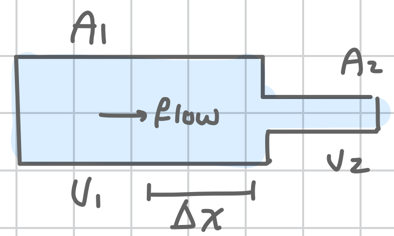
Fluid Dynamics

Sec 10.8 – 10.12

→ The Continuity Equation.



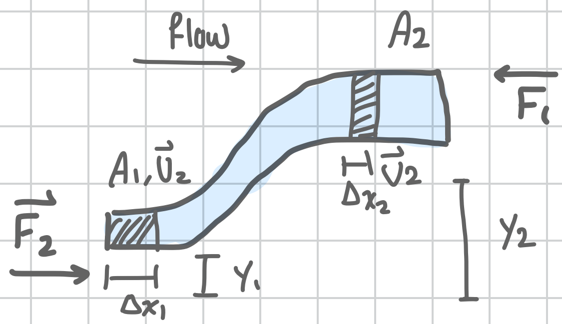
- Mass Flow Rate $\quad \dfrac{\Delta m}{\Delta t} = \text{constant}$

- $A v_1 = A v_2 \qquad A v = \text{constant}$

- Volume Flow Rate $\quad \dfrac{\Delta V}{\Delta t} = \text{constant}$

→ nonuniform tube
→ no leakage, confined
→ fluid is incompressible
" ρ doesnt change with P "

→ Bernoulli's Equation



- Work done on tube :

$$W = F_1 \Delta x_1 - F_2 \Delta x_2$$

$$W = P_1 \underbrace{A_1 \Delta x_1} - P_2 \underbrace{A_2 \Delta x_2}$$

$$W = P_1 V_1 - P_2 V_2$$
$$W = (P_1 - P_2) V$$

* $V_1 = V_2 = V$ → the fluid is incompressible

- Work changes the
  - → Kinetic Energy $\quad \dfrac{m}{2}(v_2^2 - v_1^2) \quad \Delta K.E$
  - → Potential Energy $\quad mg(y_2 - y_1) \quad \Delta P.E$

$$W = (P_1 - P_2) V = \frac{m}{2}(v_2^2 - v_1^2) + mg(y_2 - y_1) \qquad \div V$$

$$W = P_1 - P_2 = \frac{\rho}{2}(v_2^2 - v_1^2) + \rho g(y_2 - y_1)$$

$$P_1 + \frac{\rho v_1^2}{2} + \rho g y_1 = P_2 + \frac{\rho}{2} v_2^2 + \rho g y_2$$

$$P + \frac{\rho v^2}{2} + \rho g y = \text{Constant}$$

Bernoulli's Principle

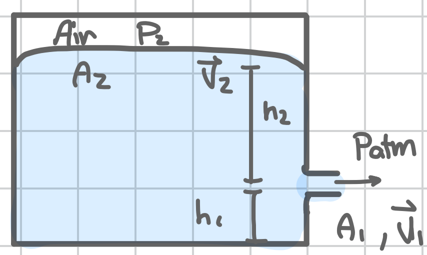
" where the velocity of the fluid is high, the Pressure is Low. and where the velocity is Low, the Pressure is high "

→ Pressure decreases when the elevation (y) increases.

→ Bernoulli's equation is an expression of the law of Energy conservation.

Applications of Bernoulli's Principle

Torricelli Law



- since $A_2 >> A_1 \rightarrow V_2 << V_1 \Rightarrow V_2 = 0$
- Bernoulli's equation:

$$P_1 + \frac{\rho v_1^2}{2} + \rho g h_1 = P_2 + 0 + \rho g (h_1 + h_2)$$

$P_1 = P_{atm}$.

→ Solve for $\underline{\underline{V_1}}$

$$V_1 = \sqrt{\frac{\rho}{2}[P_2 - P_{atm}] + 2gh_2}$$

→ then if P_2 increases ↑ V_1 increases. ↑

→ if A_2 was open to $\underline{atm}$ then $P_2 = P_{atm}$ $\Longrightarrow$ $V_1 = \sqrt{2gh_2}$ → free falling Obj

" the Liquid leaves the hole with the same speed that a freely falling Object would attain if falling from the same hieght. "

Poiseuille's Equation

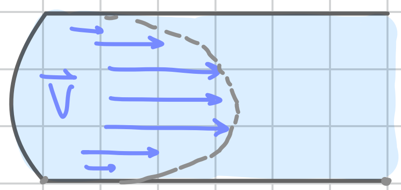
- **viscosity** → the tendency to resist flow. [similar to friction]

→ Ideal fluids have zero viscosity, speed is the same throughout the fluid.

→ Real fluids have non-zero viscosity and a flow pattern, where speed → drops to zero on the walls of the fluid.
→ reaches it's greatest in the center.

So a Force must maintain the flow of Real fluids which is provided by the pressure difference in the tube.

$(P_1 - P_2) \propto \frac{Lv}{A}$ → Lv directly proportional, A inversely proportional



P_1 Real Fluid P_2

L: Length of the tube

v: Avg speed

A: Cross-sectional Area of the tube.

$$(P_1 - P_2) = 8\pi \underbrace{\eta}_{\text{coefficient of viscosity.}} \frac{Lv}{A} \quad \text{(eq.1)}$$

coefficient of viscosity:
- SI unit: $\frac{N}{m^2} . s = Pa.s$
- non-SI unit: P (Poise)
  1 Poise = 0.1 Pa.s

Volume Flow Rate Q

$$Q = \frac{\Delta V}{\Delta t} = \underbrace{Av}_{\text{from eq 1}} = \frac{(P_1 - P_2)\pi r^4}{8 \eta L}$$

Poiseuilli's Equation

→ Q is directly proportional to Pressure

→ Q is inversely proportional to Length.

→ Q is directly proportional to 4^{th} power of r
↳ a small reduce in the radius cause a significant reduce in flow.

→ P is inversely proportional to 4^{th} power of r
↳ reducing r → $\frac{r}{2}$ increases P by 16 times.

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