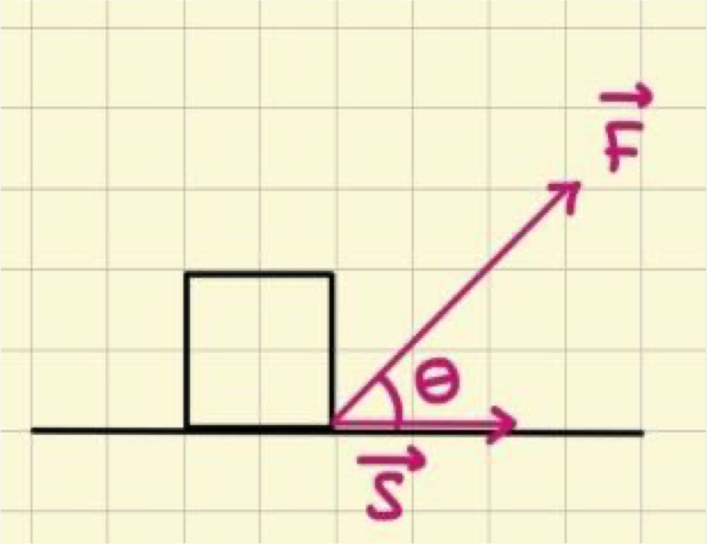
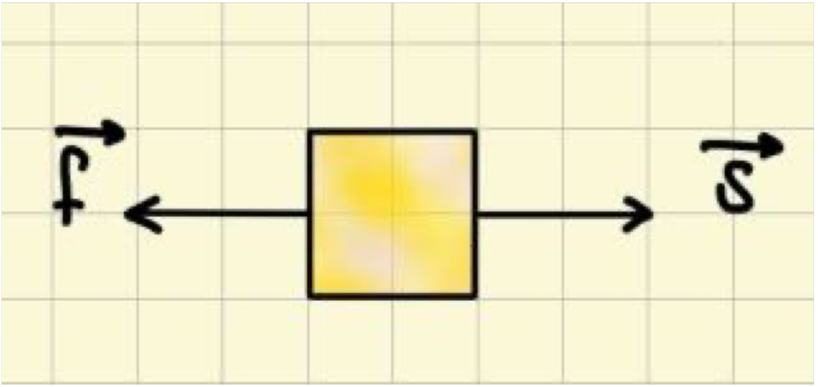
DONE BY :
BANDAR AL - SHWABKAH

work energy and power

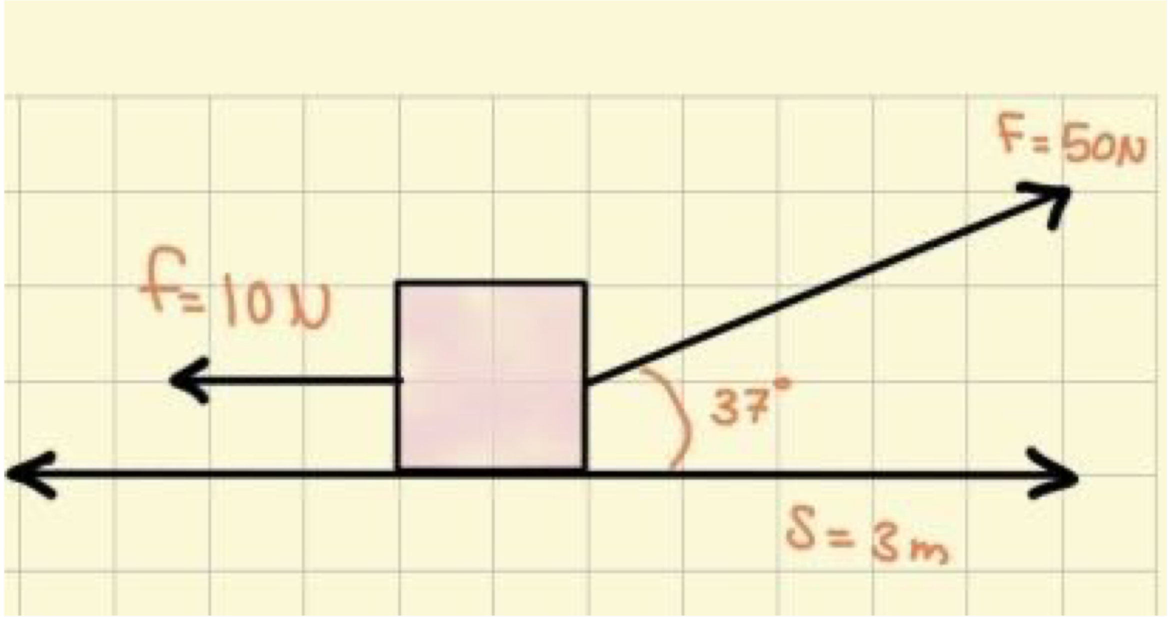
Work done by a consonant force

- The work done by a constant force $\vec{F}$ is given by :
- $W = \vec{F} * \vec{S} = FS * \cos\theta$
- The unit of work is ***joule*** (J) , where ***1J = 1n.m***
- The work done by frictional force is :
- $W_f = - FS$
- ***When $\theta = 180°$***





- **Example : (dragging a box)**
- 1) calculate the work done by the force F
- 2) calculate the work done by the fractional F
- 3) determine the net work done on the box by all forces

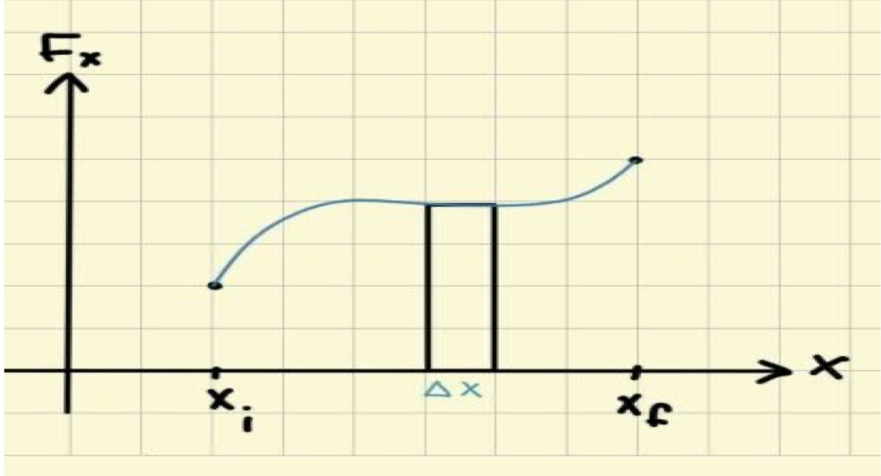


- **To be continued …..**

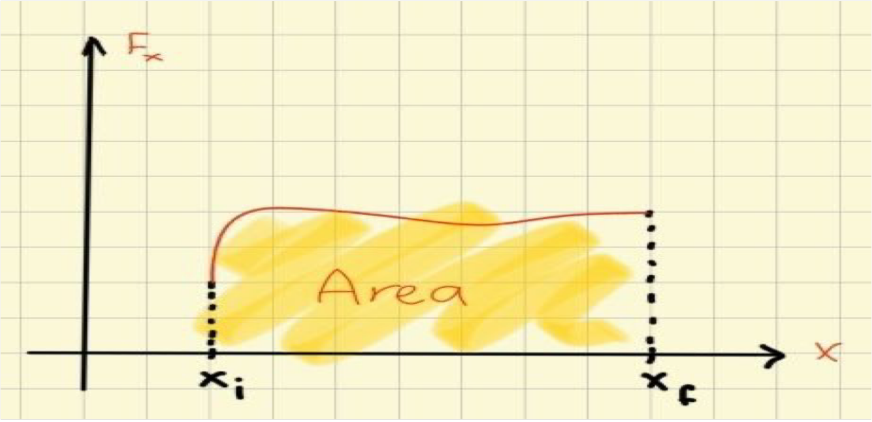
- **Solution** :
- 1) W_F = FS * cos 0 = 50 * 3 * cos 37 = 120J
- 2) W_f = FS * cos 180 = - FS = -10 * 3 = -30J
- 3) W_{net} = W_F + W_f = 120 – 30 = 90J

Work done by a varying force

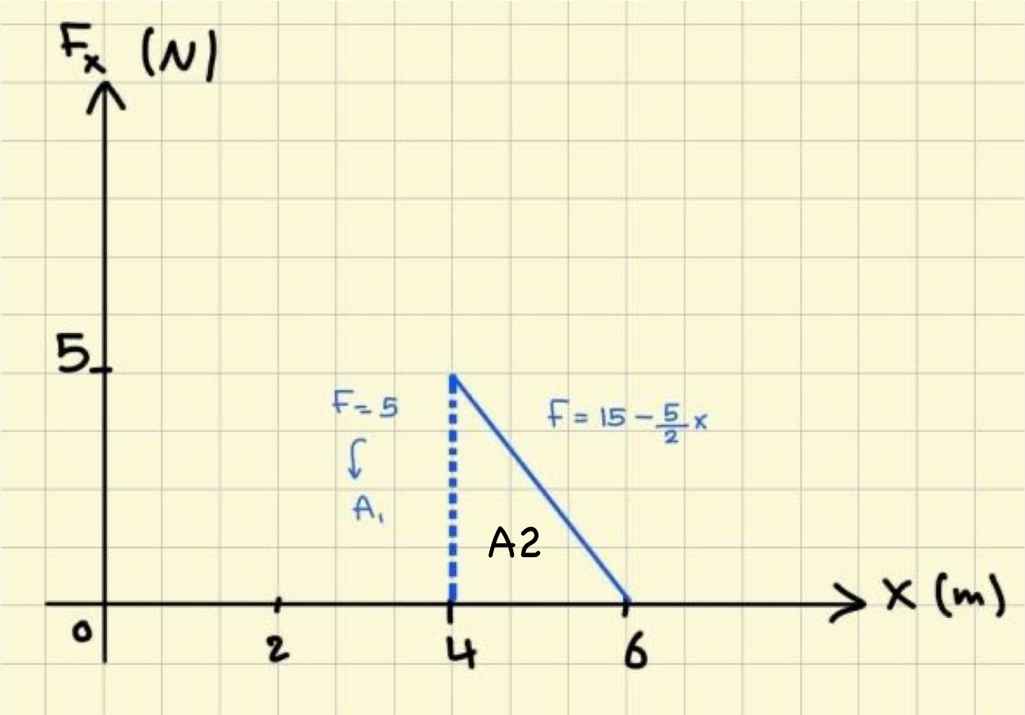
- The work done on an infinitesimal displacement Δx is :
- $\Delta w = F_x \Delta x = \Delta A$ (area)
- The total work :
- $W = \Sigma \Delta w = \sum_{x_i}^{x_f} F_x \, \Delta x$
- $W = \lim_{\Delta x \to 0} \Delta w = \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \, dx = \int_{x_i}^{x_f} F_x \, dx$



- **$W = \int_{x_i}^{x_f} F_x \, dx$ = area under the curve**



- **Example** : calculate the work done by the force F_x as the object moves from $x_1 = 0$ ***to*** $x_f = 6$ m



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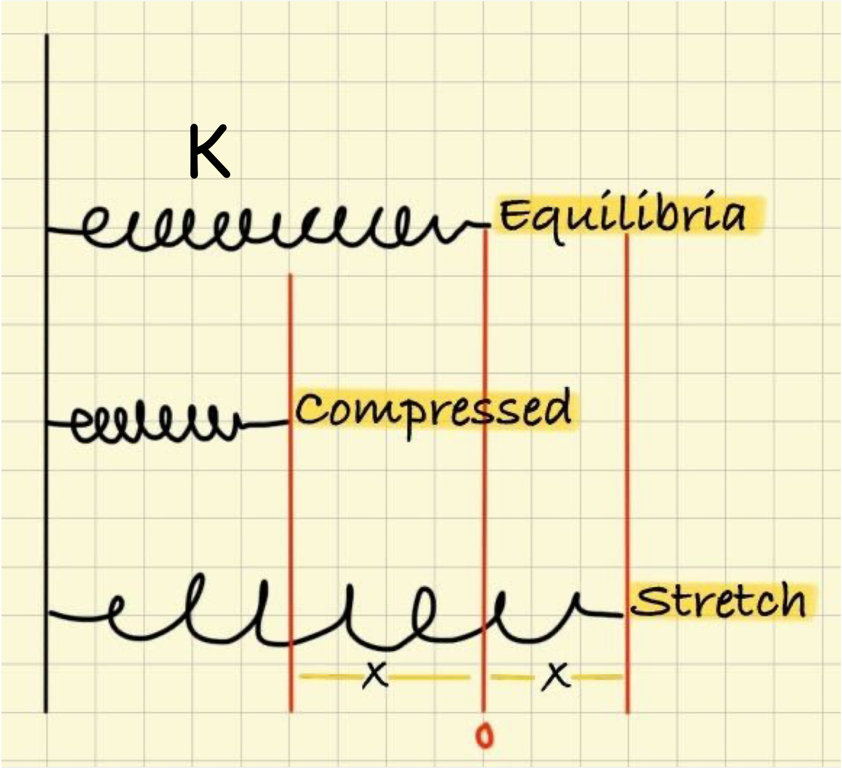
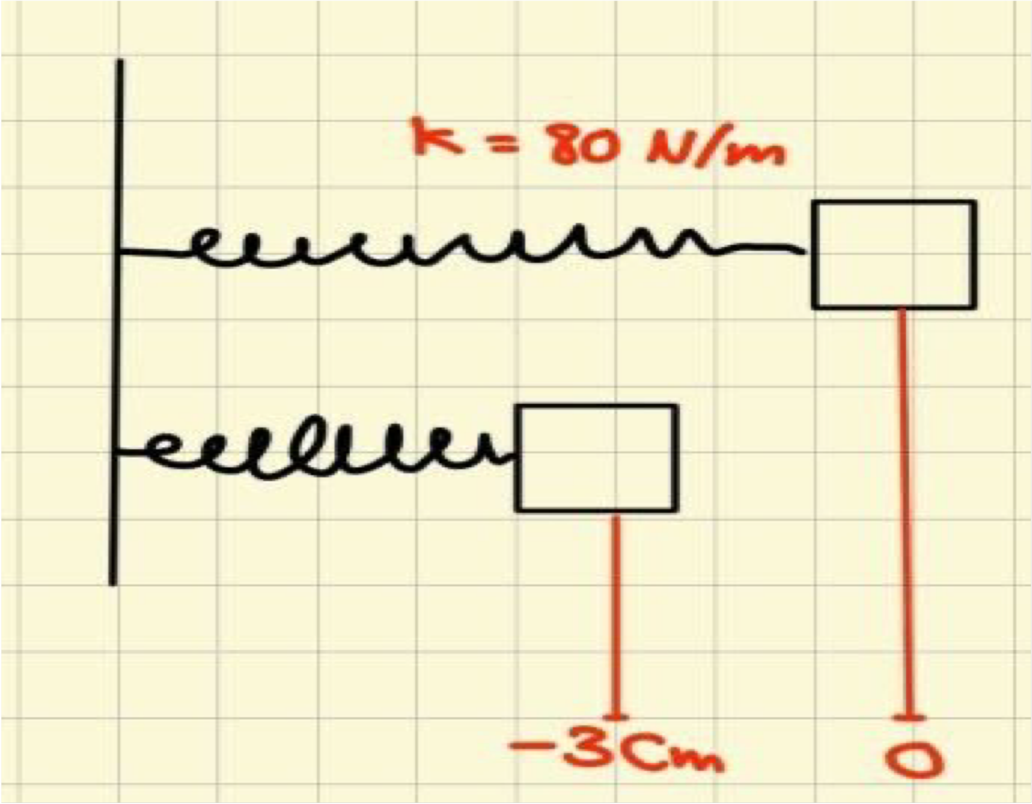
- **Solution** :
- $W = W_1 + W_2 = A_1 + A_2 = (5*4) + (\frac{1}{2} * 2 * 5) = 25J$
- Or :
- $W = \int_0^6 F_x\, dx = \int_0^4 5\, dx + \int_4^6 (15 - 5/2 * x)\, dx = 20 + 5 = 25J$

Work done by a spring

- The force done by a spring stretched or compressed a small distance **x** from equilibrium is given by :
- $F_5 = - KX$ (Hooke's law)
- Where **K** is the spring constant .
- The work done by the spring force is given by :
- $W_s = \int_{x_i}^{x_f} F_x\, dx = \int F_s\, dx = \int_{x_i}^{x_f} -KX\, dx$
- $W_s = ½\, k\, (x_i^2 – x_f^2)$



- **Example** : calculate the work done by the spring force as the block moves from X_i= -3 cm ***to*** X_f = 0



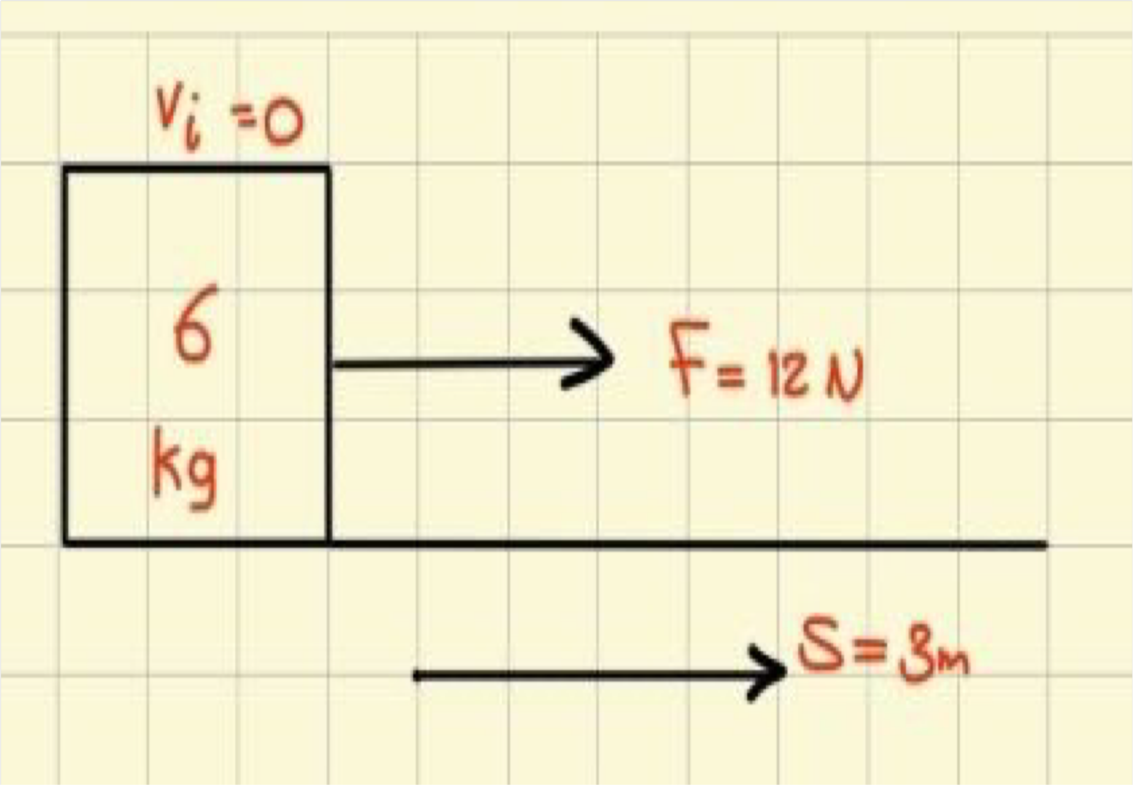
- **Solution :**

- $W_s = ½ k * (x_i^2 – x_f^2) = ½ * 80 * [(-0.03)^2 – 0] = 3.6 * 10^{-2}J$

Work and kinetic energy

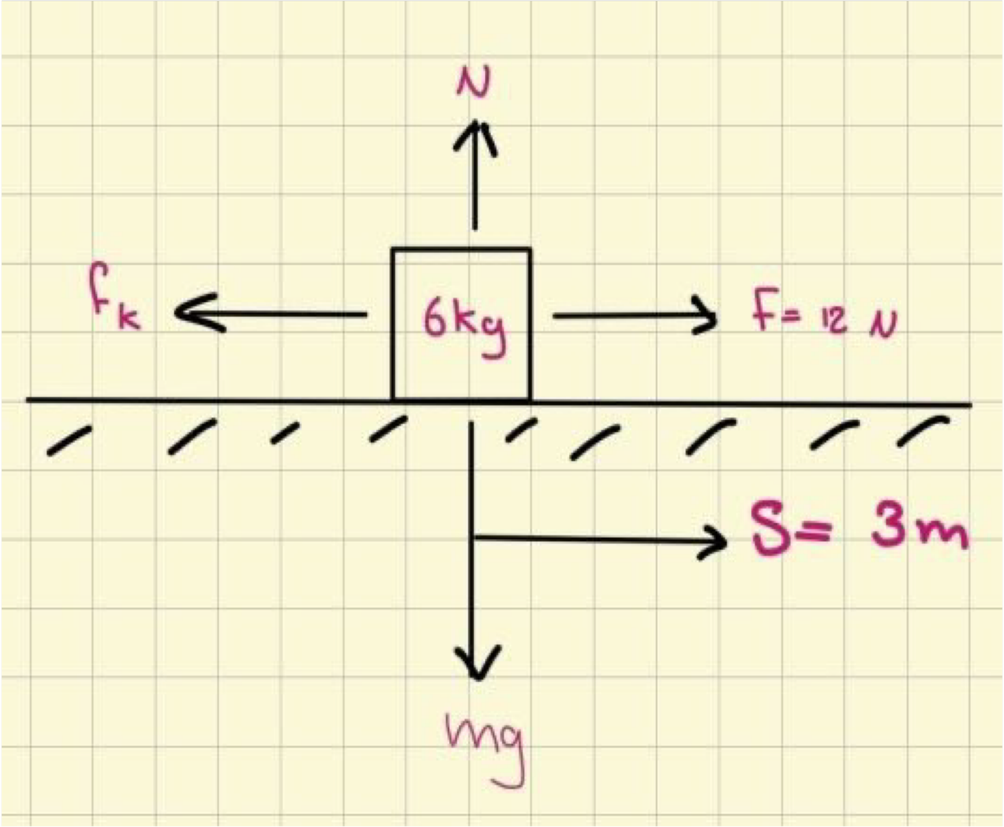
- *1 - D motion*
- $W = \int_{x_i}^{x_f} F_x\, dx$, but $F_x = ma_x = m * dv_x/dt = m * dv_x/dx * dx/dt$ **(chain rule)**
- $F_x = m * dv_x/dx * v_x$
- *Or* :
- $F_x = mv * dv/dx$
- $W = \int (mv * dv/dx)\, dx = \int_{v_i}^{v_f} mv\, dv$
- $W = ½\, m\, (v_f^2 – v_i^2)$
- We define the kinetic energy **K** as :
- $K = ½\, mv^2$
- $W = k_f – k_i = \Delta k$ (work – energy theorem)
- *3 – D motion*
- $W = \int \vec{F} * d\vec{s} = \int F_x\, dx + \int F_y\, dy + \int F_z\, dz = \Delta k = ½\, m(v_f^2 – v_i^2)$
- *Where :* $\vec{V} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

- **Example** : find the speed of the block after it moves a distance **3m** on a smooth surface .



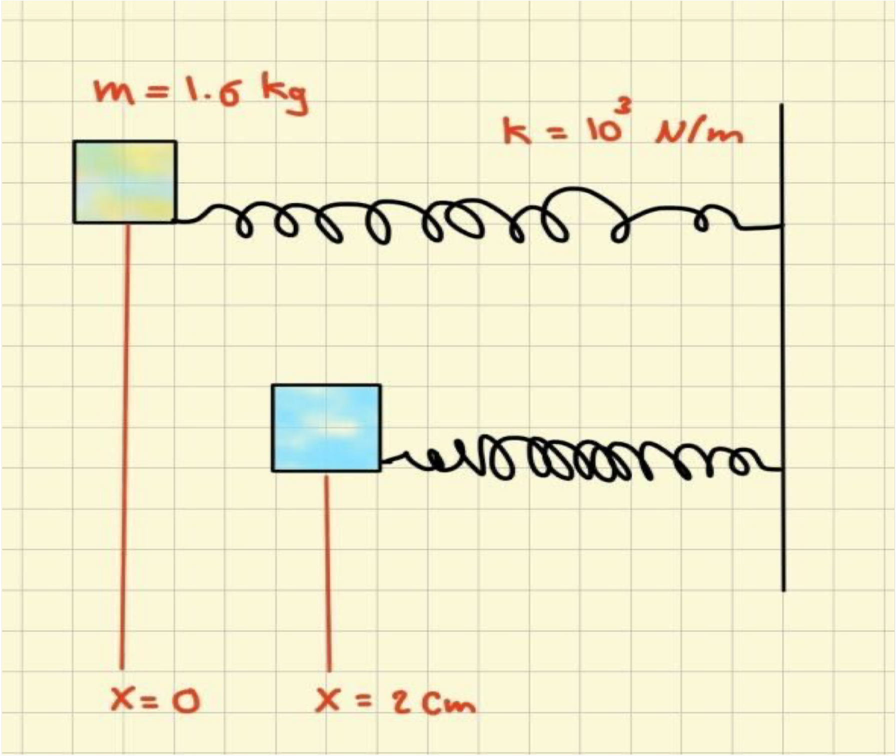
- **Solution :**
- $W = FS = 12 * 3 = 36J$
- $W = \Delta k = ½ m (v_f^2 – v_i^2)$
- $36 = ½ * 6 * [v_f^2 – 0]$
- $V_f = 3.46 m/s$

- **Example** : in the previous example , what is the speed of the block after it moves a distance ***3m*** on a rough surface ***(M_k = 0.15)*** ?



- **Solution :**
- $W_F = FS = 12 * 3 = 36J$
- $W_f = -f_k S = - M_K NS = - MK * mg * s$
- $= (0.15 * 6 * 9.8 * 3) = - 26.5J$
- $W_{net} = W_f + W_{fk} = 36 – 26.5 = 9.5J$
- $W_{net} = \Delta k = ½\ m\ (v_f^2 – v_i^2)$
- $9.5 = ½ * 6 * [v_f^2 – 0]$
- $V_f = 1.78\ m/s$

- ***Example*** : after the spring is compressed a distance of ***2cm*** to the right , the block is released from rest calculate the speed of the block as it passes through the equilibrium position **x = 0** if :
- 1) the surface is frictionless
- 2) a constant frictional force of 4N retards its motion



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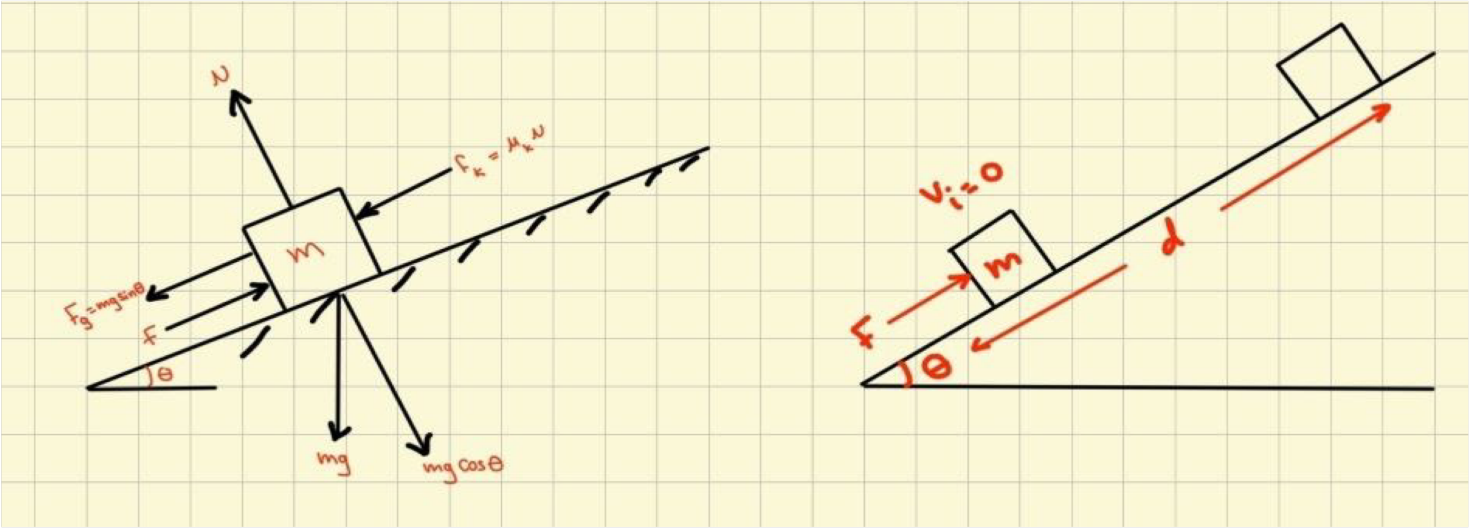
- ***To be continued…..***

- ***Solution*** :
- 1)
- $W_s = ½ k (x_i^2 – x_f^2) = ½ * 10^3 * [(0.02)^2 – 0] = 0.2J$
- $W_s = Δk = ½ m (v_f^2 – v_i^2)$
- $0.2 = ½ * 1.6 * [v_f^2 – 0]$
- $V_f = 0.5$ m/s
- 2)
- $W_s = 0.2J$
- $W_f = - f_k S = - 4 * 0.02 = - 0.08J$
- $W_{net} = W_S + W_f = 0.2 – 0.08 = 0.12J$
- $W_{net} = Δk = ½ m (v_f^2 – v_i^2)$
- $0.12 = ½ * 1.6 * [v_f^2 – 0]$
- $V_f = 0.39$ m/s

- ***Example*** : as an object moves a distance d upward an inclined rough plane :
- 1) calculate the work done by the applied force F
- 2) calculate the work done by the force of gravity F_g
- 3) calculate the work done by the frictional force F_k
- 4) find the net work
- 5) if $v_i = 0$, what is the final speed of the object ?



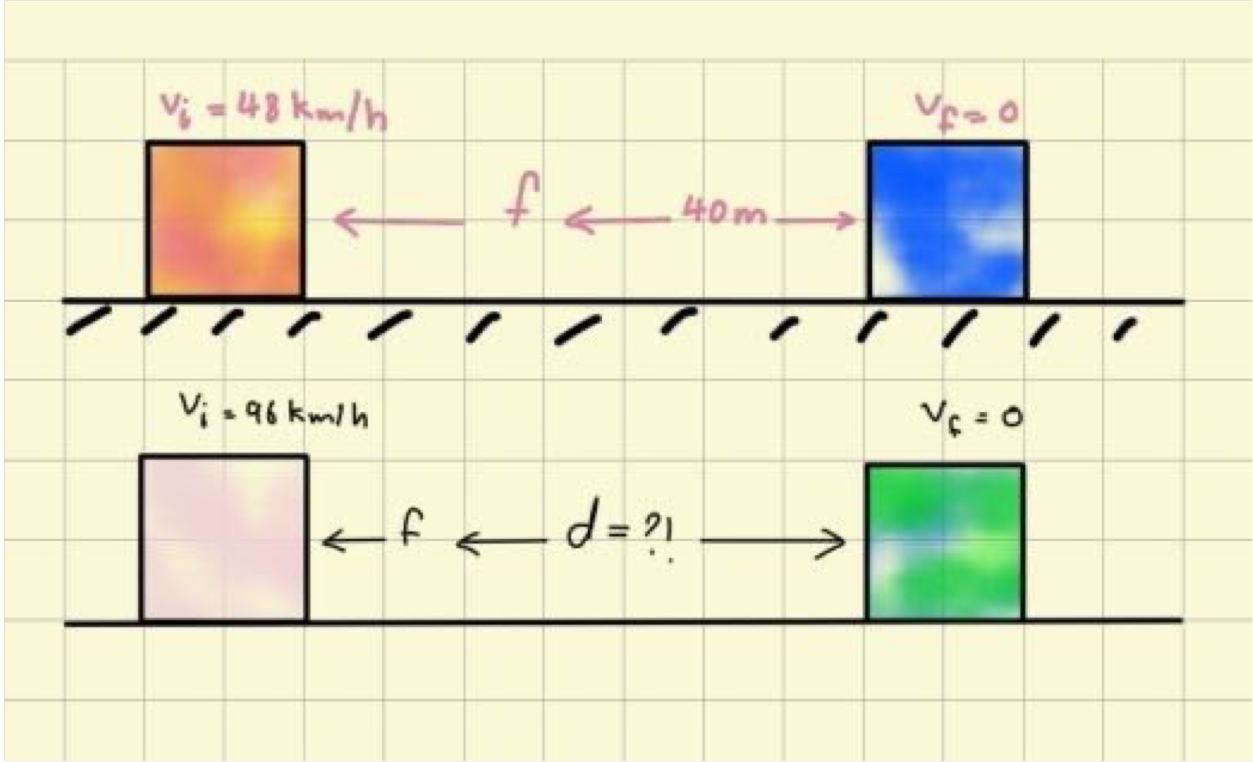
- ***To be continued …..***

- **Solution :**
- 1)
- W_f = Fd * cos(0) = Fd
- 2)
- W_g = mg*sinθ*d*cos180 = - mg*d*sinθ
- 3)
- W_f = f_kd*cos180 = -f_kd = -M_k*mg*cosθ*d
- 4)
- W_{net} = Fd – mg*d*sinθ – M_k*mg*d*cosθ
- 5)
- W_{net} = Δk = ½ m (v_f^2 – v_i^2) = ½ mv_f^2
- $V_f = \sqrt{2/m}$ * W_{net} = $\sqrt{2d/m}$ * [F – mg*sinθ – M_k*mg*cosθ]
- ***to be continued .....***

If
F = 15N
D = 1m
θ = 25°
M = 1.5 kg
M_k = 0.3

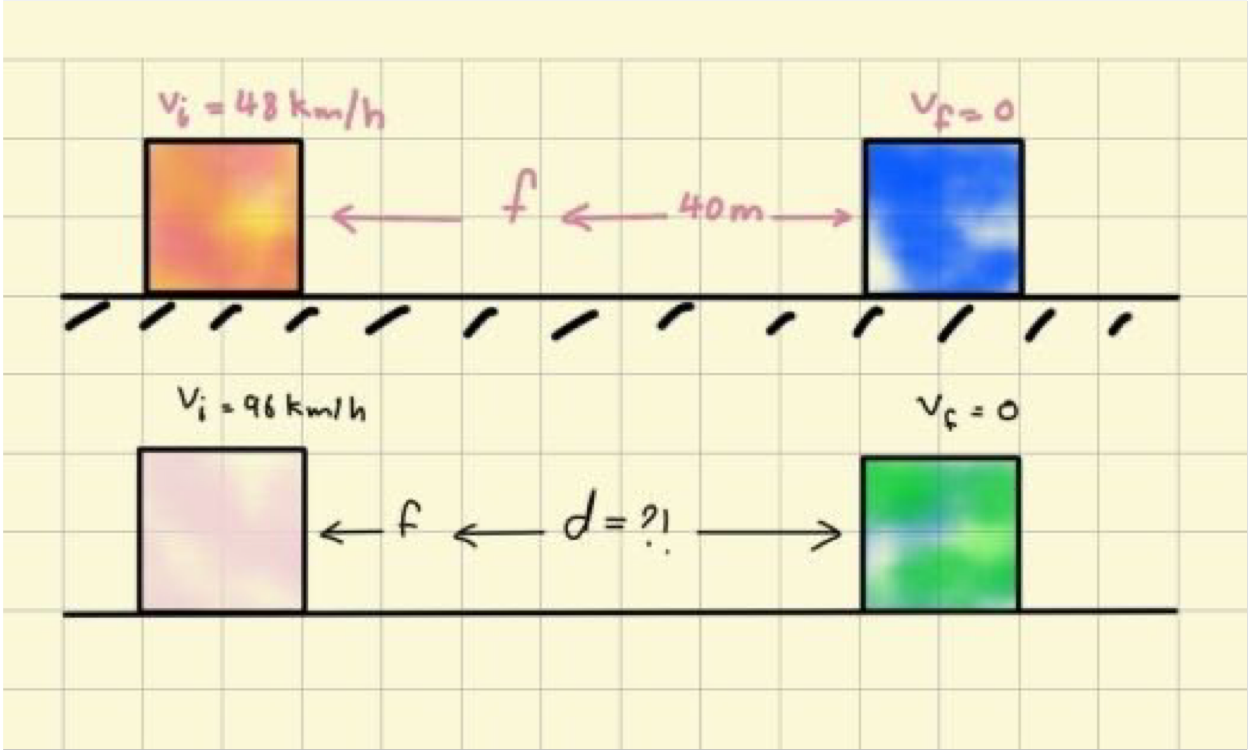
Then
W_F = 15J
W_g = 6.2J
W_f = -4J
W_{net} = 4.8J
V_f = 2.5 m/s

- **Example** : an object given an initial velocity of **48 km/hr** and passes a distance of **40m** before coming to rest due to the friction , if the initial velocity changes to **96 km/hr** , what is the distance travelled to come to rest ?



- ***To be continued .....***

- **Solution** :
- **W_f = - fd = ½ m ($v_f^2 - v_i^2$) = - ½ m * v_i^2**
- D = m/2f * v_i^2
- $d_1/d_2 = v_{i1}^2/v_2^2$
- $d_2 = v_{i2}^2/v_{i1}^2 * d_1$
- $d_2 = (96/48)^2 * 40 = 160$



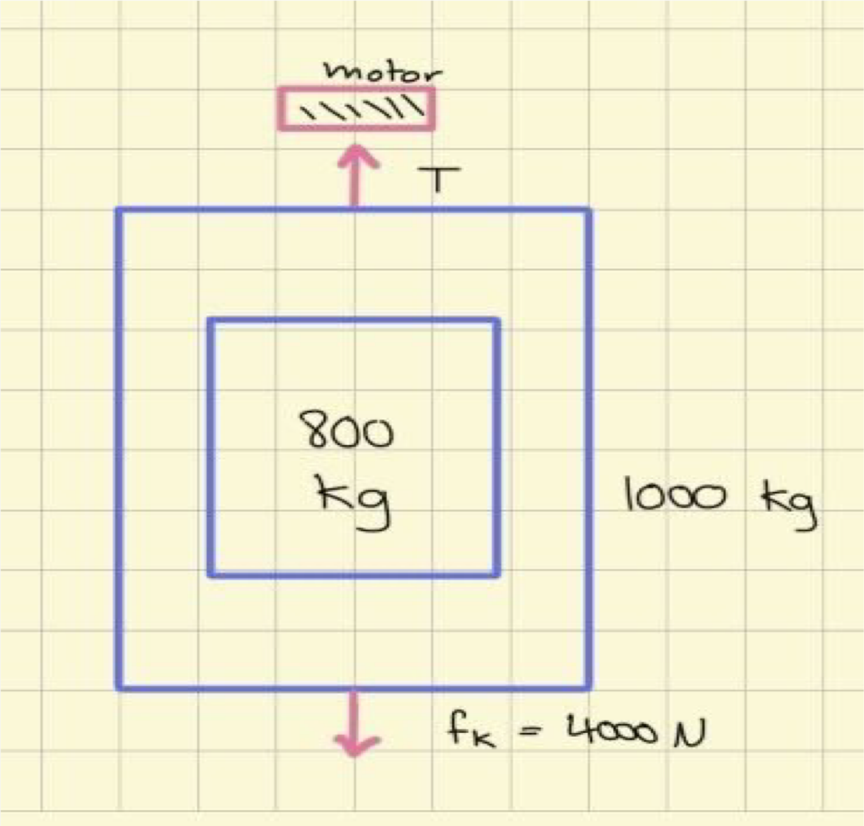
power

- ***The average power*** is defined as the ratio of the work done to the time interval.
- $\bar{P} = \Delta w / \Delta t$
- ***The instantaneous power :***
- $P = \lim_{\Delta t \to 0} \Delta w / \Delta t = dw/dt$
- $P = dw/dt = d/dt\,(\vec{F} * \vec{S}) = \vec{F} * d\vec{s}/dt = \vec{F} * \vec{V}$ ⟶ (for constant $\vec{F}$)
- The unit of power is watt (W)
- ***where 1W = J/s***

- **Example** : a **2kg** object moves in a straight line with an initial speed of **4 m/s** , it accelerates uniformly to a final speed of **7 m/s** in **15s** , calculate the average power delivered to the object ?
- **Solution** :
- $\overline{P} = \Delta w/\Delta t = \Delta k/\Delta t = ½\ m\ (v_f^2 - v_i^2)/\Delta t$
- $\overline{P} = ½ * 2 * [\ (7)^2 - (4)^2]/15 = 2.2W$
- ____
- **Example** : an object moves a long the ***x – axis*** with an instantaneous speed of **5 m/s** under the influence of a force $\vec{F} = (3\hat{i} + 4\hat{j})N$, what is the instantaneous power delivered by $\vec{F}$?
- **Solution** :
- $\vec{P} = \vec{F} * \vec{V} = (3\hat{i} + 4\hat{j}) * 5\hat{i} = 15W$

• Example : (motion of an elevator)

- 1) what must be the minimum power delivered by the motor to lift the elevator at a constant speed of **3 m/s** ?
- 2) what power must the motor deliver at any instant if it is designed to provide an upward acceleration of **1 m/s^2** ?

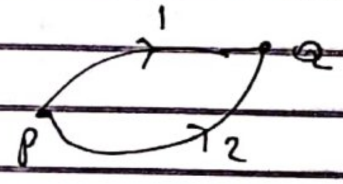


- ***To be continued ……***

- **Solution :**
- 1)
- ***V= 3 m/s = constant …… a = 0***
- $\Sigma F = ma = 0$
- $T - mg - f = 0$
- $T = mg + f = (1800 * 9.8) + 4000 = 2.16 * 10^4 N$
- $P = \vec{t} * \vec{v} = 2.16 * 10^4 * 3 = 6.49 * 10^4 W$
- 2)
- ***$\Sigma F = ma$, a = constant***
- $T - mg - f = 0$
- $T = mg + f + ma$
- $T = (1800 * 9.8) + 4000 + (1800 * 1) = 2.34 * 10^4 N$
- $P = \vec{T} * \vec{V} = 3.34 * 10^4 V$

□ Conservative Force

a Force is conservative if the work done by that force on a particle between two points is independent of the path the particle takes between the points



$$(W_{PQ})_1 = (W_{PQ})_2$$

$$(W_{PQ})_1 = -(W_{QP})_2$$

$$(W_{PQ})_1 + (W_{QP})_2 = 0$$

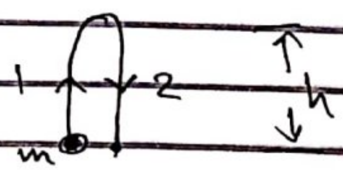
* this means that the total work done by a conservative force on a particle is zero when the particle moves around a closed path and returns to its initial position

$$W_{tot} = \oint \vec{F} \cdot d\vec{s} = 0$$

Examples of conservative forces

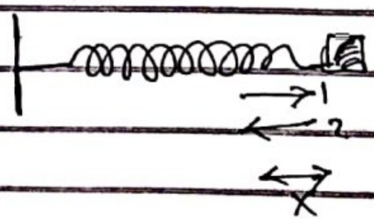
① the Force of gravity

$$W_1 + W_2 = -mgh + mgh = 0$$



② spring Force

$$W_1 + W_2 = 0$$



where

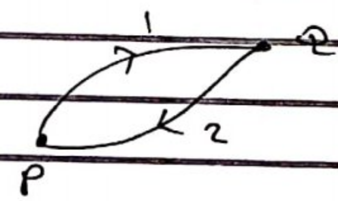
$$W_1 = \frac{1}{2}k(x_i^2 - x_f^2)$$

$$W_2 = \frac{1}{2}k(x_f^2 - x_i^2)$$

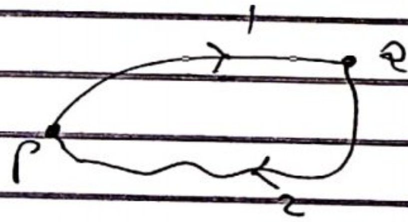
Non Conservative Force

A force is nonconservative if the work done by the force on a particle moving between two points depends on the path taken

$$(W_{PQ})_1 \neq (W_{PQ})_2$$



An Example of nonconservative force is the frictional force



$$W_1 = -f d_1$$
$$W_2 = -f d_2 \Rightarrow W_1 + W_2 \neq 0$$

here f is contant but d is not constant

Potential Energy

We define the potential energy such that the work done by the conservative force equals the decrease in potential energy

$$W_c = \int_{x_i}^{x_f} F_x \, dx = -\Delta U$$

or $$U_f - U_i = -\int_{x_i}^{x_f} F_x \, dx$$

Conservation of Mechanical Energy

From the work-energy theorem $W_c = \Delta K$
and from the definition of potential energy $W_c = -\Delta U$

That is $W_c = \Delta K = -\Delta U$

$$\boxed{\Delta K + \Delta U = 0}$$

or

$$K_f - K_i + U_f - U_i = 0$$

$$U_i + K_i = U_f + K_f$$

$$\boxed{E_i = E_f}$$ conservation law of mech. energy

where E is the total mechanical energy

$$E = U + K$$

Gravitational potential energy near the earth's surface

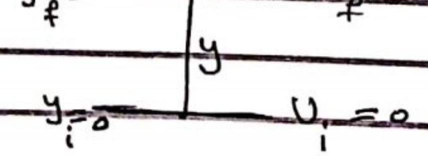
$$U_f - U_i = -\int_{y_i}^{y_f} F_y\,dy = -\int -mg\,dy$$

$$U_f - U_i = mg(y_f - y_i)$$

choose $U_i = 0$ at $y_i = 0$, then

$$U_f = mgy_f$$

or $$\boxed{U_g = mgy}$$

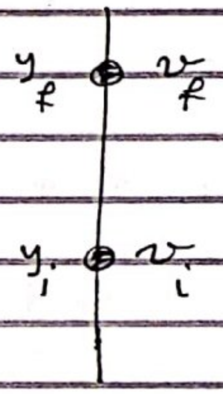


Conservation of mechanical energy of a freely falling body

$$E_i = E_f$$

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2} m v_i^2 + mgy_i = \frac{1}{2} m v_f^2 + mgy_f$$



Example

A ball of mass m is dropped from a height h above the ground. Determine

a) the speed of the ball when it is a height y above the ground

b) the speed of the ball at y if it is given an initial speed v_i at the initial altitude h

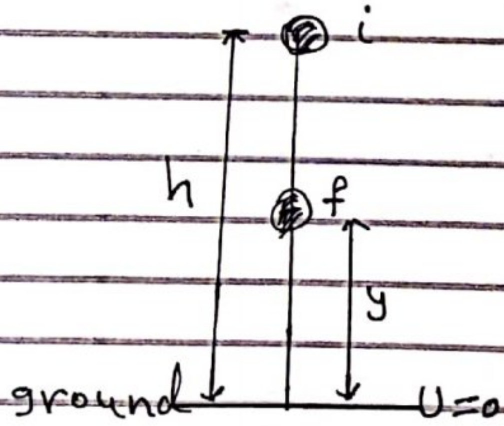
solution

a) $E_i = E_f$

$$K_i + U_i = K_f + U_f$$

$$0 + mgh = \frac{1}{2} m v_f^2 + mgy$$

$$v_f = \sqrt{2g(h-y)}$$



b) $E_i = E_f$

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2} m v_i^2 + mgh = \frac{1}{2} m v_f^2 + mgy$$

$$v_f = \sqrt{v_i^2 + 2g(h-y)}$$

Example (the Pendulum)

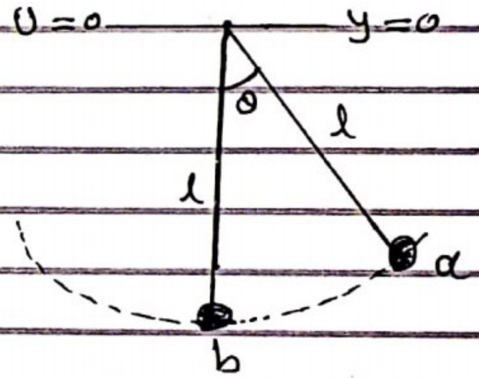
A pendulum consists of a sphere of mass m attached to a light cord of length l. The sphere is released from rest when the cord makes an angle θ with the vertical

a) find the speed of the sphere when it is at the lowest point (b)

b) what is the tension T in the cord at b

c) find the speed and tension at b when $l = 2\,m$, $m = 0.5\,kg$, $\theta = 30^\circ$

solution

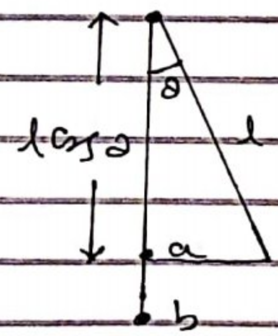


a) $E_a = E_b$

$K_a + U_a = K_b + U_b$

$$0 - mgl\cos\theta = \frac{1}{2} m v_b^2 - mgl$$

$$v_b = \sqrt{2gl(1-\cos\theta)}$$



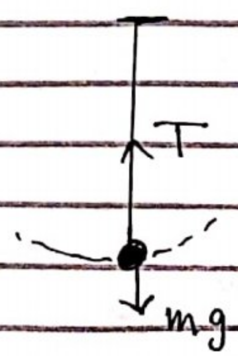
b) $\sum F_r = m a_c$

$$T_b - mg = m\frac{v_b^2}{l}$$

$$T_b = mg + m\frac{v_b^2}{l}$$

$$= mg + \frac{m}{l}\,2gl(1-\cos\theta)$$

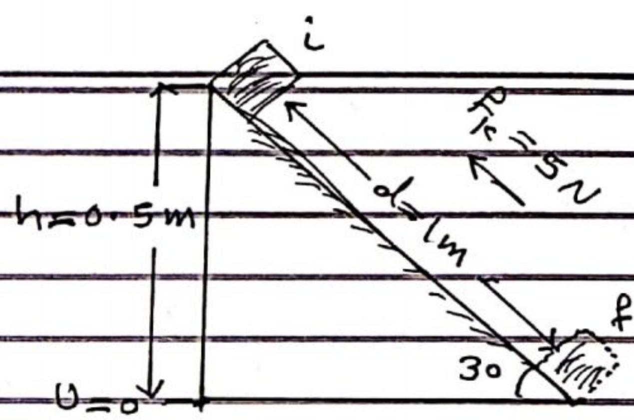
$$T_b = mg(3 - 2\cos\theta)$$



c) $v_b = \sqrt{2(9.8)(2)\left[1 - \frac{\sqrt{3}}{2}\right]} = 2.3\,m/s$

$$T_b = 0.5(9.8)\left[3 - 2\frac{\sqrt{3}}{2}\right] = 6.2\,N$$

solution



a) $E_i = U_i + K_i = mgh + 0$

$= 3(9.8)(0.5) = 14.7\ J$

$E_f = U_f + K_f = 0 + \frac{1}{2} m v_f^2$

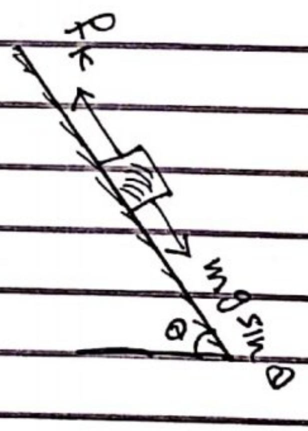
$= \frac{3}{2} v_f^2$

$W_{nc} = -f_k d = -(5)(1) = -5\ J$

$W_{nc} = E_f - E_i$

$-5 = \frac{3}{2} v_f^2 - 14.7 \Rightarrow v_f = 2.54\ m/s$

b) Newton's 2nd law : $\Sigma F = ma$



$mg \sin\theta - f_k = ma$

$(3)(9.8)(\sin 30) - 5 = 3a$

$\Rightarrow a = 3.23\ m/s^2$ = constant

$v_f^2 = v_i^2 + 2ad$

$= 0 + 2(3.23)(1)$

$v_f = 2.54\ m/s$

c) for frictionless surface $W_{nc} = 0$

$E_i = E_f$

$K_i + U_i = K_f + U_f$

$0 + mgh = \frac{1}{2} m v_f^2 + 0 \Rightarrow v_f = \sqrt{2gh}$

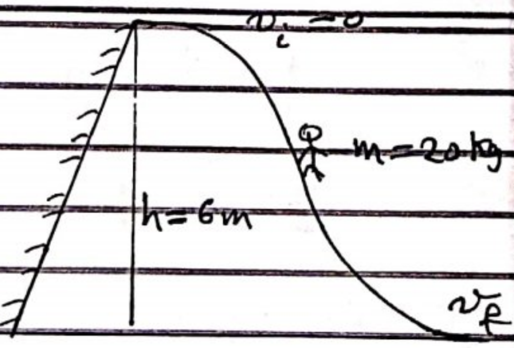
~~$(3)(9.8)(0.5) = \frac{1}{2}(3) v_f^2$~~

$= \sqrt{2(9.8)(0.5)}$

$= 3.13\ m/s$

Example

a) determine the speed of the child at the bottom



b) if there were a frictional force, what would be the work done by this force if he reaches the bottom at a speed of 8 m/s

Solution

a) $E_i = E_f$

$K_i + U_i = K_f + U_f$

$0 + mgh = \frac{1}{2} m v_f^2 + 0$

$v_f = \sqrt{2gh} = \sqrt{2(9.8)(6)} = 10.8$ m/s

b) $W_{nc} = E_f - E_i$

$E_f = \frac{1}{2} m v_f^2$

$E_i = mgh$

$W_{nc} = \frac{1}{2} m v_f^2 - mgh$

$= \frac{1}{2}(20)(8)^2 - (20)(9.8)(6)$

$W_{nc} = -536$ J