Chap 3
Kinematics in Two Dimensions (Vectors)

3-1 : Vectors and Scalars

A vector is a quantity which must have both magnitude and direction (velocity, Force etc)

A scalar is a quantity which must have magnitude only (mass, temperature, energy, time)

3-2 : Addition of Vectors - Graphical Methods

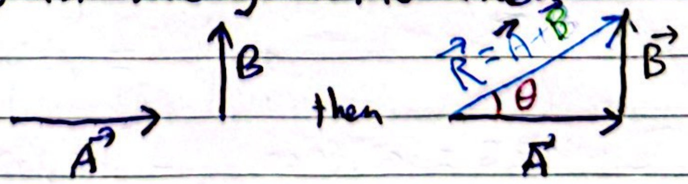
- If we have two vectors $\vec{A}$ and $\vec{B}$ in the same axis, then $\xrightarrow[\vec{A}]{3m}$, $\xrightarrow[B]{4m}$ then $\xrightarrow[\vec{A}+\vec{B}=\vec{R}]{7m}$

the result of the addition of vectors is called Resultant $\vec{R}$

but if $\xleftarrow[A]{3m}$, $\xrightarrow[\vec{B}]{4m}$ then $\xrightarrow[\vec{A}+\vec{B}=\vec{R}]{1m}$

- If one vector $\vec{A}$ in the x axis and the other vector $\vec{B}$ in the y-axis then

$\vec{R}=\vec{A}+\vec{B}$



To find the Restant $\vec{R}$ we measure the length of $\vec{R}$ and using the same scale for $\vec{A}$ and $\vec{B}$ we get the magnitude of $|\vec{R}|$, then we measure the angle θ which is the angle between $\vec{R}$ and the x-axis, then we have determined the resultant of the addition of $\vec{A}$ and $\vec{B}$, the magnitude and the direction. The magnitude can, also, be determined by using the Pythagoras theorem $|\vec{R}| = \sqrt{\vec{A} + \vec{B}}$

if $\vec{A} = 10$ Km to the east and $\vec{B} = 5$ Km to the north, then

$$R = \sqrt{(10)^2 + (5)^2} = 11.2 \text{ Km}$$

- The rules for getting the resultant byagraphical method are

(1) On a diagram, draw $\vec{A}_1$ to scale

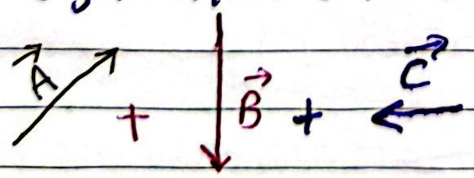
(2) Next draw the second vector $\vec{B}$ to the same scale, and placing its tail at the tip of the first vector directed at the same original direction

(3) The vector (Arrow) drawn from the tail of the first vector $(\vec{A})$ to the tip of the second vector $(\vec{B})$ represent the sum or resultant of the two vectors $\vec{R} = \vec{A} + \vec{B}$

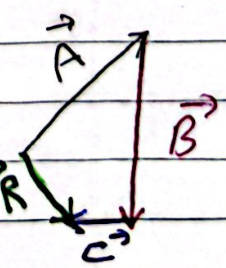
- $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

- ~~mult~~ If we have more than two vectors we do the same as for two vectors

e.g.: if we have three vectors

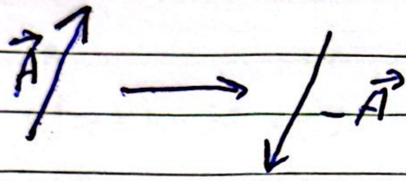


, then



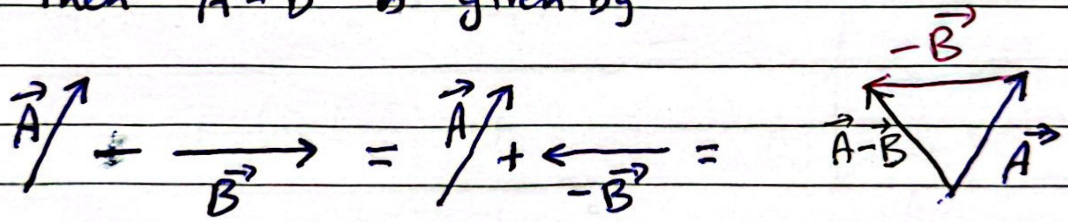
3-3 Subtraction of Vectors, and Multiplication of A Vector by A scalar.

- Negative vector: if $\vec{A}$ is a vector then $-\vec{A}$ is defined as it has the ~~[illegible]~~ same magnitude of $\vec{A}$ but is in opposite direction to it



Then subtraction of vectors is equal to the sum of first plus the negative of the second.

Then $\vec{A}-\vec{B}$ is given by

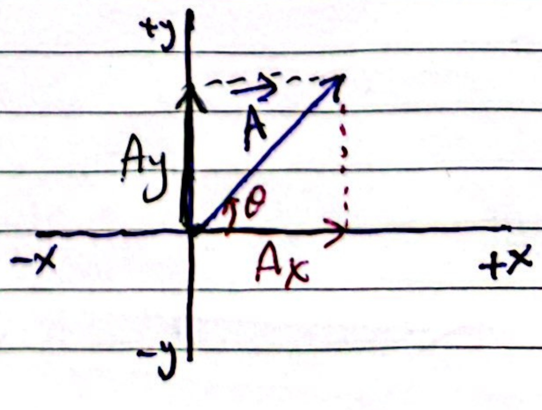


3-4 Adding Vectors by Components

Any vector can be resolved into two components along x and y axes.

If we have a vector $\vec{A}$ then its components are

if $\vec{A}$ makes an angle θ



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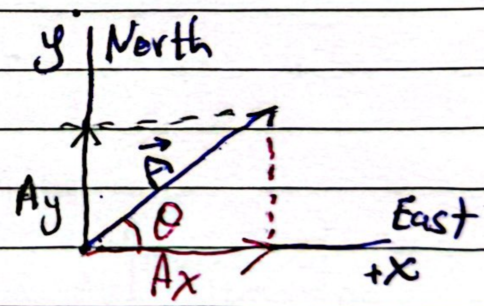
with the positive x-axis, then, the x-component is given by $\boxed{A_x = A\cos\theta}$, and the y-component is given by $\boxed{A_y = A\sin\theta}$, where $\boxed{A}$ is the magnitude of the vector $|\vec{A}|$ and $\boxed{\theta}$ must be measured from the positive x-axis counter clockwise.

Example: suppose $\vec{A}$ represents a displacement of 500 m in a direction 30° north of east, then

$$A_x = A\cos\theta = 500\cos(30^\circ)$$
$$= 500(0.866) = 433\ \text{m (east)}$$



and

$$A_y = A\sin\theta = 500\sin(30^\circ)$$
$$= 500(0.5) = 250\ \text{m (North)}$$

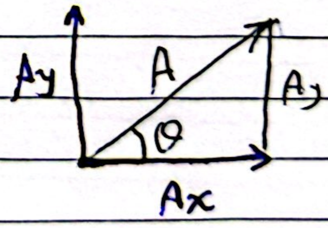
- There are two ways to specify a vector in a given coordinate system

1. We can give its components A_x and A_y
2. We can give its magnitude A and the angle θ

it makes with the positive x-axis.

- The above discussion is to shift from the second description to the first. But the following discussion is to shift from the first to the second description; that is from the components to find the magnitude and the direction of the vector. So if we have the components A_x and A_y for a vector $\vec{A}$, then we can get the magnitude and direction of the vector using the following relations

$$A = |\vec{A}| = \sqrt{A_x^2 + A_y^2} \quad \text{(magnitude)}$$



and

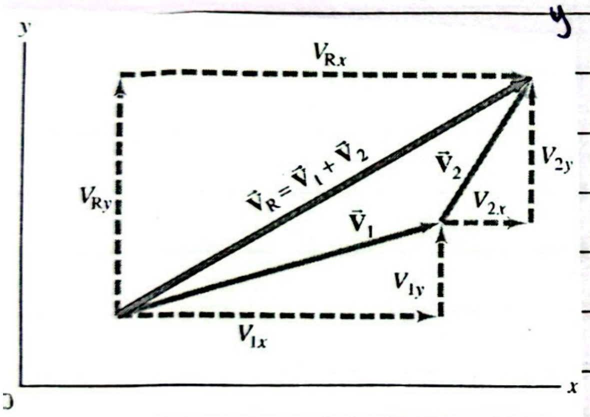
$$\tan\theta = \frac{A_y}{A_x} \quad \text{(direction)}$$

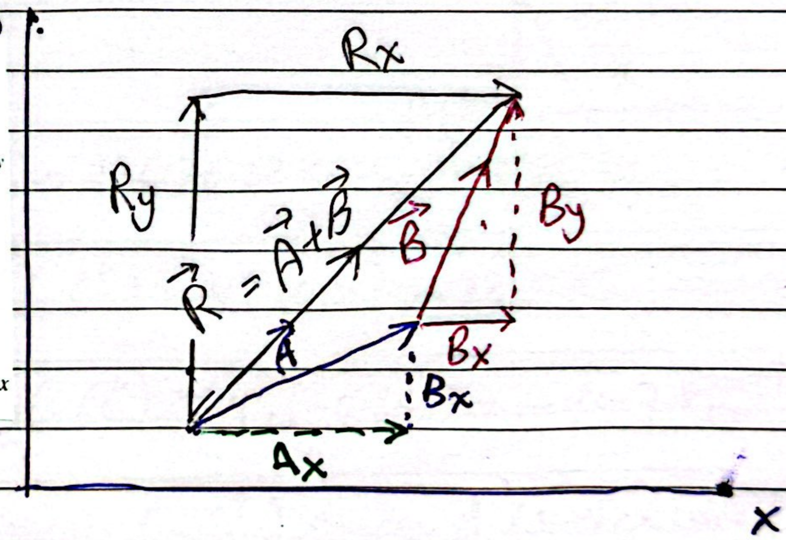
$$\Rightarrow \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) \quad , (\tan^{-1} \text{ read tan inverse})$$

Adding Vectors

We can use the components to add vectors which is called the analytical method of adding vectors.

Suppose we have two vectors $\vec{A}$ and $\vec{B}$, then the sum (resultant) $\vec{R} = \vec{A} + \vec{B}$ is given by R_x and R_y





$R_x = A_x + B_x$

and

$R_y = A_y + B_y$

or the magnitude $|\vec{R}| = R = \sqrt{R_x^2 + R_y^2}$

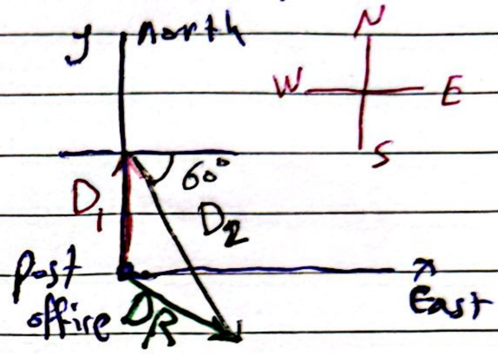
and the direction of $\vec{R}$ is $\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$

where θ is measured from the x-axis

Example 3-2: Mail carrier's Displacement:

She leaves the post office and drives 22 Km North, she then drive 47 Km in a direction of 60° South of east. What is her displacement from the post

Solution



$D_{1x} = D_1 \cos 90° = 22(0) = 0$ Km

$D_{1y} = D_1 \sin 90° = 22(1) = 22$ Km

also

$D_{2x} = D_2 \cos(-60°) = 47(0.5) = 23.5$ Km

$D_{2y} = D_2 \sin(-60°) = 47(-0.866) = -40.7$ Km

We use the angle $(-60°)$ since it is measured clockwise. Then the Resultant Components are

$D_{Rx} = D_{1x} + D_{2x} = 0 + 23.5 = 23.5$ Km

$D_{Ry} = D_{1y} + D_{2y} = 22 + (-40.7) = -18.7$ Km

This specifies the resultant displacement completely or magnitude or direction

$D_R = \sqrt{D_{Rx}^2 + D_{Ry}^2} = \sqrt{(23.5)^2 + (-18.7)^2} = 30$ Km

and

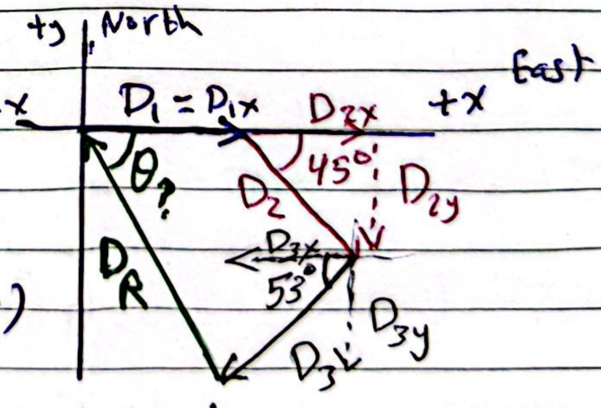
$\theta = \tan^{-1}\left(\frac{D_{Ry}}{D_{Rx}}\right) = \tan^{-1}\left(\frac{-18.7}{23.5}\right) = \tan^{-1}(-0.796) = -38.5°$

The minus sign means θ is measured clockwise or θ is below the x-axis. So the resultant displacement is 30 Km directed at 38.5° Southeast.

Example 3.3: Three Short trips

An airplane trip involves three legs. The first leg is due east for 620 Km; the second leg is southeast (45°) for 440 Km and the third leg is at 53° south of west for 550 Km, as shown. What is the plane total displacement?

Solution



1. Draw a diagram

2. Choose axex (figure)

3 and 4. Calculate the components

$\vec{D}_1$: $D_{1x} = D_1 \cos(0°) = 620\ Km$

$D_{1y} = D_1 \sin(0°) = 0\ Km$

$\vec{D}_2$: $D_{2x} = D_2 \cos(45°) = 440(0.707) = 311\ Km$

$D_{2y} = D_2 \sin(45°) = 440(-0.707) = -311\ Km$

$\vec{D}_3$: $D_{3x} = -D_3 \cos(+53°) = -550(0.602) = -331\ Km$

$D_{3y} = -D_3 \sin(53°) = -550(0.799) = -439\ Km$

Note: If the components points in the $-x$ or $-y$ direction we give a minus sign to the component

5. Add the Components

We add the x-components together and the y-components together.

$D_{Rx} = D_{1x} + D_{2x} + D_{3x} = 620 + 311 + (-331) = 600\ Km$

$D_{Ry} = D_{1y} + D_{2y} + D_{3y} = 0 + (-311) + (-439) = -750\ Km$

This is one way to give the answer.

6. Magnitude and direction: we can give the answer as

$$D_R = \sqrt{D_{Rx}^2 + D_{Ry}^2} = \sqrt{(600)^2 + (-750)^2} = 960\,Km$$

$$\theta = \tan^{-1}\left(\frac{D_{Ry}}{D_{Rx}}\right) = \tan^{-1}\left(\frac{-750}{600}\right) = \tan^{-1}(-1.25) = -51^\circ$$

Southeast