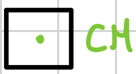
Center of Mass

CM : the point which we can imagine All the mass of an Obj is concentrated in.

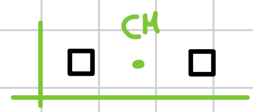
$$CM = CG$$

- **One Object** → mass density is constant → in the Geometrical Center
  uniform



- **2 Objects** → identical , $m_1 = m_2$ → On the Midway between m_1, m_2

$$CM = \vec{R} = \frac{\vec{r}_1 + \vec{r}_2}{2}$$



- **2 Objects** → $m_1 \neq m_2$ → $\vec{R} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$

- CM is closer to the massive object.

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \qquad y_{CM} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

- **Generalized Formula CM** [n of Obj]

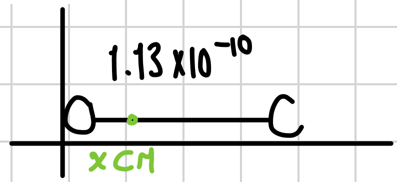
→ $CM = \vec{R} = \frac{\sum_{i=1}^{n} m_i \vec{r}_i}{M}$ → sum of masses

- $x_{CM} = \frac{\sum_{i=1}^{n} m_i x_i}{M}$ • $y_{CM} = \frac{\sum_{i=1}^{n} m_i y_i}{M}$

• Problem 7.45

(I) The distance between a carbon atom ($m = 12\,u$) and an oxygen atom ($m = 16\,u$) in the CO molecule is 1.13×10^{-10} m. How far from the carbon atom is the center of mass of the molecule?

$O_2 \to$ origin $= 1.13 \times 10^{-10}$ m



$$x_{CM} - x_C = ?$$

$x_O = 0$

$x_C = ?$

$$x_{CM} = \frac{x_O 16 + x_C 12}{28}$$

$$x_{CM} = \frac{3 x_C}{7}$$

$$x_C - x_{CM} = x_C - \frac{3}{7} x_C = \frac{4}{7} x_C$$

$$= \frac{4}{7}(1.13 \times 10^{-10})$$

$$= 6.46 \times 10^{-11}\ \text{m.}$$

Carbon $\to$ Origin

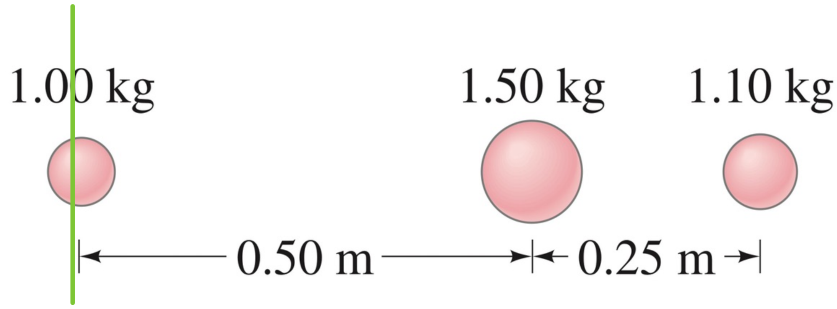
$$x_{CM} - x_C =$$

$$x_{CM} = \frac{x_O(m_O) + x_C m_C}{m_C + m_O} = \frac{1.13 \times 10^{-10}\,(16)\,u}{28\,u} = 6.45 \times 10^{-11}\ \text{m}$$

$$x_{CM} - x_C = x \Rightarrow x_{distance} = 6.45 \times 10^{-11}\ \text{m.}$$

Problem 7.46

6. (I) Find the center of mass of the three-mass system shown in Fig. 7–37 relative to the 1.00-kg mass.



$$x_{CM} = \frac{0 + (1.5)(0.5) + (1.1)(0.75)}{3.6} = 0.44\,m$$

Problem 7.56

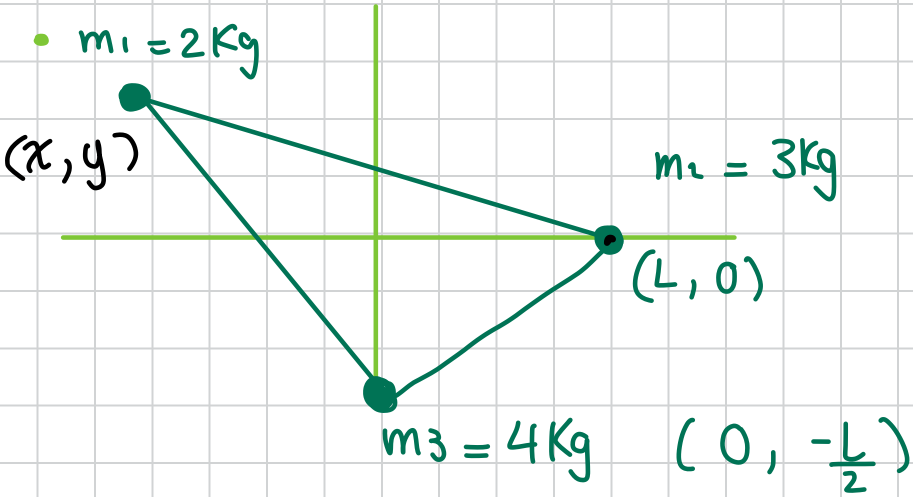
6. (II) The masses of the Earth and Moon are 5.98×10^{24} kg and 7.35×10^{22} kg, respectively, and their centers are separated by 3.84×10^{8} m. (*a*) Where is the CM of the Earth–Moon system located? (*b*) What can you say about the motion of the Earth–Moon system about the Sun, and of the Earth and Moon separately about the Sun?

Eearth (origin)

a. $$CM = \frac{m_E x_E + m_M x_M}{m_E + m_M} = \frac{0 + (3.84 \times 10^{8})(7.35 \times 10^{22})}{7.35 \times 10^{22} + 5.98 \times 10^{24}}$$

$$= 4.6 \times 10^{30} \times 10^{-24} = 4.6 \times 10^{6}\,m. = 4.6 \times 10^{3}\,Km$$

- Closer to M_E, Bcz $M_E > M_M$
- less than the raduis of Earth.



$CM : (+\frac{L}{4}, -\frac{L}{5})$

$(-x, y+)$

- $x_{CM} = \frac{L}{4} = \frac{3L + 2x + 0}{9} \Rightarrow \frac{9L}{4} = 3L + 2x$

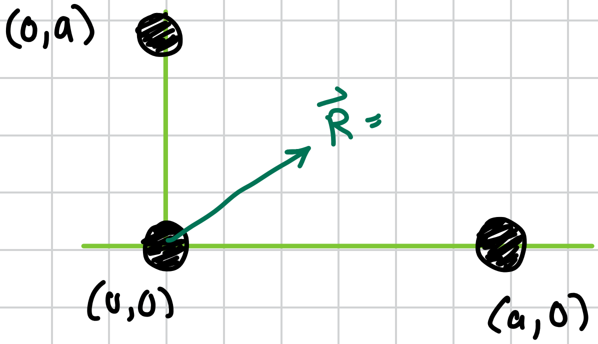
$\frac{9L - 12L}{4} = 2x \Rightarrow -\frac{3}{4} L = 2x \Rightarrow x = -\frac{3}{8} L.$

- $y_{CM} = -\frac{L}{5} = \frac{0 + -2L + 2y}{9}$

$-\frac{9}{5} L = -2L + 2y \rightarrow 2y = \frac{10L - 9L}{5} = \frac{L}{5}$

$y = \frac{L}{10}$

$m = 2Kg \rightarrow (-\frac{3}{8} L, \frac{L}{10}).$

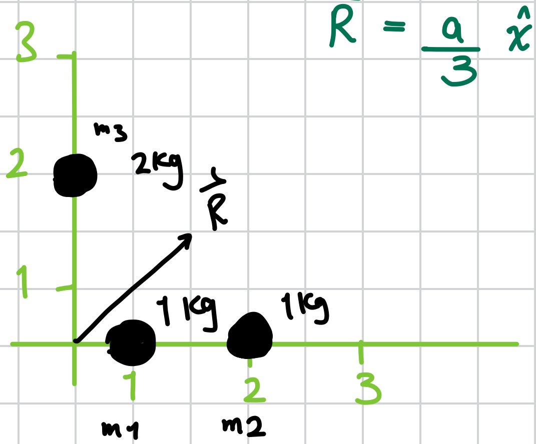
- 

- 3 identical Balls.

Q find CM

- $x_{CM} = \frac{ma}{3m} = \frac{a}{3}$
- $y_{CM} = \frac{ma}{3m} = \frac{a}{3}$

$$\vec{R} = \frac{a}{3}\hat{x} + \frac{a}{3}\hat{y}$$

- 

Q find location of CM.

$$x_{CM} = \frac{1+2+0}{4} = \frac{3}{4} = 0.75$$

$$y_{CM} = \frac{4}{4} = 1$$

$$R = (0.75\,\hat{x} + 1\,\hat{y})\ m$$