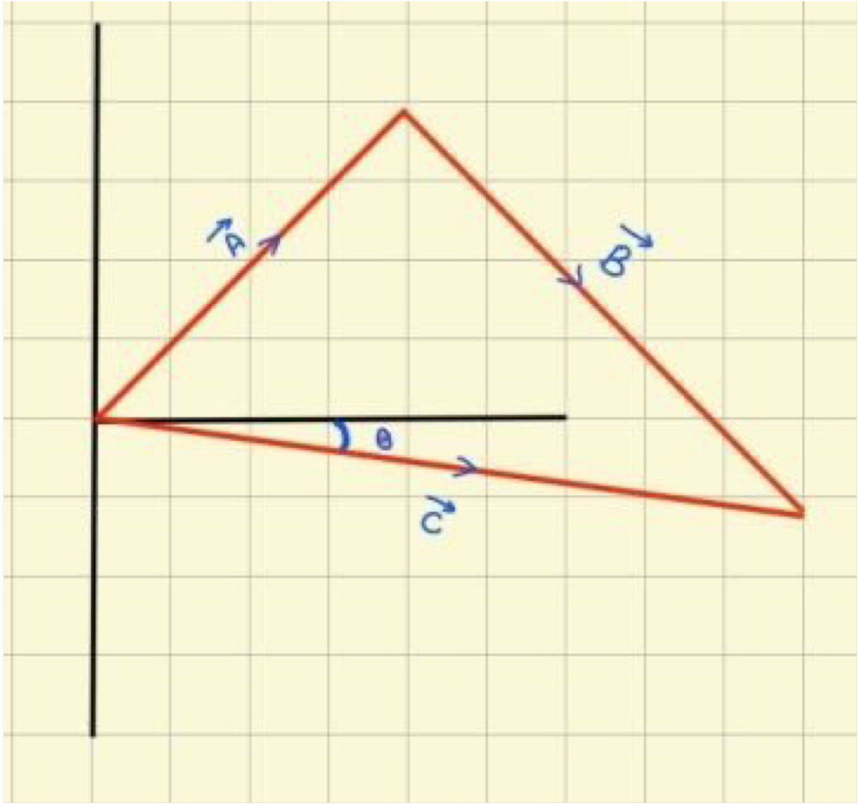




• Solution :

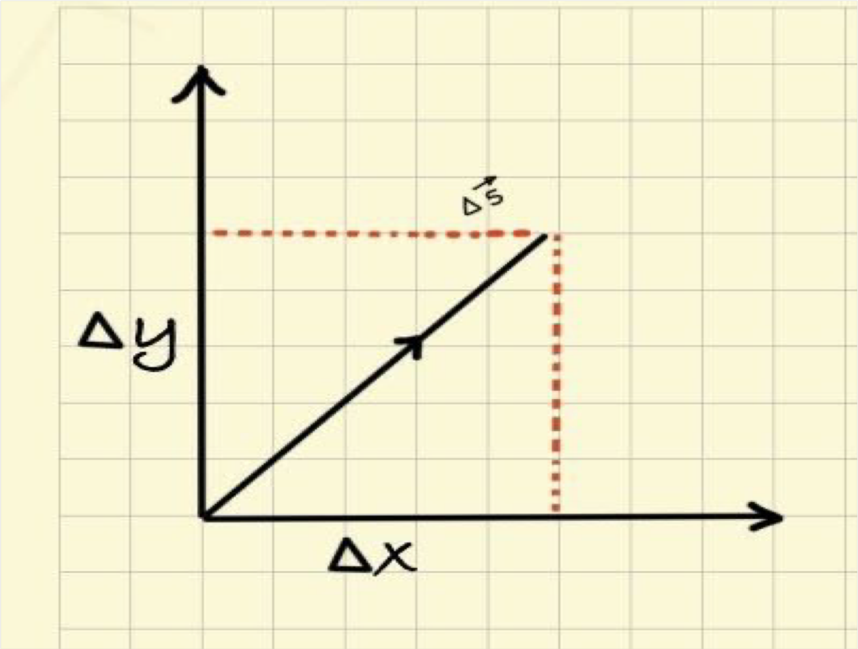
- 1)
- $A_x = A * \cos \theta_1 = 2 \cos 30 = 1.73$
- $A_y = A * \sin \theta_1 = 2 \sin 30 = 1$
- $B_x = B * \cos \theta_2 = 3 \sin(-45) = 2.12$
- $B_y = B * \sin \theta_2 = 3 \sin(-45) = -2.12$
- 2)
- $\vec{C} = \vec{A} + \vec{B} = (A_X + B_X) * \hat{X} + (A_Y + B_Y) * \hat{Y} \ldots. = (1.73 + 2.12)*\hat{X} + (1-2.12)*\hat{Y}$
- $= 3.85*\hat{X} - 1.12*\hat{Y}$
- $C = |\vec{C}| = \sqrt{(3.85)^2 + (-1.12)^2} = 4$
- $\theta = \tan^{-1} c_y/c_x = \tan^{-1} -1.12/3.85 = -16.2^{\circ}$

- **Example** : given $\vec{A} = 2\hat{X} + \hat{Y}$, $\vec{B} = 4\hat{X} + 7\hat{Y}$, Find :
- 1) the components of $\vec{C} = \vec{A} + \vec{B}$
- 2) the magnitude and direction of $\vec{C}$
- **Solution :**
- 1)
- $\vec{C} = (2 + 4) * \hat{X} + (1 + 7) * \hat{Y} = 6\hat{X} + 8\hat{Y}$
- $C_X = 6$, $C_Y = 8$ ......$|\vec{C}| = \sqrt{36+64} = 10$
- 2)
- $\theta = \tan^{-1} c_y/c_x = \tan^{-1} 8/6 = 53^{\circ}$

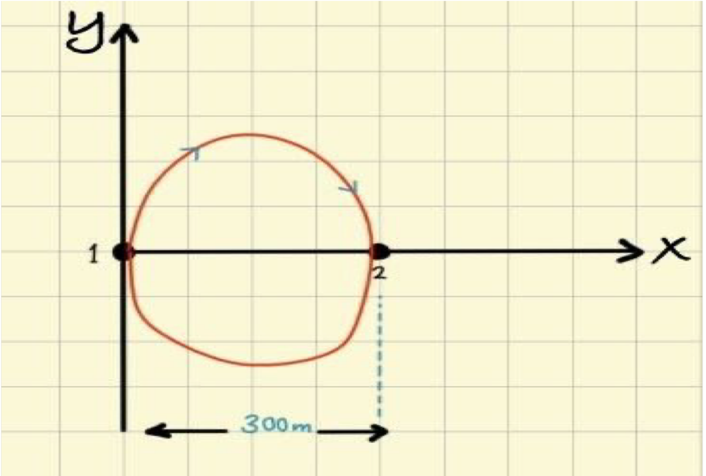


The velocity in two dimension

- **The displacement $\overrightarrow{\Delta S}$ is given by** : $\overrightarrow{\Delta S} = (\Delta x * \hat{x}) + (\Delta y * \hat{y})$
- **The average velocity is :**
- $\overrightarrow{\bar{V}} = \overrightarrow{\Delta S}/\Delta t = (\Delta x/\Delta t * \hat{x}) + (\Delta y/\Delta t * \hat{y})$
- $\overrightarrow{\bar{V}} = (\bar{v}_x * \hat{x}) + (\bar{v}_y * \hat{y})$



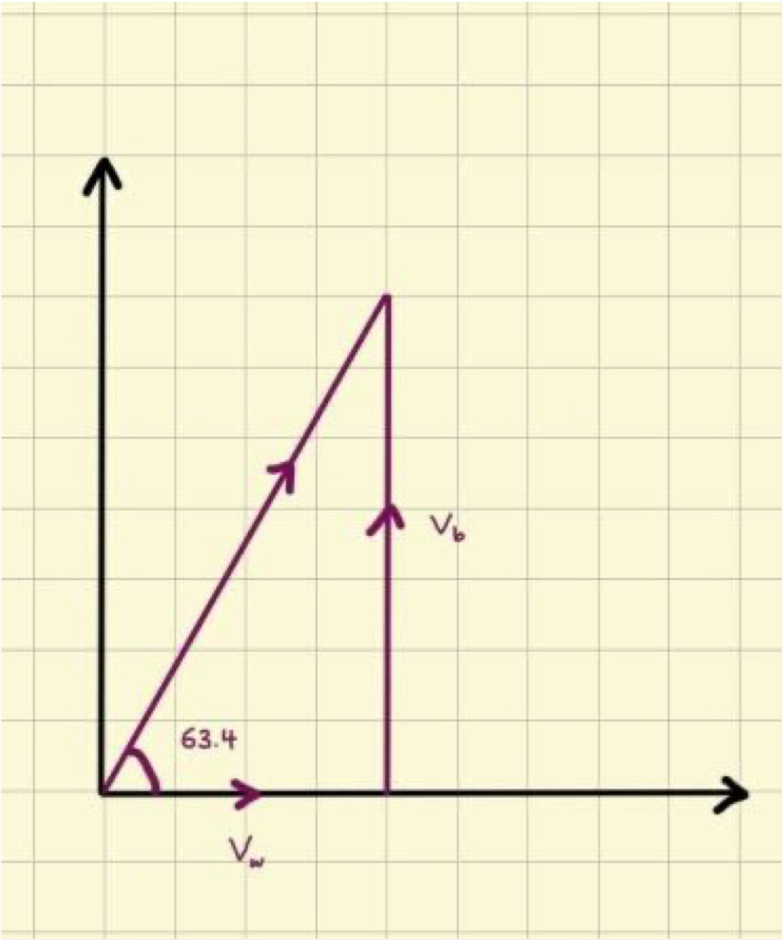
- **Example** : a car travels halfway around an oval race track at constant speed of 30 m/s :
- 1) what are it v_{ins} at points 1 and 2 ?
- 2) it takes 40s to go from 1 to 2 which are 300m apart , what is the average velocity during this time interval ?



- **Solution :**
- 1)
- $\vec{V}_{ins(1)} = 30\hat{y}$ m/s
- $\vec{V}_{ins(2)} = -30\hat{y}$ m/s
- 2)
- $\vec{\overline{V}} = \Delta\vec{S}/\Delta t = (\Delta x * \hat{x} + \Delta y * \hat{y})/\Delta t = (300\hat{x} + 0\hat{y})/40 = 300\hat{x}/40 = 7.5\hat{x}$ m/s

- **Example** : a boat moves at 10m/s relative to the water toward the shore , the velocity of the water current is 5m/s to the right , find the velocity of the boat relative to the shore ?

- **Solution :**
- $|\vec{V}| = \sqrt{v_b^2 + v_w^2} = \sqrt{10^2 + 5^2} = 11.18$ m/s
- $\theta = \tan^{-1} v_b/v_w = \tan^{-1} 10/5 = 63.4^\circ$



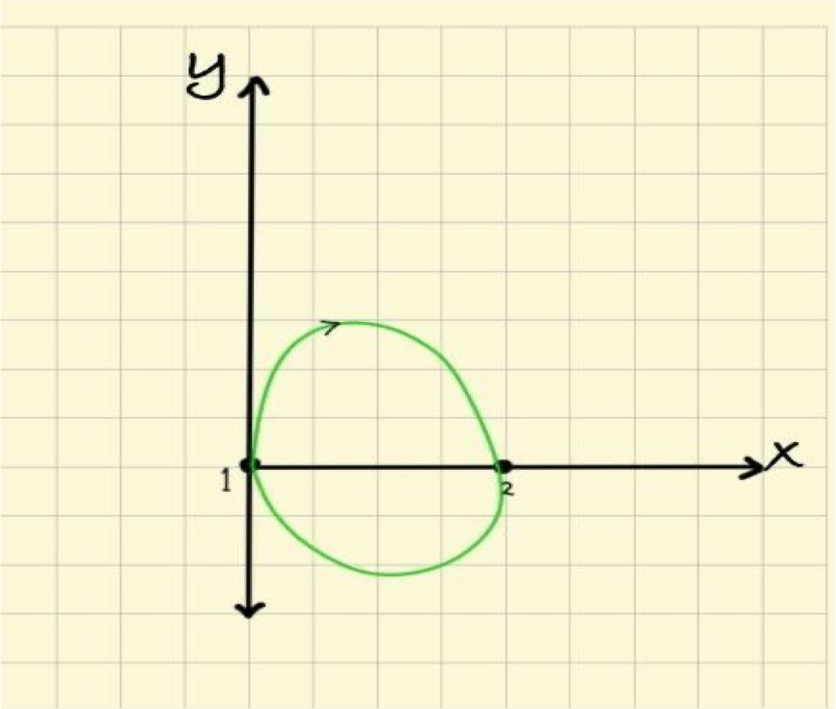
Acceleration in two dimension

- **Average acceleration** : $\vec{a} = \Delta\vec{v}/\Delta t = (\vec{v}_2 - \vec{v}_1)/\Delta t$
- where $\vec{V} = (v_x * \hat{x}) + (v_y * \hat{y})$:
- $\vec{a} = (\Delta v_x/\Delta t * \hat{x}) + (\Delta v_y/\Delta t * \hat{y}) = (a_x * \hat{x}) + (a_y * \hat{y})$
- **Instantaneous acceleration** : $\vec{a} = d\vec{v}/dt$
- $\vec{a} = (dv_x/dt * \hat{x}) + (dv_y/dt * \hat{y}) = (a_x * \hat{x}) + (a_y * \hat{y})$
-

- **Example** : the example that is in slide 15 , the velocity of the car changes from $\vec{v}=30\hat{y}$ m/s to $-30\hat{y}$ m/s , what was the average acceleration ?

- **Solution :**

- $\vec{\bar{A}} = \Delta\vec{v}/\Delta t = (\vec{v}_2 - \vec{v}_1)/\Delta t = (-30\hat{y} - 30\hat{y})/40 = 1.5\hat{y}$ m/s^2



Finding the motion of an object

- Motion in two dimensions can be considered as two separate motions , the first in the x – direction and the second in the y – direction .
- Using the same kinematical equation for one dimensional motion:
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$V_x = v_{0x} + a_x t$	$\Delta x = v_{0x} t + \frac{1}{2} a_x t^2$
$V_y = v_{0y} + a_y t$	$\Delta y = v_{0y} t + \frac{1}{2} a_y t^2$

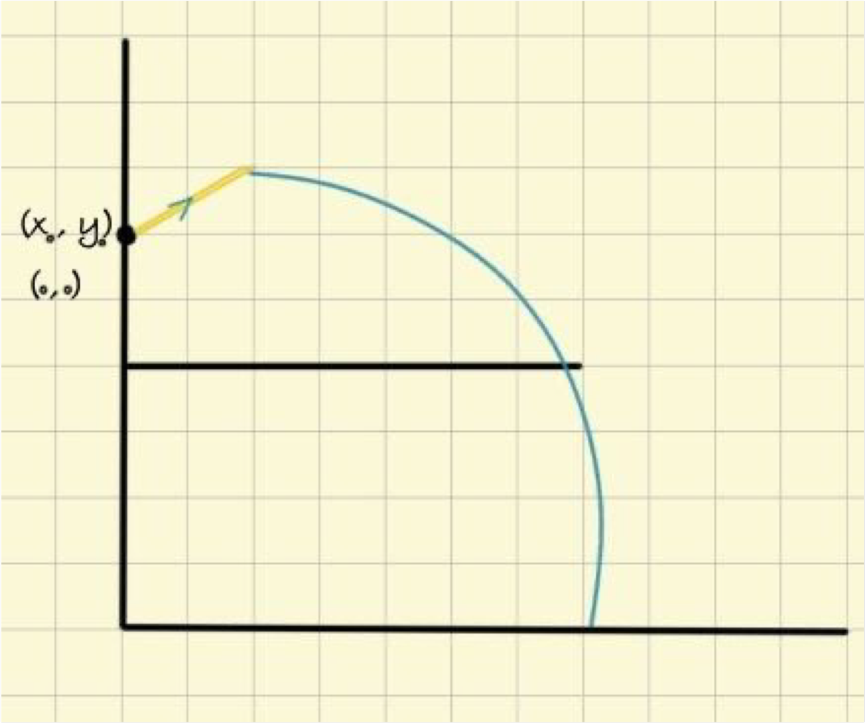
projectiles

- ***Two conditions must be satisfied :***
- 1) air resistance is neglected
- 2) $\vec{g}$ is constant and the only force acting on the object is the gravitational force

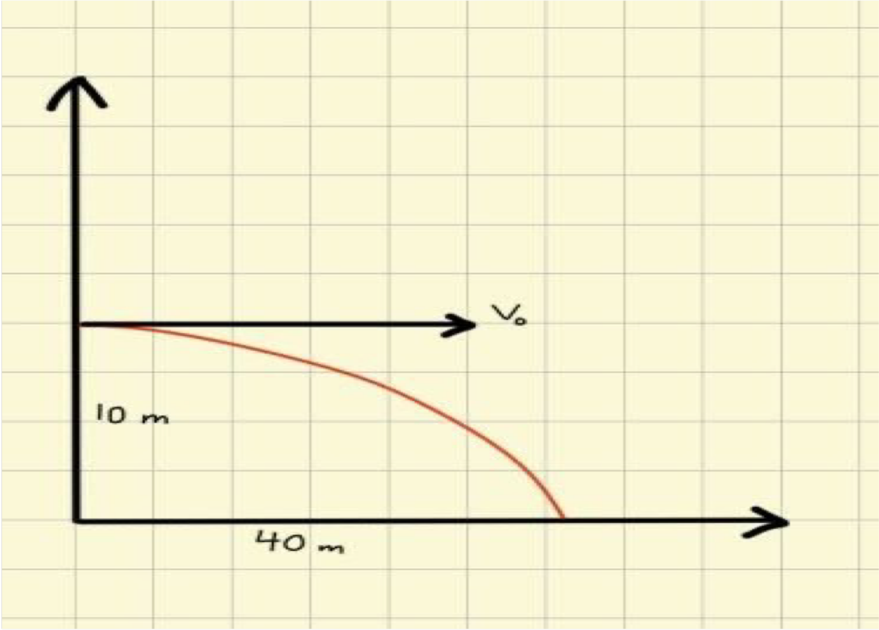
- ***Projectile motion can be considered as two separate motions :***
- 1) uniform motion in the x – direction ($a_x = 0$)
- 2) free falling motion in the y – direction ($a_y = -g$)

Uniform motion	Free falling
$V_x = v_{0x}$	$V_y = v_{0y} - gt$
$\Delta x = v_{0x}t$	$\Delta y = v_{0y}t - \frac{1}{2} gt^2$

- **Example** : a diver leaps from a tower with $v_{0x} = +7$ m/s and $v_{0y} = +3$ m/s , find :
- 1) the components of her position and velocity 1s later ?
- 2) the position and velocity
- **Solution :**
- 1)
- $V_x = v_{0x} = 7$ m/s
- $V_y = v_{0y} - gt = 3 - (9.8 * 1) = -6.8$ m/s
- $X = v_{0x}t = 7$ m
- $Y = v_{0y}t - ½ gt^2 = 3 - (½ * 9.8 * 1^2) = -1.9$ m
- 2)
- $V = \sqrt{v_x^2 + v_y^2} = \sqrt{7^2 + (-6.8)^2} = 9.7$
- $θ = \tan^{-1} v_y/v_x = -44.1°$
- $|\vec{V}| = \sqrt{x^2 + y^2} = \sqrt{7^2 + (-1.9)^2} = 7.25$ m
- $θ = \tan^{-1} y/x = -15°$



- **Example** : a ball is thrown horizontally from a window 10m above the ground and hits the ground 40m away , how fast was the ball thrown ?
- **Solution :**
- $Y = v_{0y}t - ½\, gt^2 = 0 - ½ * 9.8 * t^2$ …… $- 10 = -1/2 * 9.8 * t^2$ …… $t = 1.429$ s
- $X = v_{0x}t$ …… $v_{0x} = x/t = 40/1.429 = 28$ m/s



- ***Note that :*** ***$v_{0x} = v_0 = 28$ m/s***

- **Example** : a ball is kicked with $v_0 = 25$ m/s at an angle of 30° to the horizontal :
- 1) when does it reach its greatest height ?
- 2) where is it at that time ?
- **Solution :**
- 1)
- $V_{0x} = v_0 \cos\theta = 25 \cos 30 = 21.7$ m/s
- $V_{0y} = v_0 \sin\theta = 25 \sin 30 = 12.5$ m/s
- ***At the highest distance …. $v_y = 0$***
- $V_y = v_{0y} - gt$ …… $0 = 12.5 - 9.8t$ ……. $t = 1.28$ s
- 2)
- $X = v_{0x}t = 21.7 * 1.28 = 27.8$ m
- $Y = v_{0y}t - \frac{1}{2} gt^2 = (12.5 * 1.28) - (\frac{1}{2} * 9.8 * 1.28^2) = 7.97$ m
- ***Position*** : $\vec{v} = (x * \hat{x}) + (y * \hat{y}) = (27.8 * \hat{x} + 7.97 * \hat{y})$ m