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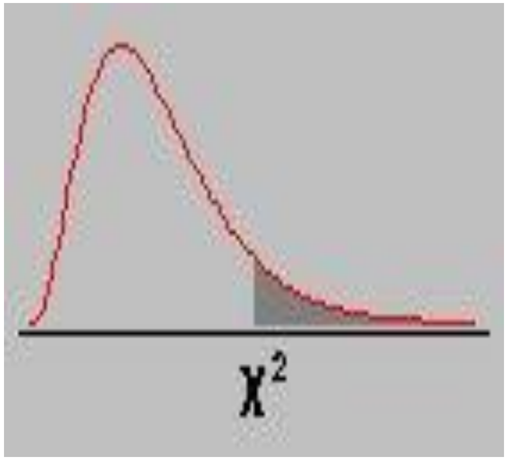


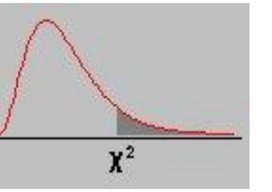
LX

Chi Square (χ^2) test

@ August 18- 2024

PART 1

- **Prof. Dr. Waqar AL-Kubaisy**



SPECIFIC LEARNING OUTCOMES

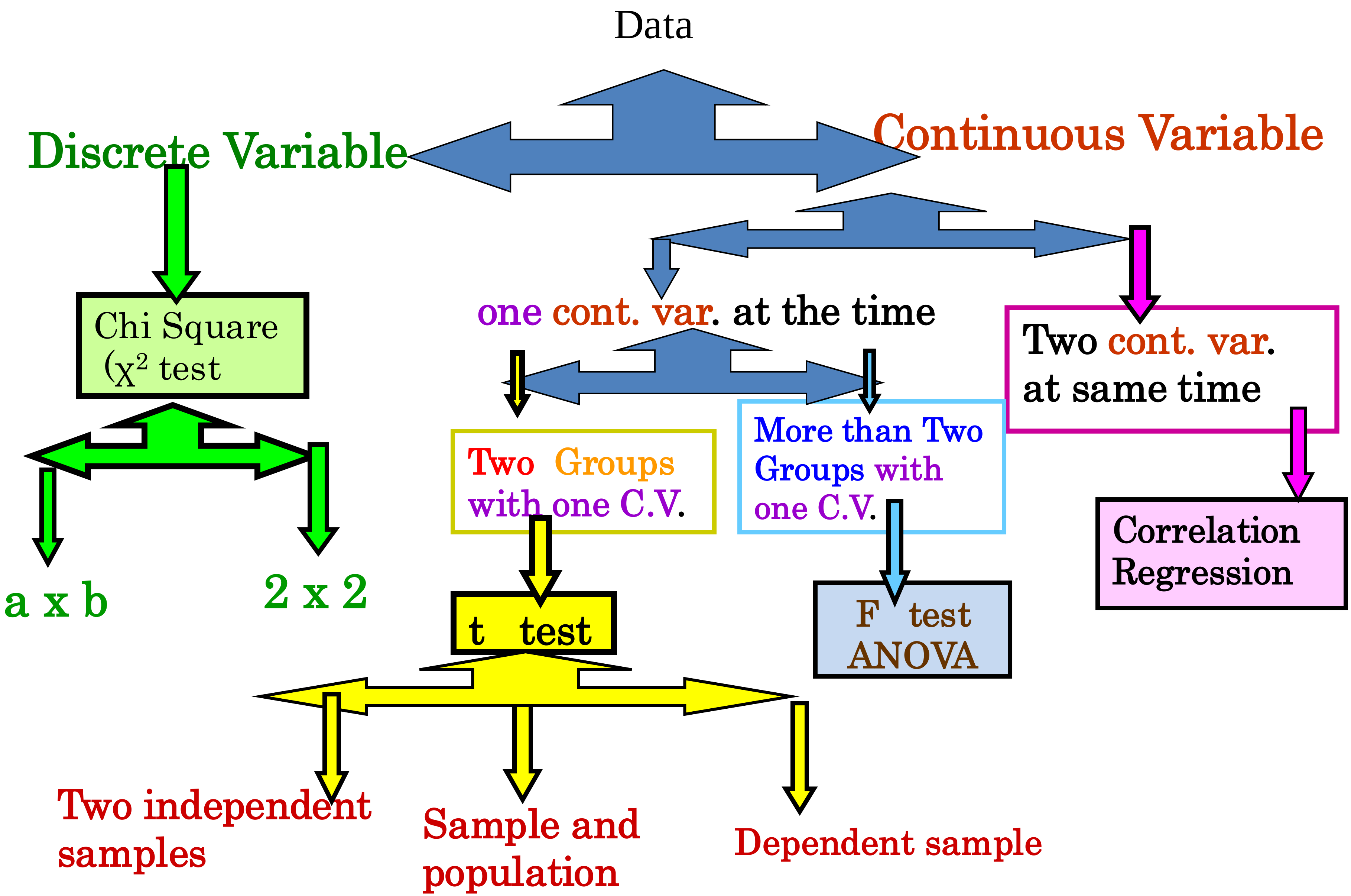
On completion of this lecture, you should be able to:

1. Explain the basis for the use of Chi square tests
2. Explain the **limitations of the Chi square** tests
3. **Carry out the** Chi square tests
4. **Interpret the findings** from the Chi square tests of significance
5. Interpret degrees **of freedom and critical** values of Chi square statistics from **Chi square table**

CONTENTS

1. **Explanation of the basis for** the use of Chi square tests on **qualitative data**
2. Explanation of the limitations of the Chi square tests
3. Calculation of Chi square
4. Chi square table
5. Interpretation of the findings from the Chi square tests of significance

An important thing is the type of the variable concerned.



An important thing is the type of the variable concerned.

when the data measurement is continuous

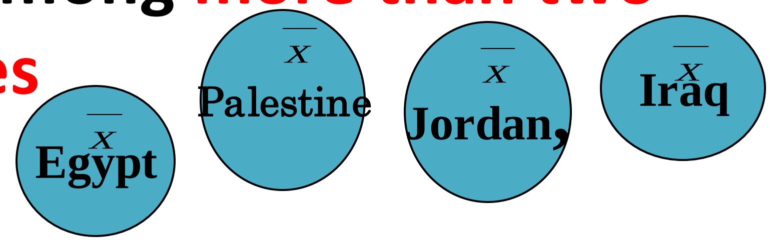
t test be applied

to test significance difference between two means

Body weight,

ANOVA (F test) be applied

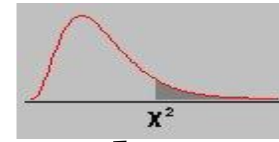
to test significance difference among more than two means Body weight adult males



Numbers of students who were succeeded

succeeded

An important thing is the type of the variable concerned.



The data we have here is only enumerative data or counting data .

Counting No. of individuals falling in one category, class, group or another

Sex

30 +

Age groups

.<20

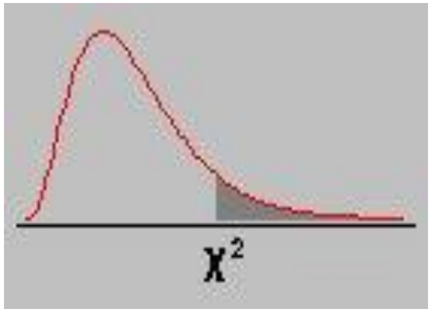
20 – 24

25 – 29

occupation

The data consist of counting No. in each sample or group

An important thing is the type of the variable concerned.



170

Mutah

180

Baghdad

100 ♀

75 ♂

Numbers of students who were succeeded

??????

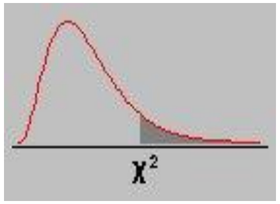
????????

cause could be

??

??

	succeeded
Baghdad	180
Mutah	170



????? cause could be

	succeeded
Baghdad	180
UiTM	220
Syria	200
Mutah	170

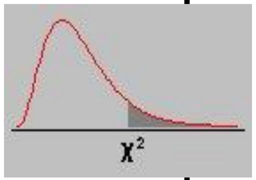
?????

Numbers of students who were succeeded

cause could be

Therefore

	Total	succeeded	%	Not succeeded
Baghdad	240	180	75%	60
Mutah	200	170	85%	30
	440	350		90



Proportion succeeded

350/440=0.80

Proportion succeeded at Baghdad ??

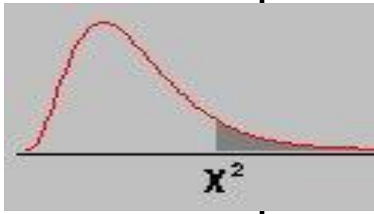
Proportion succeeded at Mutah ??

0.8X240=192

0.8X200=160

cause could be

	Total	succeeded	%	Not succeeded
Baghdad	220	180	82%	40
Mutah	200	170	85%	30
Syria	320	200	62.5%	120
UiTM	380	220	57.9%	160
	1120	770		350



770/1120 = 0.687

350/1120 = 0.3125

770/1120 X 100 = 68.7%

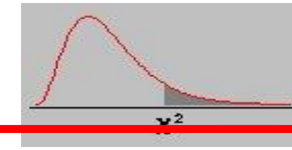
350/1120 X100 = 31.25%

Proportion succeeded at Mutah ??

Proportion succeeded at Baghdad ??

Proportion succeeded at Syria ??

Proportion succeeded at UiTM ??



When data measurement is

Qualitative data
counting data
Categorical data
Discrete.

The data consist of proportion of individuals in each group or sample,

- ❖ We have absolute numbers
- ❖ We have counting numbers — so

❑ comparing between

❑ Rates , proportions of individuals in each group

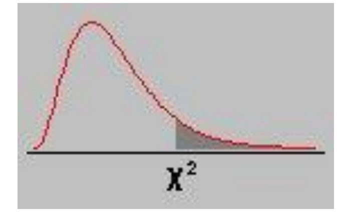
- ❖ Two groups
- ❖ More than two groups

statistical inference are made
in term of <u>difference in proportions</u>

$$Ho = P_1 = P_2 = P_0$$

$$H_A = P_1 \neq P_2 \neq P_0$$

We classify persons into categories such as



- male female
- Smoker not smoker
- Succeeded and not succeeded…. etc
- smoker, not smoker and X smoker
- then

	male	female	tota
Present			
Absent			
total			

➢count the number of observation fall in each category

The result is frequency data

enumerative data because we
enumerate the No. of person in each category

Categorical data , because we
count the No. of person in each category

When measurement

When measurement is

merely the presence or absence of certain condition,

Absolute No X

✓ Proportion

The Population Parameter is

P: :the proportion of condition in population

which is estimated by

P: the proportion of condition in the sample

So

Testing hypothesis about population proportion "P"

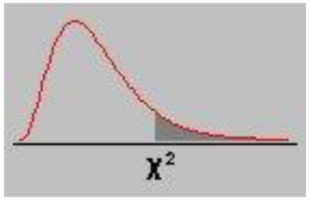
based on sample proportion P

is similar to testing hypothesis about μ .

The techniques for testing hypothesis concerning

Qualitative data
counting data
Categorical data
Discrete

is known as chi square (χ^2) test .



Chi square is

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

used in testing difference in proportions

$$Ho = P_1 = P_2 = P_0$$

$$H_A = P_1 \neq P_2 \neq P_0$$

while t test and F test are used in testing difference in means .

Also classification could be more than 2 groups, could be three, four, five ………. K groups .

P1 P2 P3 P4 P5 ………… Pk

Tumour stage I II III ……..

Class stage level I II III IV V

P1 P2 P3 P4 P5 ………… Pk

In this case

$$Ho = P_1 = P_2 = P_3 = P_4 = P_5 = P_0$$

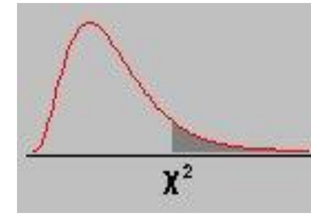
$$H_A = P_1 \neq P_2 \neq P_3 \neq P_4 \neq P_5 \neq P_0$$

	Jordanian	Iraqi	Syrian	Egyptian	total
smoker					
Not smoker					
total					

When measurement is

merely the presence or absence of certain condition,

Absolute No X



✓ Proportion

the population parameter is

P: :the proportion of condition in population

which is estimated by

P: the proportion of condition in the sample

So

Testing hypothesis about population proportion "P"

based on sample proportion P

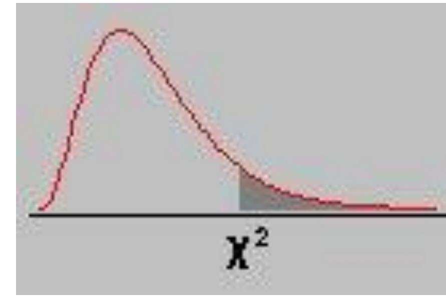
If the true population proportion of condition is Po

and sample size is N, So

Po N = Total No. of condition that expected (E) in population .

Chi square test denoted X^2

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$



This has two common applications:

first as test

whether two categorical variables are independent or not;

second as a test of

whether two proportions are equal or not

$$Ho = P_1 = P_2 = P_0$$

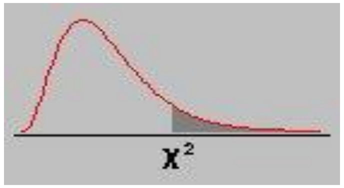
$$H_A = P_1 \neq P_2 \neq P_0$$

$$Ho = P_1 = P_2 = P_3 = P_4 = P_5 = P_0$$

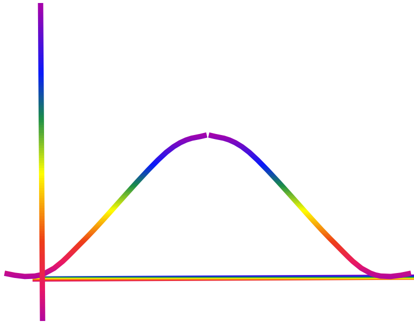
$$H_A = P_1 \neq P_2 \neq P_3 \neq P_4 \neq P_5 \neq P_0$$

contingency table

The chi square test is applied to frequency data in form of a contingency table
i.e. a table of cross- tabulations) with
the rows represent categories of one variable
and
the columns categories of a second variable.



	♂	♀	total
succeeded	70	90	160
not succeeded	10	30	40
Total	80	120	200

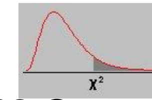


The null hypothesis
is that the two variables are unrelated

the rows represent categories of one variable and the columns categories of a second variable

Sex	succeeded	not succeeded	Total
♂	70	10	80
♀	90	30	120
Total	160	40	200

The H0; is that the two variables are unrelated
The HA ???????????????



If the variables display are Exposure and outcome. Then

usually we arrange the table with

Exposure as the row variable and

Out come as the column variable .

and display % corresponding the exposure variable

Exposure	Out come +ve	Out come -ve	total
yes			
no			
Total			

Example

smoking during pregnancy and relation to small birth weight

smoker or non smoked mother during pregnancy??

small birth weight no small birth weight ???

	Small birth weight	no small birth weight	Total
smoker mother during pregnancy			
non smoked mother during pregnancy			
Total			

	♂	♀	total
succeeded	70	90	160
not succeeded	10	30	40
Total	80	120	200

SEX	succeeded	not succeeded	Total
♂	70	10	80
♀	90	30	120
Total	160	40	200

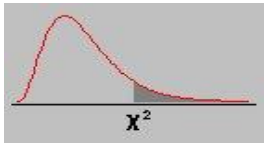
	♂	♀	total
succeeded	70	90	160
not succeeded	10	30	40
Total	80	120	200

????

merely the presence or absence of certain condition,

Absolute No X

✓ Proportion



	♂	♀	total
succeeded	70 87.5%	90 75%	160 80%
not succeeded	10 12.5%	30 25%	40
Total	80	120	200

If the true population proportion of condition is

160/200 =0.8 40/200 = 0.2

Po =0.8 and

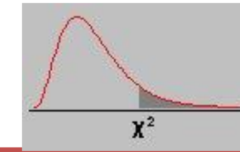
Rate (proportion) of succeeded ♂ (p_1)=70/80= 87.5%

Rate(proportion) of succeeded ♀ (p_2)= =90/120= 75%

$$Ho = P_1 = P_2 = P_0$$

$$H_A = P_1 \neq P_2 \neq P_0$$

????



	♂	♀	total
succeeded	70 (87.5%)	90 (75%)	160 80%
not succeeded	10 (12.5%)	30 (25%)	40
Total	80	120	200

If the true population proportion of condition is

160/200 =0.8 40/200 = 0.2

Po =0.8 and

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

sample size is N, (200) So

Po N =Total No. of condition that expected (E)

in Each population .

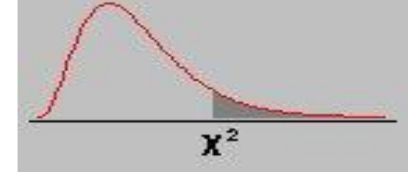
♂ 80X 0.8= 80X 0.2 =

♀ 120X 0.8= 120X 0.2=

expected (E)

♂	80X.8=	80X.2 =
♀	120X.8=	120X.2=

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$



	♂ O	♂ E	♀ O	♀ E	total
succeeded	70	64	90	96	160
not succeeded	10	16	30	24	40
Total	80		120		200

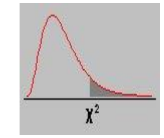
$$\sum O - E = Zero$$

$$\sum \frac{O-E}{E} = Zero$$

- the actual observed No. of subject with condition (O)
- and the expected No. of condition (E)
- Looking for the difference between the observed and expected frequencies

$$\sum O - E = Zero$$

$$\sum \frac{O-E}{E} = Zero$$



So if the actual No. of subject with condition observed No.(O) is close to the expected No. (E) then
the Ho will be not rejected (   ).

This mean that P=Po .

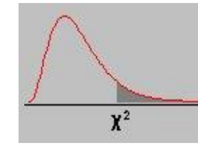
Usually summation $\sum O - E = Zero$

So $\sum \frac{O - E}{E} = Zero$

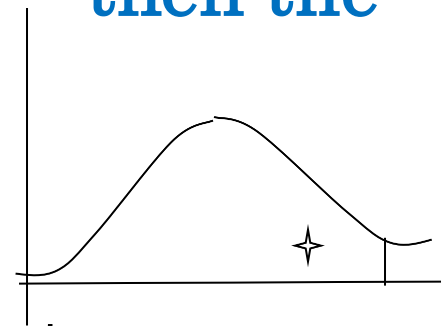
To overcome this result, we have to square O-E make it as $(O-E)^2$ then divided by E $\frac{(O-E)^2}{E}$ for each cell

Then we have to do the summation $\chi^2 = \sum \frac{(O-E)^2}{E}$

Therefore, χ^2 is always UPPER ONE SIDED TEST



❖ When O and E are close together, then the computed χ^2 is small and Ho is not Rejected .



❖ When O and E values are far apart
Then O-E is great, $(O-E)^2$ be more great
This will lead to Reject Ho .

In Enumerate (Discrete) value variable, we classified individuals into :

Those having the condition P1

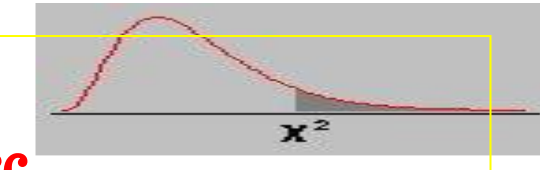
Those having no condition P2

	male	female	total
Present			
Absent			
total			

sign. Difference in proportion

$$Ho = P_1 = P_2 = P_0$$

$$H_A = P_1 \neq P_2 \neq P_0$$



Chi square (χ^2)

It is the **sum** of the **squared difference** between the **observed** frequency and **expected** frequency, divided by the **expected** frequency .

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

sign. Difference in proportion

Comparing calculated $\chi 2$ with tabulated χ^2 in relation to critical region

Critical region;

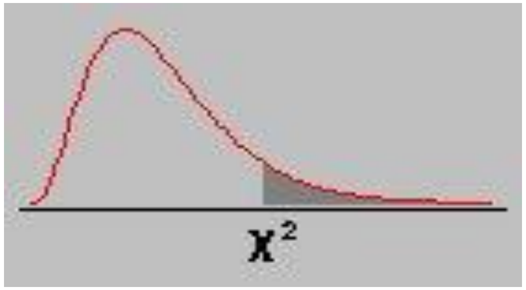
- **Level of significance 0.95, $\alpha = 0.05$**
- **d.F = (No. of rows – 1) (No. of column – 1)**

= (r – 1) (c – 1)

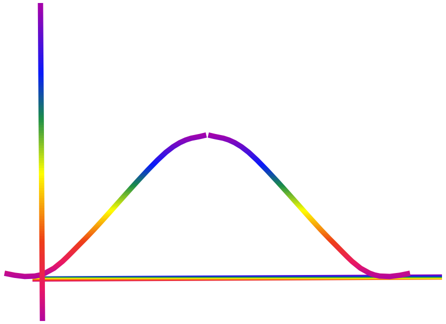
(2 – 1) (2 – 1) = 1

	male	female	total
Present			
Absent			
total			

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

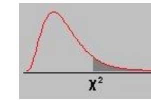


Therefore, χ2 is always UPPER ONE SIDED TEST



Comparing calculated χ2 with tabulated χ² in relation to critical region

sign. Difference in proportion



Chi square is

used in testing difference in proportions while t test and F test are used in testing difference in means .

$$Ho = P_1 = P_2 = P_0$$

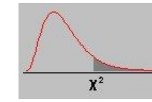
$$H_A = P_1 \neq P_2 \neq P_0$$

Chi square (χ^2)

It is the sum of the squared difference between the observed frequency and expected frequency, divided by the expected frequency .

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

Comparing calculated χ^2 with tabulated χ^2
in relation to critical region

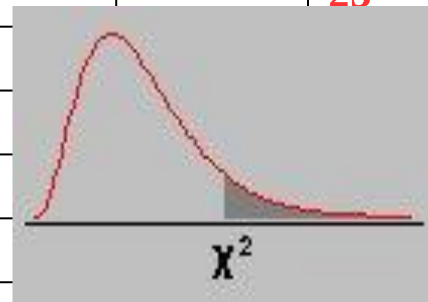


If the variables display are Exposure and outcome.
Then
we usually we arrange the table with
exposure as the row variable and
out come as the column variable .
and display % corresponding the exposure variable

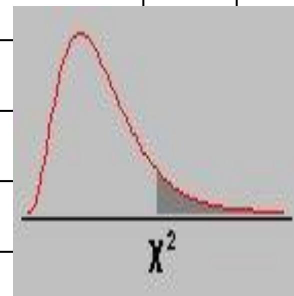
Exposure	Out come +ve	Out come -ve	total
yes			
no			
Total			

Table of Chi-square statistics

df	P =0.05	P = 0.01	P = 0.001
1	3.84	6.64	10.83
2	5.99	9.21	13.82
3	7.82	11.35	16.27
4	9.49	13.28	18.47
5	11.07	15.09	20.52
6	12.59	16.81	22.46
7	14.07	18.48	24.32
8	15.51	20.09	26.13
9	16.92	21.67	27.88
10	18.31	23.21	29.59
11	19.68	24.73	31.26
12	21.03	26.22	32.91
13	22.36	27.69	34.53
14	23.69	29.14	36.12
15	25.00	30.58	37.70
16	26.30	32.00	39.25
17	27.59	33.41	40.79
18	28.87	34.81	42.31
19	30.14	36.19	43.82
20	31.41	37.57	45.32
21	32.67	38.93	46.80
22	33.92	40.29	48.27
23	35.17	41.64	49.73
[illegible]	36.42	42.98	51.18
[illegible]	37.65	44.31	52.62
[illegible]	38.89	45.64	54.05
[illegible]	40.11	46.96	55.48
28	41.34	48.28	56.89
29	42.56	49.59	58.30
30	43.77	50.89	59.70
31	44.99	52.19	61.10
32	46.19	53.49	62.49
33	47.40	54.78	63.87
34	48.60	56.06	65.25
35	49.80	57.34	66.62
36	51.00	58.62	67.99
37	52.19	59.89	69.35
38	53.38	61.16	70.71
39	54.57	62.43	72.06
40	55.76	63.69	73.41

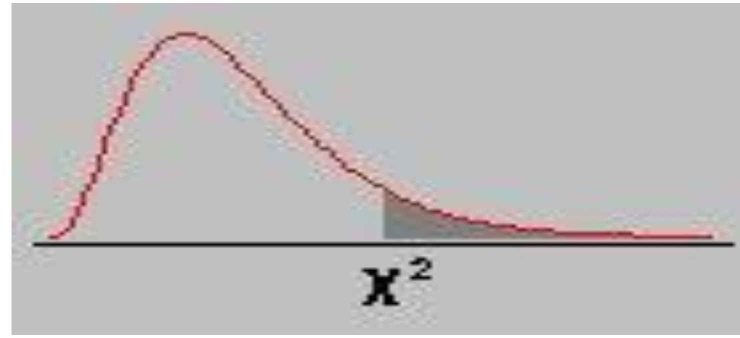


41	56.94	64.95	74.75
42	58.12	66.21	76.09
43	59.30	67.46	77.42
44	60.48	68.71	78.75
45	61.66	69.96	80.08
46	62.83	71.20	81.40
47	64.00	72.44	82.72
48	65.17	73.68	84.03
49	66.34	74.92	85.35
50	67.51	76.15	86.66
51	68.67	77.39	87.97
52	69.83	78.62	89.27
53	70.99	79.84	90.57
54	72.15	81.07	91.88
55	73.31	82.29	93.17
56	74.47	83.52	94.47
57	75.62	84.73	95.75
58	76.78	85.95	97.03
59	77.93	87.17	98.34
60	79.08	88.38	99.62



61	80.23	89.59	100.88
62	81.38	90.80	102.15
[illegible]	82.53	92.01	103.46
[illegible]	83.68	93.22	104.72
[illegible]	84.82	94.42	105.97
[illegible]	85.97	95.63	107.26
[illegible]	87.11	96.83	108.54
68	88.25	98.03	109.79
69	89.39	99.23	111.06
70	90.53	100.42	112.31
71	91.67	101.62	113.56
72	92.81	102.82	114.84
73	93.95	104.01	116.08
74	95.08	105.20	117.35
75	96.22	106.39	118.60
76	97.35	107.58	119.85
77	98.49	108.77	121.11
78	99.62	109.96	122.36
79	100.75	111.15	123.60
80	101.88	112.33	124.84

81	103.01	113.51	126.09
82	104.14	114.70	127.33
83	105.27	115.88	128.57
84	106.40	117.06	129.80
85	107.52	118.24	131.04
86	108.65	119.41	132.28
87	109.77	120.59	133.51
88	110.90	121.77	134.74
89	112.02	122.94	135.96
90	113.15	124.12	137.19
91	114.27	125.29	138.45
92	115.39	126.46	139.66
93	116.51	127.63	140.90

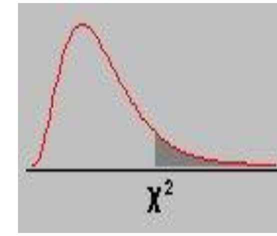


93	116.51	127.63	140.90
94	117.63	128.80	142.12
95	118.75	129.97	143.32
96	119.87	131.14	144.55
97	120.99	132.31	145.78
98	122.11	133.47	146.99
99	123.23	134.64	148.21
100	124.34	135.81	149.48

Application of χ2.

1. 2×2 table .
2. $a \times b$ table .

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$



2 × 2 table

The application of χ^2 is to test the significance association between outcome and certain factor that we are interested in .

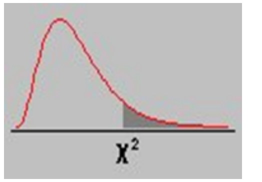
Here we have

two groups with two outcome for each group

two groups each group with two outcome for each group .

In this case we use what we call it 2 × 2 table .

In this case we are going to compare between two proportion of two groups of population .



2 × 2 table

Example

A sample of 671 diseased person were subjected to treatment, 354 individuals of them, were given drug A. Of those given drug A only 240 patients were survived. On the other hand only 212 patients *who's given drug B were survived can we conclude that the effectiveness of treatment differ* between two drugs (A&B) ????.

Let α 0.05

Out come	**Drug A**	**Drug B**	Total
Survived	240	212	?????
Died	??????	????	?????
Total	354	?????	671

(**also known as a cross tabulation or crosstab**)

Out come	Drug A	Drug B	Total
Survived	240	212	452
Died	114	105	219
Total	354	317	671

$$Ho = P_1 = P_2 = P_0$$
$$H_A = P_1 \neq P_2 \neq P_0$$

We would like to see if there is a **significance difference in** the **survival rate between the two drugs** . Let α 0.05

$$\text{Total Survival rate} = \frac{452}{671} \times 100 = 67.4\%$$

Survival rate for A $= \frac{240}{354} \times 100 = 67.8\%$

Survival rate for B $= \frac{212}{317} \times 100 = 66.9\%$

$$Ho = P_1 = P_2 = P_o$$
$$H_A = P_1 \neq P_2 \neq P_o$$

There is an **observed difference** in the survival rate between drug A (67.8%) and B (66.9%) .

Is this difference in survival rate due to :

- Drug Effectiveness .
- Chance Factor .

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

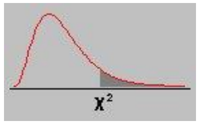
Out come	Drug A	Drug B	Total
Survived	240 (67.5%)	212(66.9%)	452(67.4%)
Died	114	105	219
Total	354	317	671

Data

Data consist of sample of patients divided into two groups, group A and group B .

Survival rate in group treated by drug **A was 67.8 %, and**

Survival rate in group treated by drug B was 66.8% .



Assumption

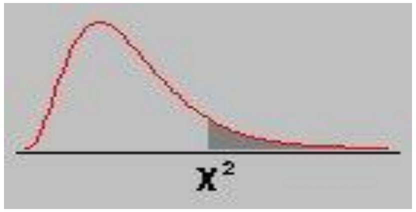
Two independent group of patients given two different type of treatment chosen randomly from normal distribution population .

Formulation of Hypothesis

Ho ???????????????

HA ???????????????????

Formulation of Hypothesis



Ho

There is no significance difference in the proportion (rate) of survival between two groups .

survival rate group treated by drug A was 67.8% &

survival rate group treated by drug B was 66.9%

There is no significance association between survival rate and type of treatment .

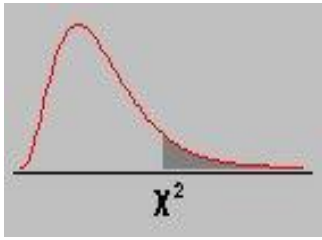
P1 = P2 = P0 .

HA

There is a **significance difference** in the survival **rate** between two type of treatment .

P1¬ ≠ P2 ≠ P0 .

Survival rate is **higher among** group of patients treated by drug A .



Critical region

Level of significance 0.95, α = 0.05

d.F =

(No. of rows – 1) (No. of column – 1)

= (r – 1) (c – 1)

(2 – 1) (2 – 1) = 1

Outcome	Drug A	Drug B	Total
Survived	240	212	**452**
Died	114	105	**219**
Total	**354**	**317**	**671**

tabulated χ^2 of d.F =1 with α 0.05 = 3.841

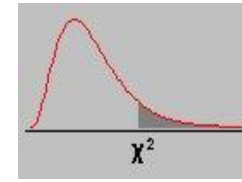
Proper test

χ^2 , 2 × 2 table

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$E = \frac{total\ column \times total\ rows}{Grand\quad total} \quad for\ each\ cell$$

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$



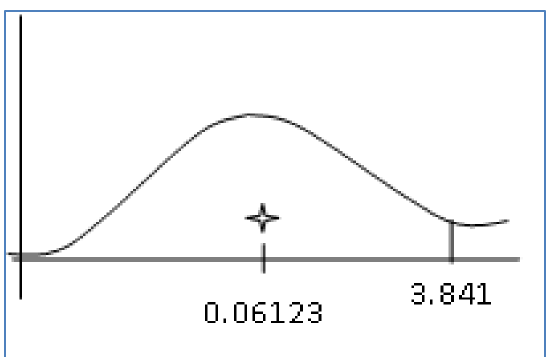
$$E_{240} = \frac{354 \times 452}{671} = 238.5$$

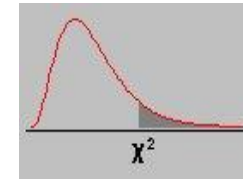
$$E_{114} = \frac{354 \times 219}{671} = 115.5$$

$$E_{212} = \frac{452 \times 317}{671} = 213.5$$

$$E_{105} = \frac{317 \times 219}{671} = 103.5$$

Outcome	Drug A		Drug B		Total
	O	E	O	E	
Survived	240	238.5	212	213.5	452
Died	114	115.5	105	103.5	219
Total	354		317		671





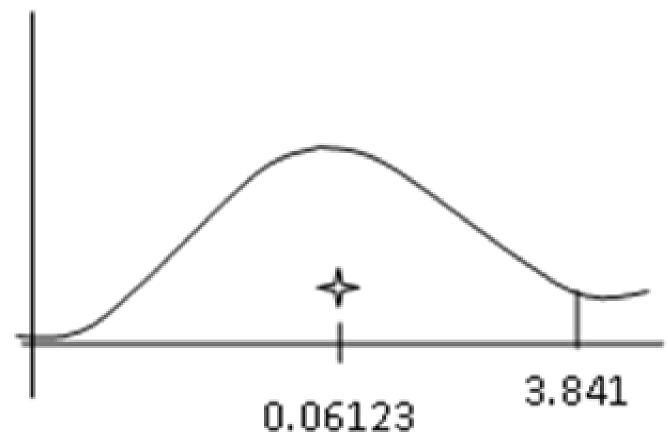
$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$= \frac{(240-238.5)^2}{238.5} + \frac{(114-115.5)^2}{115.5} + \frac{(212-213.5)^2}{213.5} + \frac{(105-103.5)^2}{103.5}$$

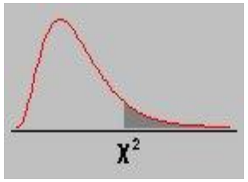
$$= \frac{(1.5)^2}{238.5} + \frac{(1.5)^2}{115.5} + \frac{(-1.5)^2}{213.5} + \frac{(1.5)^2}{103.5} = \frac{2.25}{238.5} + \frac{2.25}{115.5} + \frac{2.25}{213.5} + \frac{2.25}{103.5}$$

$$= 0.009434 + 0.0195 + 0.01056 + 0.02174$$

$$= 0.061234$$



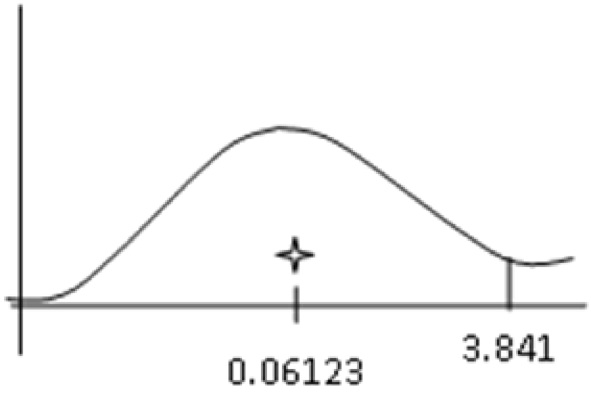
Calculated χ² fall in Accept Region → so
We **not reject** (accept) **Ho** .



There is no significance difference in proportion of survival rate between two drugs

P > 0.05

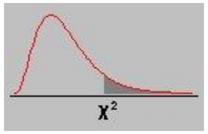
Calculated χ² less than tabulated χ²
chance factor increases,
influencing factor decrease



There is no significance effect of drug A to increase survival rate .

.**P > 0.05**

P > 0.05 .



Example

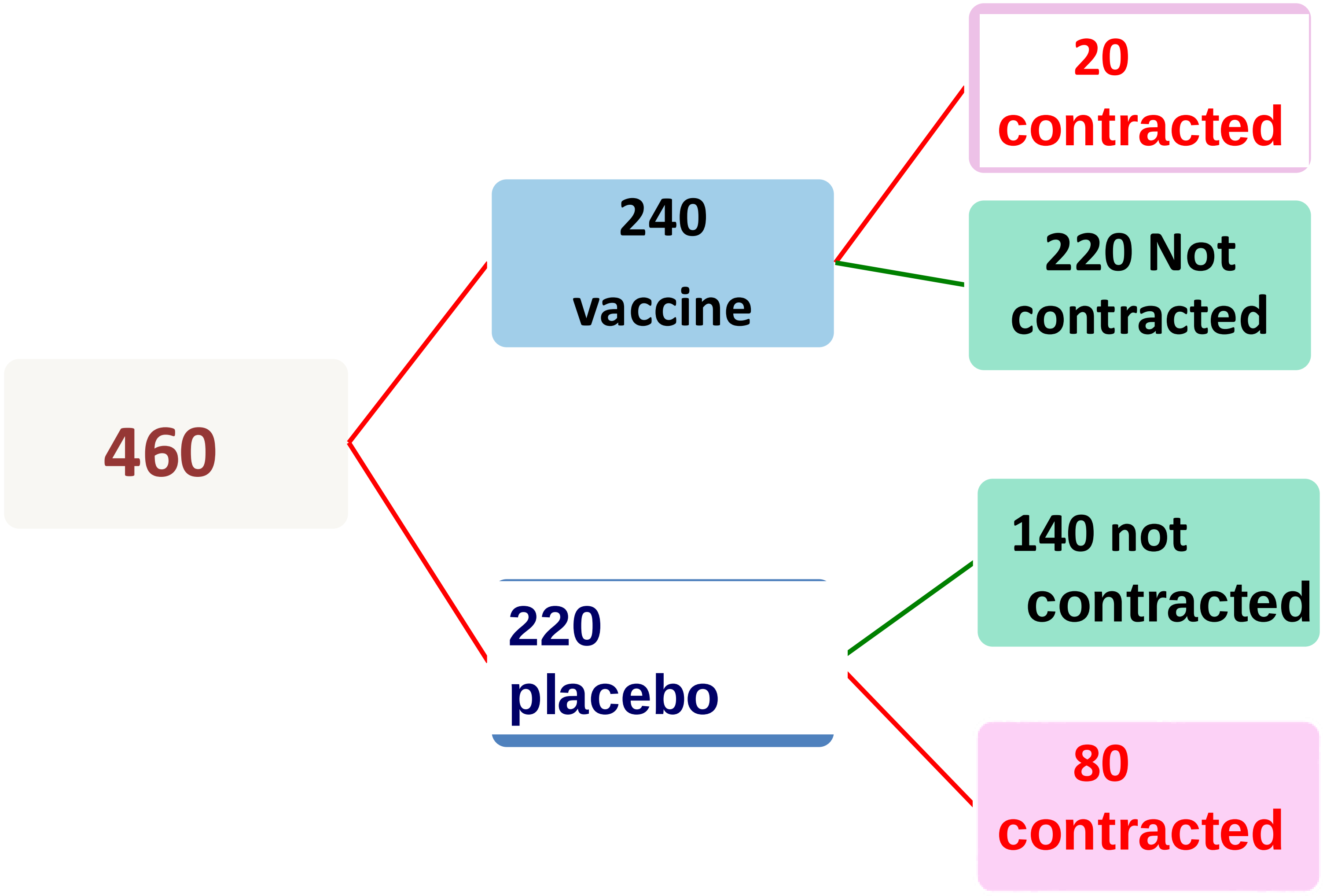
A sample of 460 adult was chosen , 240 were given influenza vaccine while the remaining given placebo Overall 100 persons contracted influenza, of whom 20 were in vaccine group .

we would like to assess the strength of evidence that vaccination affect the probability of contracting disease

is there any evidence that vaccine have an effect on contracting the disease ??

Total 460 → 100 persons contracted influenza

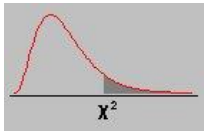
240 vaccinated → 20 contracted influenza



Total 460 → 100 persons contracted influenza

240 vaccinated → 20 contracted influenza

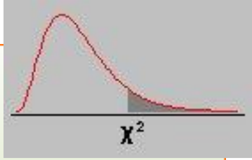
We start by display data in 2X2 table .



- **The exposure is vaccination (the row variable) and**
- **the outcome is contracting influenza (the column variable)**
- **we therefore include row % in the table**

Exposure	Out come +ve	Out come -ve	total
yes			
no			
Total			

(also known as a cross tabulation or crosstab)

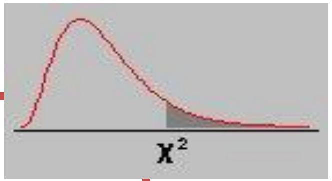


We start by display data in 2X2 table .
The exposure is vaccination (the row variable) and
the outcome is contracting influenza (the column variable)
we therefore include row % in the table

Given	Contract influenza		Not contract influenza		Total
	N	%	N	%	
Vaccine	**20**		220		**240**
placebo	80		140		220
Total	**100**		360		**460**

Total **460** → **100 persons contracted influenza**

240 vaccinated → **20 contracted influenza**



	Contract influenza N	(%)	Not contract influenza N	(%)	Total
Vaccine	**20**	**(8.3)**	**220**	**(91.7)**	**240**
placebo	**80**	**(36)**	**140**	**(63.6)**	**220**
Total	**100**	**(21.7)**	**360**	**(78.3)**	**460**

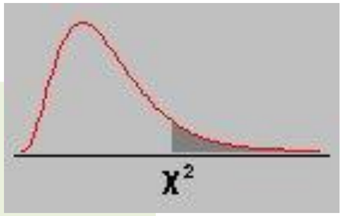
Overall persons contracting influenza

100/460= 21.7%

The chi square compare
the observed number in each of four categories
with the number expected

$$E = \frac{\text{Total row X total column}}{\text{Over all total frequency}}$$

E expected (E) $= \frac{\text{total column X total row}}{\text{Grand total}}$



E20 = $\frac{240X\ 100=}{460}$

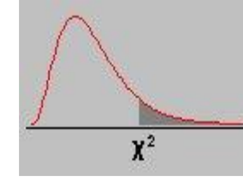
E220 = $\frac{240X\ 360=}{460}$

E80 = $\frac{220X\ 100=}{460}$

E140 = $\frac{220X\ 360=}{460}$

	Contract influenza N (%)	Not contract influenza N (%)	Total
Vaccine	20 (8.3)	220 (91.7)	240
placebo	80 (36)	140 (63.6)	220
Total	100(21.7)	360 (78.3)	460

E expected (E) $= \dfrac{\text{total column X total row}}{\text{Grand total}}$



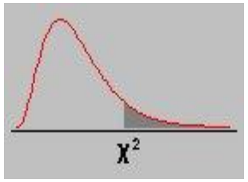
The chi square compare

the observed number in each of four categories

with the number expected

	Contract influenza		Not contract influenza		total
	O	E	O	E	
Vaccine	**20**	**52.2**	**220**	**187.8**	**240**
placebo	**80**	**47.8**	**140**	**172.2**	**220**
Total	**100**		**360**		**460**

Then chi square be calculated by calculating E. frequencies



if there were no difference in the efficacy
between vaccine and placebo.

if the vaccine and placebo having same efficiency then
we expect to have same proportion in each group
that is in the
vaccine group 100/460 X 240= 52.2
in placebo group 100/460 X 220 = 47.8

would have contract influenza. .

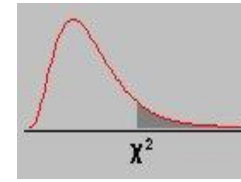
H0= 52.2 =47.8

Similarly
360/ 460 x240=187.8 in vaccine group
360/460 X 220 =172.2 in placebo group

will escape influenza

Then chi square be calculated by calculating E. frequencies

$$X^2 = \sum \frac{(O - E)^2}{E} \qquad d.f. = 1$$



	Contracting Influenza O	Contracting Influenza E	Not contract influenza O	Not contract influenza E	total
Vaccine	20	52.2	220	187.8	240
placebo	80	47.8	140	172.2	220
Total	100		360		460

$$X^2 = \frac{(20-52.2)^2}{52.2} + \frac{(80-47.8)^2}{47.8} + \frac{(220-187.8)^2}{187.8} + \frac{(140-172.2)^2}{172.2}$$

$$19.86 + 21.69 + 5.52 + 6.02 = 53.99$$

compare calcul. With tabulated

Critical region

$d.F = (C - 1)\ (r - 1)$

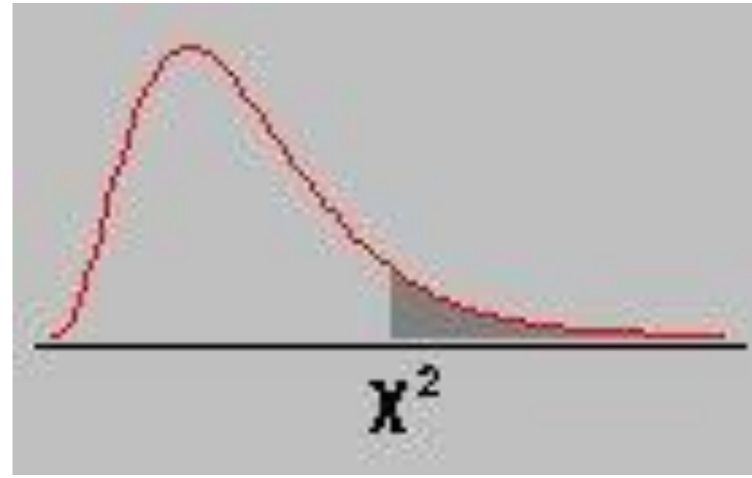
$\quad = (2 - 1)\ (2 - 1) = 1$

$\alpha = 0.05$

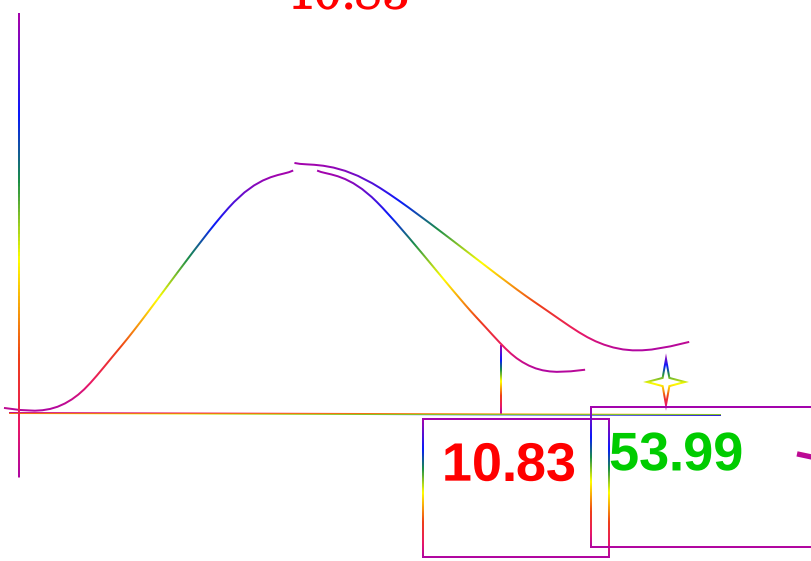
tabulated X^2 = 3.84

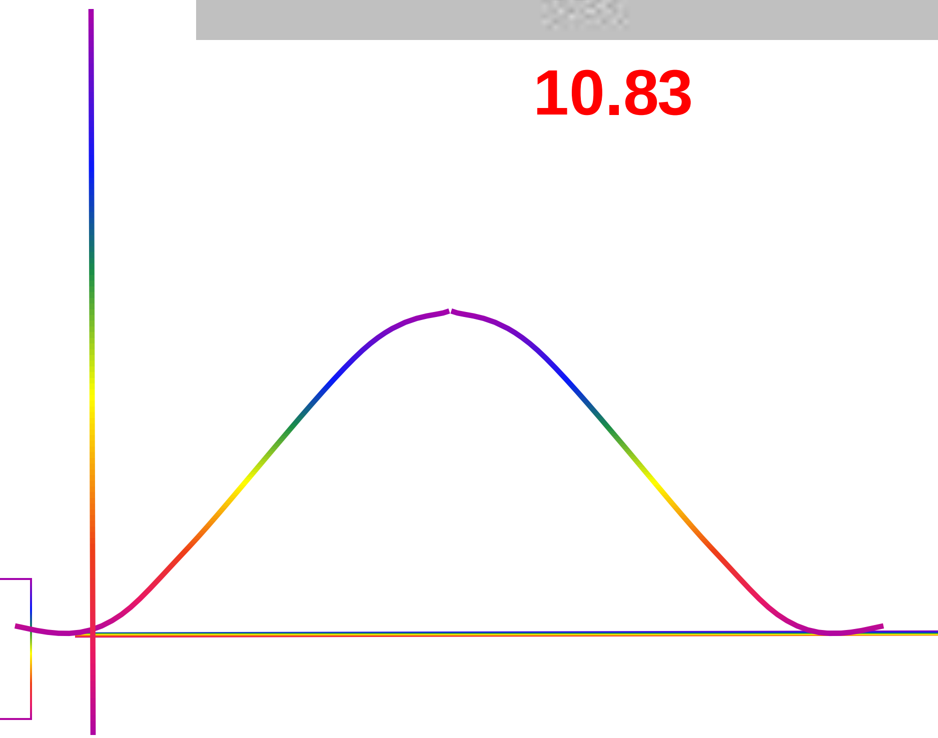
6.64

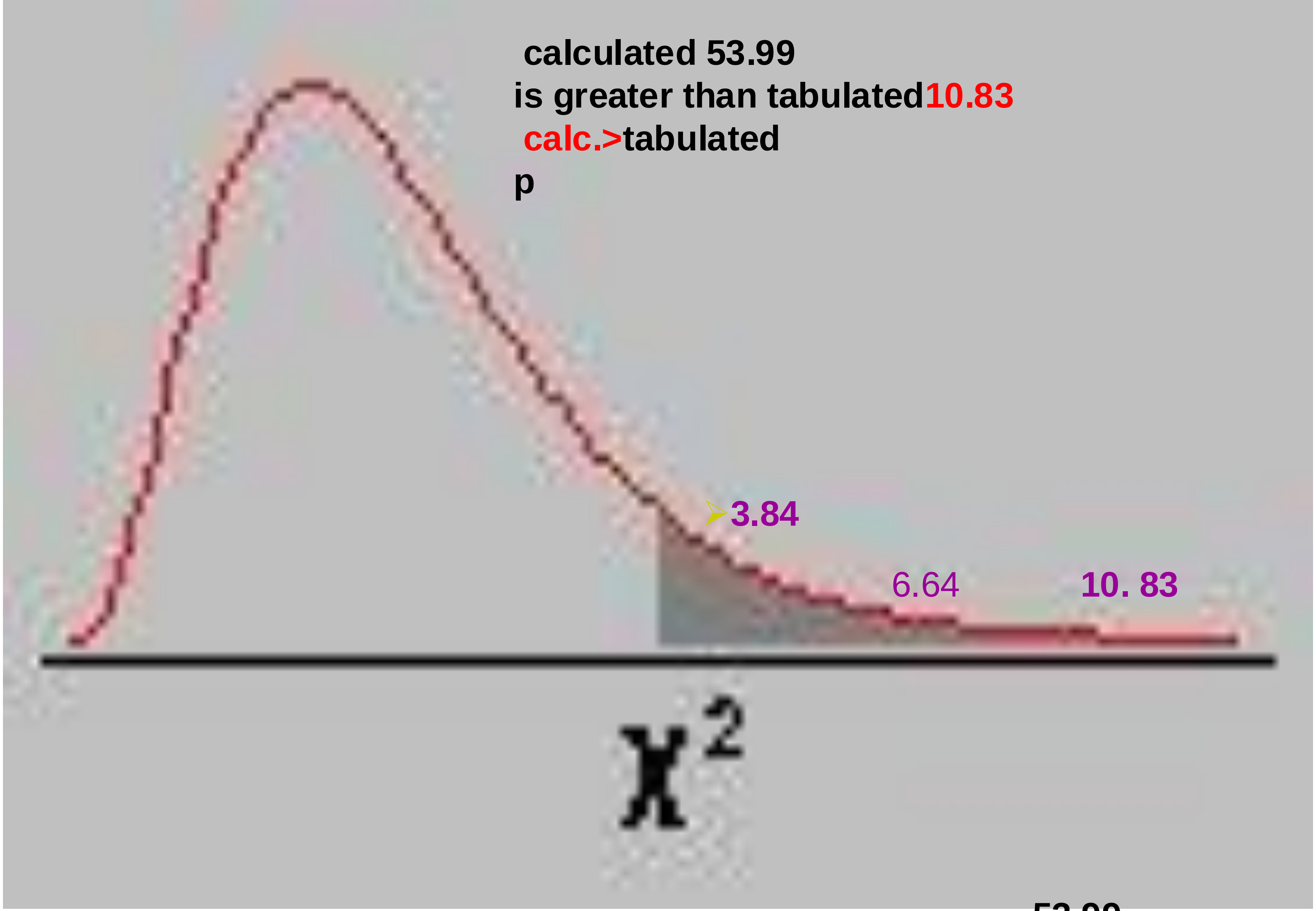
10.83



10.83



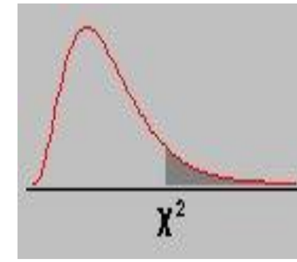



calculated 53.99
is greater than tabulated10.83
calc.>tabulated
p
3.84
6.64
10. 83
χ^2

53.99

p is ?????????

- This mean that
- the probability is less than 0.001
- that this difference is due to chance factor

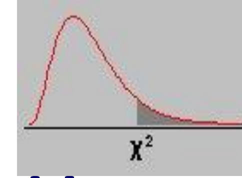


- and more than 99.999 that this difference
- due to vaccine

- Thus there is a strong evidence against null hypotheses that is saying no effect of vaccine on the probability of contracting influenza .

- there is a strong evidence that vaccine is effective

- Therefore it is concluded that vaccine is effective



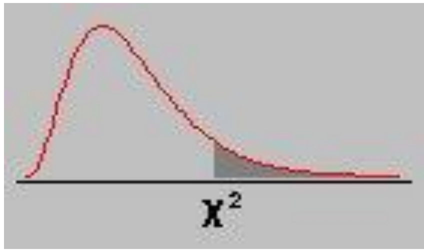
Continuity Correction

The chi square test for 2X2 table can be improved by using continuity correction we call it Yates continuity correction the formula become

$$X^2 = \sum \frac{\left[(O - E) - 0.5\right]^2}{E} \quad d.f. = 1$$

Pearson's chi-squared test by subtracting 0.5 from the difference between each observed value and its expected value in a 2 × 2

Resulting in small value for chi square
(the value of O –E) ignoring the sig



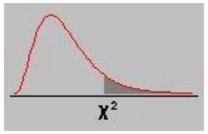
Chi square calculation procedure

- ✓ Calculate the expected values E for each cell
- ✓ Calculate the value O- E for each cell
- ✓ O is the observed
- ✓ Square O-E
- ✓ Divide each squared O- E by E for each cell
- ✓ Sum all of the values in previous step

this result is called test statistic

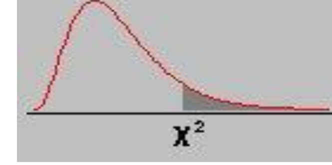
- ✓ identify the critical chi-square obtained
- ✓ from the chi square table.
- ❑ To reject the null hypothesis of equal proportion i.e. of independent variables the value of the test statistics must exceed the critical chi-square obtained from the chi square table.

Example



A sample of 84 mother chosen randomly
20 were smoker who delivered **14** babies with small birth weight (BW) and 6 normal BW.
On the other hand **64** non smoker women deliver **20** small BW babies and **44** normal BW babies
can we conclude that maternal smoking has a relation to small birth weight ?

mother	Small BW	Normal BW	total
Smoker	14	6	20
Non smoker	20	44	64
Total	**34**	**50**	84



Example

A sample of 84 mother chosen randomly 20 were smoker who delivered 14 babies with small birth weight (BW) and 6 normal BW. On the other hand 64 non smoker women deliver 20 small BW babies and 44 normal BW babies can we conclude that maternal smoking has a relation to small birth weight ?

	Small BW	Normal BW	total
Smoker	14 (70%)	6 (30%)	20
Non smoker	20 31.3 %)	44 68.7%	64
Total	34 (40.5%)	50	84

Ho ;

small BW and smoking status during pregnancy are **not related** in the population.

The Two variables are independent

H1:

Small BW and smoking status during pregnancy are **related** in the population .

The Two variable are Dependent

$$Ho = P_1 = P_2 = P_0$$

$$H_A = P_1 \neq P_2 \neq P_0$$

If the two variables are unrelated(H0)

then there is no reason why the proportion of small BW among smokers should be different to the proportion of small BW among non smokers mothers (H0)

In another ward these two proportions should be equal

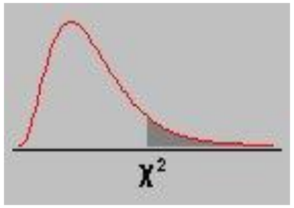
P1 =P2 70% = 31.3%

this difference could be due to chance (H0)

	Small BW	Normal BW	total
Smoker	14 **70%**	6 **30%**	20
Non smoker	20 **31.3 %**	44 **68.7%**	64
Total	34 **40.5%**	50	84

The question is that what proportion would we expect to find if null hypothesis of unrelated variable is true ??

The answer is that



since we got 34 small BW in a total of 84.

34/84= **0.405** **40.5%**

so we expect in smokers group to have ;0.405 x 20=8.1

in nonsmokers 0.405 x 64 = 25.92

An easer way to calculate Expected cell frequency

<u>Total row X total column</u>

Over all total frequency

$$\frac{34 \times 20}{84} = 8.094$$

$$\frac{34 \times 64}{84} = 25.904$$

	Small BW	Normal BW	total
Smoker	14	6	20
Non smoker	20	44	64
Total	34	50	84

Expected freq. = $\dfrac{\text{Total row X total column}}{\text{Over all total frequency}}$

	Small BW O	Small BW E	Normal BW O	Normal BW E	total
Smoker	14	8.1	6	11.9	20
Non smoker	20	25.1	44	30.1	64
Total	34		50		84

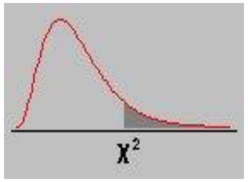
2

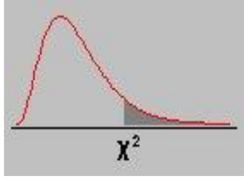
$$\frac{(14-8.1)^2}{8.1} + \frac{(6-11.9)^2}{11.9} + \frac{(20-25.1)^2}{25.1} + \frac{(44-30.1)^2}{30.1}$$

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

χ^2 = 4,3+2.9+1+6.4 = **14.6**

compare calculated χ^2 **with** tabulated χ^2





Critical region

d.F = (C – 1) (r – 1)

= (2 – 1) (2 – 1) = 1

$\alpha = 0.05$

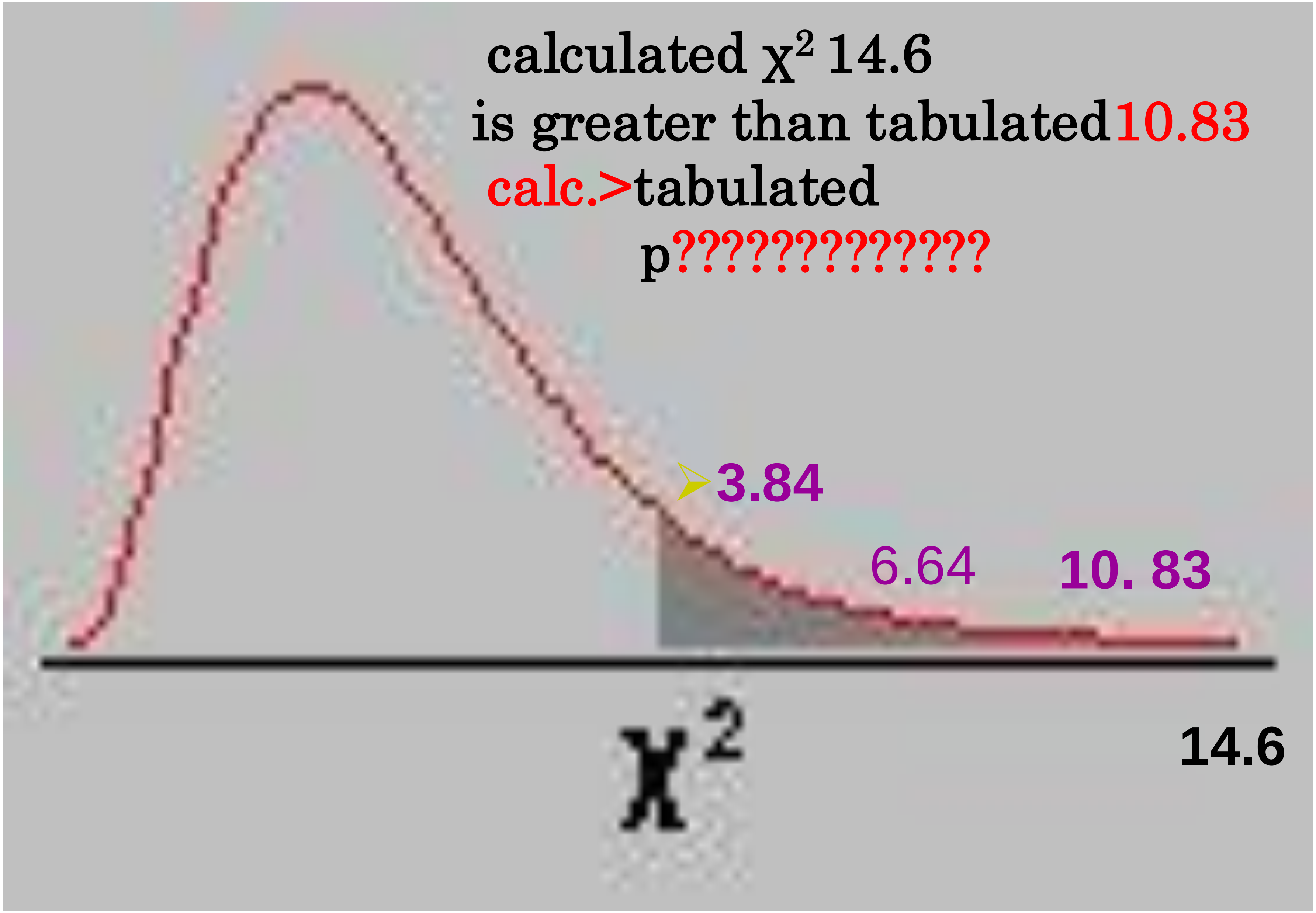
tabulated χ^2 = 3.84

6.64

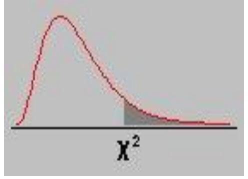
10.83

calculatedχ^2 =14.6

??????


calculated χ2 14.6
is greater than tabulated10.83
calc.>tabulated
p?????????????
3.84
6.64
10. 83
χ2
14.6

p is ?????????



- This mean that
- the probability is less than 0.001 that this difference is due to chance factor
  - And more than 99.999 that this difference due to smoking
- Thus there is a strong evidence against null hypotheses that is saying no effect of smoking on the probability of LBW.
- there is a strong evidence that LBW is related to smoking
- Therefore it is concluded that smoking is risk

p is ?????????

P > 0.05	**P > 0.01**	**P > 0.001**
p < 0.05	**p < 0.01**	**p < 0.001**

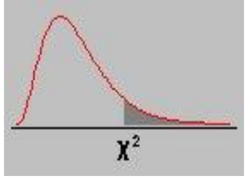
You can answer

if p-value associated with chi square is **less than 0.05 or** less than 0.01 → you **reject** null hypoth.

And conclude that

- the two variable are **not independent** or

- there is a **statistically significant difference** in the **proportions**



Continuity Correction

The chi square test for 2X2 table can be improved by using continuity correction we call it **Yates continuity correction** the formula become

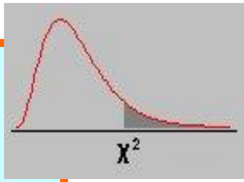
$$X^2 = \sum \frac{\left[(O - E) - 0.5 \right]^2}{E} \qquad d.f. = 1$$

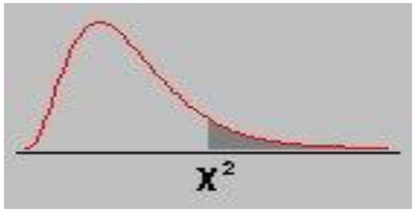
Resulting in small value for chi square

$$X^2 = \frac{(14 - 8.1\ ) - 0.5\ )^2}{8.1} + \frac{(\ 6 - 11.9\ )\ - 0.5)^2}{11.9} +$$

$$\frac{(20 - 25.1) - 0.5\ )^2}{25.1} + \frac{(44 - 30.1) - 0.5)^2}{30.1}$$

P < 0.001





Validity of χ^2

When the expected numbers are very small the chi square test is not good enough

We recommended other test (Exact Test)

Thus χ^2 is valid

- when the overall total is more than 40 , regardless the expected values

and

- when the overall total between 20 and 40 provided that all expected values are at least 5

Thank You