Chap 6

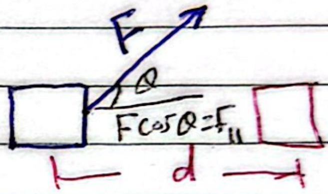
Work and Energy

6-1 Work Done by a Constant Force

There are two condition for any force to do work

1. the object must move a displacement
2. the force must have at least a component parallel to the direction of motion

The Work is given by



$$W = F_{\parallel} d = F d \cos\theta$$

Where F is the force applied, d is the displacement of the object and θ is the angle between them.

- Work is a scalar quantity.
- Work is $\boxed{\text{positive}}$ if the force and displacement are in the same direction and $\boxed{\text{negative}}$ if the force and displacement are in opposite directions

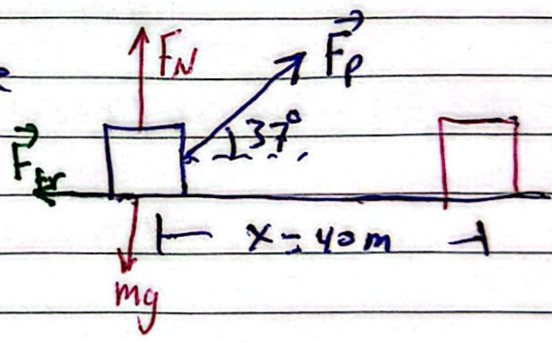
- The unit for work ~~are~~ SI system is the Joule

1 Joule = 1 N.m — SI system
1 erge = 1 dyne.cm — cgs system
foot-pound — British system

- If the force and displacement are perpendicular to each other then no work is done ($W=0$)
- If we have more than one force, then the total work is the algebraic sum of the work of each force, $W_{total} = W_1 + W_2 + W_3 \cdots$

Example 6-1. Work done on a crate (box)

A person pulls a crate of mass 50 Kg a distance of 40 m on



a rough horizontal floor with friction force of $\vec{F}_{Fr} = 50\,N$. If the force exerted by the person $\vec{F}_P = 100\,N$ which make angle $\theta = 37°$ with respect to the x-axis

Determine (a) the work done by each force

(b) the net work done on the crate

Solution.

We have four forces as shown in the figure

$W_{person} = W_p = F_p x \cos\theta = (100N)(40m)\cos 37^\circ = 3200J$

$W_{friction} = F_{fr} x \cos 180^\circ = (50)(40m)(-1) = -2000J$

$W_{gravity} = W_G = mg x \cos 90^\circ = 0$

$W_{normal\ force} = W_N = F_N x \cos 90^\circ = 0$

(b) The net work is

(1) $W_{net} = W_p + W_{fr} + W_G + W_N$

$= 3200J + (-2000J) + 0 + 0 = 1200J$

or

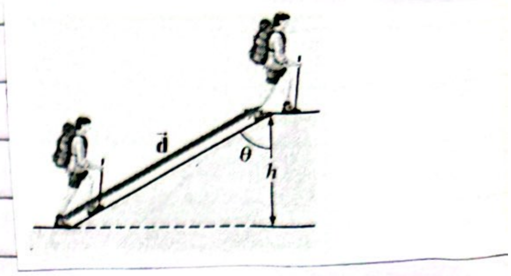
(2) Adding the forces then calculating the work done by the net force

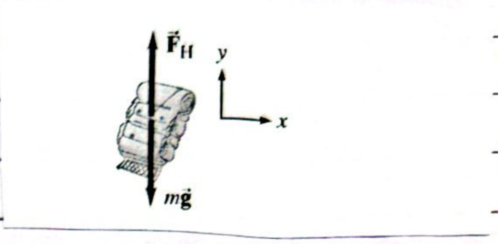
$F_{net} = F_p - F_{fr} = 100N\cos(37^\circ) - 50N$

$\rightarrow W_{net} = (100N\cos(37^\circ) - 50N)(40) = 1200J$

Example 6-2 Work on a backpack.

(a) Determine the work a hicker must do on a 15 Kg backpack to carry it up a hill of hight h = 10m as shown



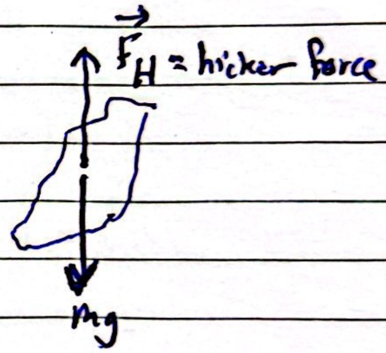


(b) Determine the W_G on the backpack

(c) Determine the net Work

Solution

1. Draw a free-body diagram



$F_G = mg$, the force of gravity

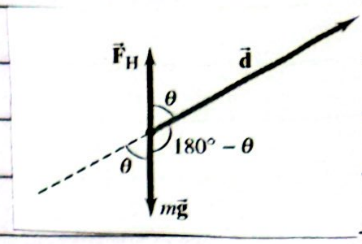
and F_H is the force the hicker must exert to support the Backpack.

the horizonal force on the backpack is zero since the acceleration is zero

2- Choose a cordinate system; we interested in the vertical motion so we choose the y-direction

3. $\boxed{\text{Apply Newtons Laws}}$

$$\sum F_y = ma_y = 0 \quad \text{since } a_y = 0$$

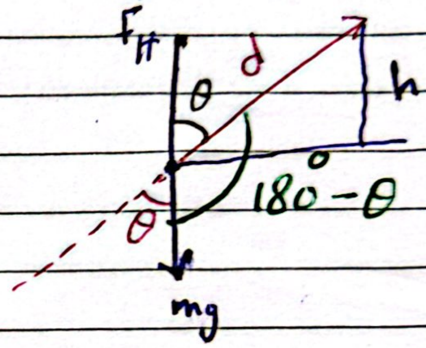


$$F_H - mg = 0 \Rightarrow$$

$$F_H = mg = (15\,Kg)(9.8\,m/s^2) = 147\,N$$

4. $\boxed{\text{Work done by a specific force}}$

(a) Work done by the hicker



$$W_H = F_H (d \cos\theta)$$

but $h = d\cos\theta$ then

$$W_H = F_H h = mgh = (147\,N)(10\,m) = 1470\,J$$

W_H depends on the hight of the hill.

(b) Work done by gravity

$$W_G = mgd \cos(180° - \theta) = mgd(-\cos\theta)$$

$$= -mgh = -(147\,N)(10\,m) = -1470\,J$$

5. $\boxed{\text{Net Work done}}$

$$W_{net} = W_H + W_G = 1470 + (-1470) = 0$$

Eventhough $W_{net} = 0$ but the hicker does work $= 1470\,J$

6-3 Kinetic Energy, and the Work-Energy Principle

- Energy is defined as the ability to do work
- Kinetic energy is the energy of motion, the cause of motion. Mathematically it is given by

$$KE = \frac{1}{2}mv^2 \quad \text{translational K.E}$$

- The work-Energy principle (theorem) can be stated as " the net work done on an object is equal to the change in the object's Kinetic energy" . Mathematically

$$W_{net} = \Delta K.E = K_f - K_i$$

$$W_{net} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Example 6-4 : How much work must be done to increase the velocity of a car from 20 m/s to 30 m/s for a car of mass 1000 kg

Solution :

$$W = KE_2 - KE_1 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$= \frac{1}{2}(1000\,kg)(30)^2 - \frac{1}{2}(1000)(20)^2 = 2.5 \times 10^5 \; J$$

Example 6-5 Work to stop a car

A car traveling 60 Km/h can brake to stop in a distance d of 20 m. If the car is going twice as fast, 120 Km/h. What is its stopping distance?

Solution: using the work-energy principle

$$W_1 = Fd\cos 180^\circ = -Fd_1$$

$$-Fd_1 = \frac{1}{2}m(0)^2 - \frac{1}{2}mv_1^2$$

$$\Rightarrow Fd_1 = \frac{1}{2}mv_1^2 \quad (1)$$

also

$$Fd_2 = \frac{1}{2}mv_2^2 \quad (2)$$

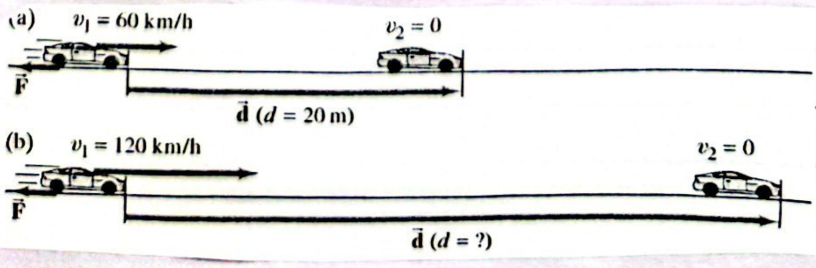
F is constant (assume)

divide (1) by (2)

$$\frac{Fd_1}{Fd_2} = \frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2}$$

$$\frac{d_1}{d_2} = \frac{v_1^2}{v_2^2} \Rightarrow$$

$$d_2 = \frac{d_1 v_2^2}{v_1^2} = \frac{(20m)(120\ Km/h)^2}{(60\ Km/h)^2} = 80m$$

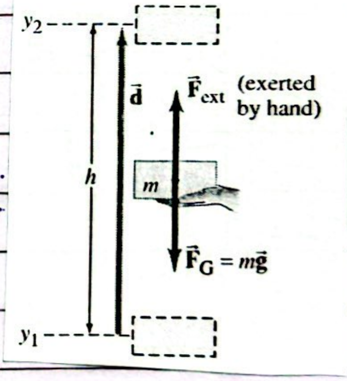


6-4 Potential Energy (P.E)

Potential energy which is associated with forces that depend on the position or configuration of an object relative to the surroundings.

Gravitational Potential Energy

Let us find the form for the gravitational potential energy of an object near the surface of the earth.



The work done by the person W_{ext} is

$$W_{ext} = F_{ext}\, d \cos 0^\circ$$

$$= mgh = mg(y_2 - y_1) \quad (1)$$

and the work done by gravity is

$$W_G = F_G\, d \cos(180^\circ) = -mgh = -mg(y_2 - y_1) \quad (2)$$

We define the gravitational potential energy of an object due to the earth's gravity force as

$$PE_G = mgy$$

where y is the height of the object above some reference. From equation (2)

$$W_G = -mg(y_2 - y_1) = -mgy_2 + mgy_1$$

$$= -(PE_2 - PE_1) = -\Delta PE_G$$

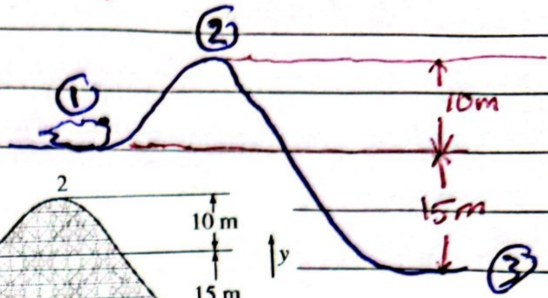
That is the change in the gravitational potential energy is equal to negative of the work done by gravity itself.

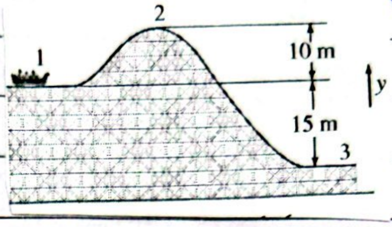
$$\Delta PE_G = -W_G$$

- Therefore what is physically important is the change in the potential energy.
- To calculate the potential energy of an object at any point, we must use a reference level (for example the ground's level).

Example 6-6 Potential energy changes for a roller coaster

A 1000 kg roller coaster car moves from point 1 to point 2 to point 3





(a) What is $(PE)_G$ at point 2 and 3 relative to 1 ?

take $y = 0$ (reference point at point 1

(b) What is the change in P.E when the car goes from point 2 to point 3

(c) Repeat part (a) and (b) by takeing $y = 0$ at point 3 as a reference point

Solution,

At the reference point the potential energy is zero (at point 1) $y_1 = 0$, then

$(PE)_2 = mgy_2 = (1000\,kg)(9.8\,m/s^2)(10\,m) = 9.8 \times 10^4\,J$

and

$(PE)_3 = mgy_2 = (1000\,kg)(9.8\,m/s^2)(-15) = -1.5 \times 10^5\,J$

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(b)

$$\Delta PE = PE_{③} - PE_{②} = -1.5\times10^{5}\,J - 9.8\times10^{4}\,J$$

$$= -2.5\times10^{5}\,J$$

(c) Now at $y_3 = 0$ the P.E is zero then $y_1 = 15m$

and $y_2 = 25m$ so

$$(P.E)_{①} = (1000\,kg)(9.8\,m/s^2)(15) = 1.5\times10^{5}\,J$$

and

$$(P.E)_{②} = (1000\,kg)(9.8\,m/s^2)(25) = 2.5\times10^{5}\,J$$

∴ The potential energy's change between ② and ③

$$(PE)_3 - (PE)_{②} = 0 - 2.5\times10^{5}\,J = -2.5\times10^{5}\,J$$

Which is the same as in part (b).

Note: the change in the potential energy between two points is the same regardless of the reference point.

6-5 Conservative and NonConservative Forces

Conservative force is defined as the force for which the work done by it does not depend on the path taken between any two points but depend on the initial and final position.

e.g: gravitational force, electric force, magnetic force, spring force ...

Non conservative force is defined as the force ~~wh~~ for which the work done by it depend on the path taken between any two points (e.g: friction force, pull, push ...)

- Potential energy is associated with conservative forces only, so we have gravitational potential energy, electrical potential energy, magnetic potential energy but no frictional potential energy ($W_c = -\Delta P.E$)

Work-Energy Extended

If some of the forces acting on an object are non conservative, then the net work is the sume of work due to all forces

$$W_{net} = W_c + W_{nc} = \Delta KE$$

but $W_c = -\Delta P.E \Rightarrow$

$$-\Delta PE + W_{nc} = \Delta KE$$

$\Rightarrow$

$$\boxed{W_{nc} = \Delta PE + \Delta KE}$$

that is the work done by the non conservative forces equals the change of the potential energy plus the change in the kinetic energy of the moving object. This statement is the extended work-energy principle

6-6 Mechanical Energy and Its Conservation

If all forces acting on an object are Conservative than from the work-energy principle $W_{nc} = \Delta PE + \Delta KE = 0$

$\Rightarrow$

$$(PE_2 - PE_1) + KE_2 - KE_1 = 0$$

$\Rightarrow$

$$PE_2 + KE_2 = PE_1 + KE_1$$

$\Rightarrow$

$$E_2 = E_1 = \text{Constant}$$

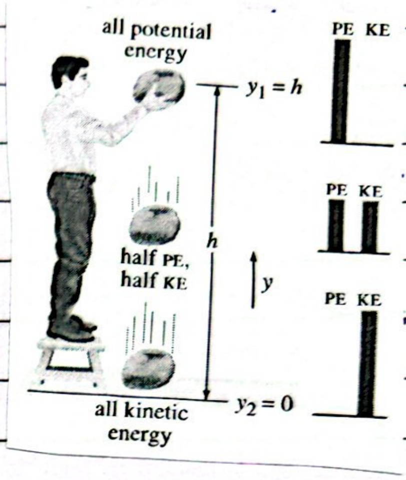
where E is defined as the sum of the potential energy plus the kinetic energy and E is called the mechanical energy.

That is mechanical energy is conserved if all forces acting on the object are conservative

$$\left.\begin{array}{c} E_1 = E_2 \\ \Delta KE + \Delta PE = 0 \\ \Delta E = 0 \end{array}\right\} \text{conservation of mechanical energy principle}$$

6-7 Problem Solving Using Conservation of Mechanical Energy

A simple example of the conservation of mechanical energy (neglecting air resistance) is a rock allowed to fall from a height h due to F_G only.



at position y_1: P.E is maximum $= mgh$, but KE is zero since the rock is at rest,

at position y: to reach position y, the P.E decreases and K.E increases but the total mechanical energy E is constant $E_{y_1} = E_y = mg$

at position y_2: P.E is zero and K.E is maximum and equals mgh.

$$E_{y_1} = E_{y_2}$$

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2$$

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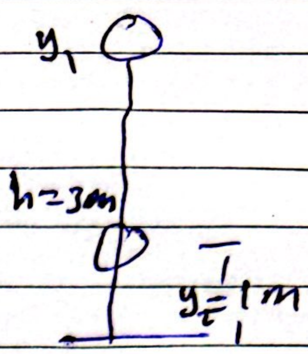
$\boxed{\text{Example 6-7}}$ Falling rock



If the initial height of the rock is $h = 3m$, calculate the rock's velocity when it reach (fall) to 1 m above the ground if $v_1 = 0$ (rest)

Solution

1. We must use a reference level $(y=0)$ at which P.E is zero.

2. Applying the conservation of mechanical energy principle $E_1 = E_2 \Rightarrow$

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2$$

$$0 + gy_1 = \tfrac{1}{2}v_2^2 + gy_2 \Rightarrow$$

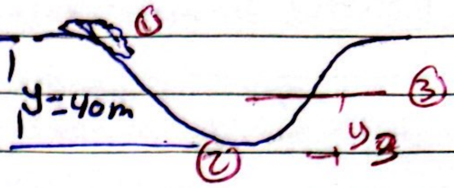
$$v_2^2 = 2g(y_1 - y_2) \Rightarrow$$

$$v_2 = \sqrt{2g(y_1 - y_2)} = \sqrt{2(9.8\,m/s^2)(3m - 1m)} = 6.3\,m/s$$

Not the velocity doesn't depend on the mass of the rock

$\boxed{\text{EX 6.8}}$ Roller-Coaster car speed using energy conservation

Assume the height of the hill is $y = 40m$, the car starts from rest at the top



(a) Calculate the speed of the car at the bottom

(b) At what height it will have half this speed

Solution:

take the reference level $y=0$ at the bottom

using ~~mechanical~~ mechanical energy conservation

$$E_1 = E_2 \Rightarrow KE_1 + PE_1 = KE_2 + PE_2$$

$$\Rightarrow \frac{1}{2}\cancel{m v_1^2}^{0} + mgy_1 = \frac{1}{2}mv_2^2 + \cancel{mg y_2}^{0}$$

$$\Rightarrow mgy_1 = \frac{1}{2}mv_2^2$$

$$v_2 = \sqrt{2gy_1} = \sqrt{2(9.8 m/s^2)(40m)} = 28 m/s$$

(b) $E_1 = E_3 \Rightarrow \frac{1}{2}\cancel{m v_1^2}^{0} + mgy_1 = \frac{1}{2}m(14 m/s)^2 + mgy_3$

$$\Rightarrow y_3 = y_1 - \frac{(14 m/s)^2}{2g} = 40m - \frac{(14 m/s)^2}{2(9.8 m/s^2)} = 30m$$

The car velocity 14 m/s when it at 30 m above the ground both when descending and ascending

6-8 Other Forms of Energy and Energy transformation; the Law of Conservation of Energy

We many other energies in life such as electric, magnetic, nuclear, thermal and chemical energy

So we have the Conservation of energy law which states " the total energy is neither increased nor decreased in any process, Energy Can be transformed from one form to another and transferred from object to another, but the total amount remains the Same

6-9 Energy Conservation with Dissipative Forces: Solving problems

frictional forces are called dissipative forces

because the system must lose energy to overcome

the negative work done by the dissipative force.

The general form of the Work-Energy principle is

$$W_{NC} = \Delta KE + \Delta PE$$

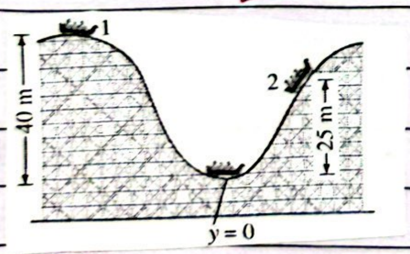
$$= KE_2 - KE_1 + PE_2 - PE_2$$

$\Rightarrow$

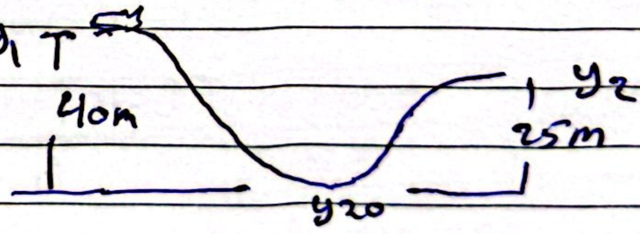
$$KE_2 + PE_2 = KE_1 + PE_1 + W_{NC}$$

Where $W_{NC} = -F_{fr}\, d$ for friction force (estimation)

$\boxed{\text{Example 6-12}}$ Friction on the roller-coaster car



$m = 1000\,kg$



Since we have friction force we need to use the general form of Work-energy principle

Solution

1. Draw a picture . 2. The system has gravitational force and frictional force

3. Choose initial and final position, $y_1 = 40$ and $y_2 = 25m$

4. choose a reference level : $y = 0$ at which P.E = 0

5. Mechanical energy is not conserved (friction)

6. Apply $W_{nc} = \Delta KE + \Delta PE \Rightarrow$

$$KE_2 + PE_2 = KE_1 + PE_1 + W_{NC}$$

$$\cancel{\frac{1}{2}mv_2^2}^{0} + mgy_2 = \cancel{\frac{1}{2}mv_1^2}^{0} + mgy_1 - F_{fri}d$$

but $v_1 = v_2 = 0$ $y_1 = 40m$, $y_2 = 25m$, $d = 400m$,

$\Rightarrow$

$$F_{fri}d = mg(y_1 - y_2)$$

$$= (1000\,kg)(9.8\,m/s^2)(40m - 25m) = 147000\,J$$

then $\bar{F}_{fri}$ average $= \frac{147000\,J}{d} = \frac{147000}{400} = 370\,N$

This result is a rough estimate average value since F_{fri} depends on the normal force which changes from point to point

6-10 power (P)

power is defined as the rate at which work is done

$$P_{average} = \frac{\Delta W}{\Delta t} = \frac{\Delta E}{\Delta t}$$

FIGURE 6–28 Example 6–13.

- It is a scalar quantity
- The unit of power is the Watt W

$1 W = J/s$ SI system

horsepower (hp) British system

$1 hp = 550 ft.lb/s = 746 W$

Example 6-13 Stair-climbing power

A 60-kg jogger runs up a long stair in 4s. the vertical height of the stair is 4.5m (a) estimate the jogger's power in W and hp (b) How much energy is required

Solution:

(a) $\bar{P} = \frac{W}{t} = \frac{mgy}{t} = \frac{(60kg)(9.8 m/s^2)(4.5)}{4} = 660 W$

$\bar{P} = \frac{660}{760} = 0.87 hp$ very small

(b) $E = \bar{P}t = (660 J/s)(4 s) = 2600 J = mgy$

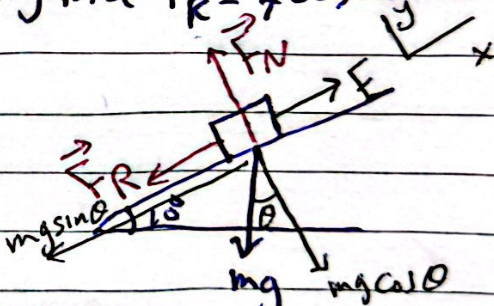
Also $\boxed{\bar{P} = \frac{W}{t} = \frac{Fd}{t} = F\bar{v}}$, $\bar{v} = \frac{d}{t}$

$\boxed{\text{Example 6-14}}$ Power needs of a car

Calculate the power required of a 1400-kg car under the following circumstances (a) the car climbs a 10° hill at a steady 80 km/h,

(b) The car accelerates along a level road from 90 to 110 km/h in 6 s to pass another car.

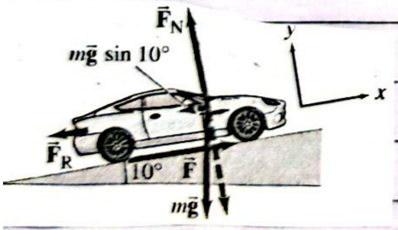
Assuming the average retarding force $F_R = 700\,N$.



Solution

(a) Newtons Law (First)

$F = F_R + mg\sin\theta$



$= 700 + (1400\,kg)(9.8\,m/s^2)(0.174)$

$= 3100\,N$, then

$\bar{v} = 80\,km/h = 80\,\frac{km}{h} \times \frac{1000\,m}{1\,km} \times \frac{hr}{3600\,s} = 22\,m/s$

The power

$\bar{P} = F\bar{v} = (3100\,N)(22\,m/s) = 6.8 \times 10^4\,W$

$= 68\,kW = \frac{6800}{760} = 91\,hp$

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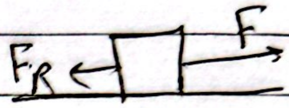
(b) the acceleration $\bar{a} = \frac{v_2 - v_1}{t}$

$v_2 = 110\,km/h = 30.6\,m/s$

$v_1 = 90\,km/h = 25\,m/s$ $\Rightarrow$

$\bar{a}_x = \frac{30.6-25}{6s} = 0.93\,m/s^2$



Newtons Law

$\sum F_x = \vec{F} - F_R = m a_x$

$\Rightarrow F = m a_x + F_R$

$= (1400kg)(0.93\,m/s^2) + 700 =$

$= 1300N + 700N = 2000N$

$\bar{p} = F\bar{v} = (2000N)(30.6\,m/s) = 6.1\times10^4\,W$

$= 61\,kW = 82\,hp$

Since the required power increases with speed and the motor must provide the maximum power so we used the final speed.

efficiency $= e = \frac{P_{out}}{P_{in}}$