

# Hypothesis Testing

**Example:** Average intake in children for dietary fat is 70 g of fat per day. Suppose we want to study children who eat a vegetarian diet. Possible hypotheses are

1. Average fat intake is 70 g per day
2. Average fat intake is less than 70 g per day

**Def:** One-sample problem—a single distribution.

**Def:** Hypothesis—statement about parameters in a population or populations. We want to know how likely this is to be true, given the evidence (data). For example,

1. Average number of beds filled per day in the hospital
2. Average number of minutes per day the doctor spends with a patient
3. Average lead content of water for a housing project

Def: Null hypothesis— $H_0$ —the hypothesis to be tested. **This is usually a statement of no difference.** The population value of the parameter is not different from some specified value.

Def: Alternative hypothesis— $H_1$  or  $H_A$ —This is the statement we will accept if we reject the null hypothesis.

$H_0$ : Mean fat intake in vegetarian children is 70 g per day.  $H_0: \mu = \mu_0$  or  $\mu \geq \mu_0$

$H_1$ : Mean fat intake in vegetarian children is < 70 g per day.  $H_1: \mu < \mu_0$

Possible decisions:

1. Accept  $H_0$  (really, fail to reject  $H_0$ )
2. Reject  $H_0$

## Possible Scenarios:

<b>Jury Trial</b>		
	<b>Truth</b>	
<b>Verdict</b>	<b>Innocent</b>	<b>Guilty</b>
<b>Innocent</b>	<b>Correct decision</b>	<b>Error</b>
<b>Guilty</b>	<b>Error</b>	<b>Correct decision</b>

<b>Test of hypothesis</b>		
	<b>Truth</b>	
<b>Results of Test</b>	<b>H<sub>0</sub></b>	<b>H<sub>1</sub></b>
<b>Accept H<sub>0</sub></b>	<b>Correct decision</b>	<b>Type II error</b>
<b>Reject H<sub>0</sub></b>	<b>Type 1 error</b>	<b>Correct decision</b>

**Def:** Type I error is the probability of rejecting H<sub>0</sub> when H<sub>0</sub> is true.

**Def:** Type II error is the probability of accepting H<sub>0</sub> when H<sub>1</sub> is true.

**Example:** We have developed a new procedure to improve survival of premature infants. If the hospital adopts these procedures, there will have to be new rooms and new equipment purchased. This is very costly.

1. What does a Type I error imply?
2. What does a Type II error implies?

**Def:** Level of significance:  $\alpha$  = Probability of a Type I error. This is the area under the curve below (or above) the critical value. This is the probability of rejecting  $H_0$  when  $H_0$  is true.

**Def:**  $\beta$ : Probability of a Type II error.

**Def:**  $1-\beta$ —Power of a test. This is the Pr (rejecting  $H_0|H_1$  is true).

**Goal:** Make  $\alpha, \beta$  as small as possible. Usually, as  $\alpha \uparrow, \beta \downarrow$  and as  $\alpha \downarrow, \beta \uparrow$ .

**Fix  $\alpha$  (0.05 or 0.01). Find a test to minimize  $\beta$ .**

Best test for the fat experiment is one based on  $\bar{X}$ .

Def: Acceptance Region—These are the values of  $\bar{X}$  for which  $H_0$  is accepted.

Def: Rejection Region—These are the values of  $\bar{X}$  for which  $H_0$  is rejected.

Note: For this example, we are conducting a one-sided or one-tailed test. We will only reject  $H_0$  for values of  $\bar{X}$  that are low.

Def: One-tailed—this is a test in which values of parameter under  $H_1$  either  $>$  or  $<$  values under  $H_0$  but not both.

The picture behind the two sided test

