Hypothesis Testing

Example: Average intake in children for dietary fat is 70 g of fat per day. Suppose we want to study children who eat a vegetarian diet. Possible hypotheses are

1. **Average fat intake is 70 g per day**

2. **Average fat intake is less than 70 g per day**

Def: <u>One-sample problem</u>—a single distribution.

Def: <u>Hypothesis</u>—statement about parameters in a population or populations. We want to know how likely this is to be true, given the evidence (data). For example,

1. **Average number of beds filled per day in the hospital**

2. **Average number of minutes per day the doctor spends with a patient**

3. **Average lead content of water for a housing project**

Def: Null hypothesis—H_o—the hypothesis to be tested. This is usually a statement of no difference. The population value of the parameter is not different from some specified value.

Def: Alternative hypothesis—H_1 or H_A—This is the statement we will accept if we reject the null hypothesis.

H_o: Mean fat intake in vegetarian children is 70 g per day. H_o: $\mu = \mu_0$ or $\mu \geq \mu_0$

H_1: Mean fat intake in vegetarian children is < 70 g per day. H_1: $\mu < \mu_0$

Possible decisions:

1. **Accept H_o (really, fail to reject H_o)**
2. **Reject H_o**

Possible Scenarios:

Jury Trial		
	Truth	
Verdict	Innocent	Guilty
Innocent	Correct decision	Error
Guilty	Error	Correct decision

Test of hypothesis		
	Truth	
Results of Test	H_o	H_1
Accept H_o	Correct decision	Type II error
Reject H_o	Type 1 error	Correct decision

Def: <u>Type I error</u> is the probability of rejecting H_o when H_o is true.

Def: <u>Type II error</u> is the probability of accepting H_o when H_1 is true.

Example: We have developed a new procedure to improve survival of premature infants. If the hospital adopts these procedures, there will have to be new rooms and new equipment purchased. This is very costly.

1. **What does a Type I error imply?**

2. **What does a Type II error implies?**

Def: Level of significance: α = Probability of a Type I error. This is the area under the curve below (or above) the critical value. This is the probability of rejecting H_o when H_o is true.

Def: β: Probability of a Type II error.

Def: $1-\beta$—Power of a test. This is the Pr (rejecting $H_o | H_1$ is true).

Goal: Make α, β as small as possible. Usually, as $\alpha\uparrow$, $\beta\downarrow$ and as $\alpha\downarrow$, $\beta\uparrow$.

Fix α (0.05 or 0.01). Find a test to minimize β.

Best test for the fat experiment is one based on $\overline{X}$.

Def: <u>Acceptance Region</u>—These are the values of $\overline{X}$ for which H_o is accepted.

Def: <u>Rejection Region</u>—These are the values of $\overline{X}$ for which H_o is rejected.

Note: For this example, we are conducting a one-sided or one-tailed test. We will only reject H_o for values of $\overline{X}$ that are low.

Def: <u>One-tailed</u>—this is a test in which values of parameter under H_1 either > or < values under H_o <u>but not both</u>.

The picture behind the two sided test